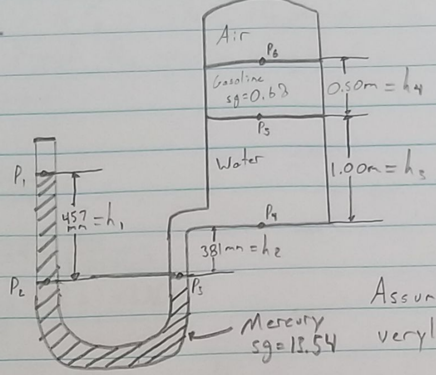
Foster Gibbs

Test 1 Problem 1

Purpose: - Calculate Air pressure above gasoline
- Find air pressure when Hg reading drops from 457mm to 0mm

Drawings and Diagrams



Assume tank is very large

Sources

Mott + Untener. Applied Fluid Mechanics 7th Edition. Pearson 2015

Design Considerations

Based on the problem description, I assume the following:
- The fluids are incompressible
- Steady State
- Isothermal problem
- Water, gasoline, Mercury, and Air do not mix

Data + Variables

Part 1) $\gamma_{Hg} = 132.83 kN/m^3$ $h_1 = 457mm$
$\gamma_w = 9.81 kN/m^3$ $h_2 = 381mm$
$\gamma_g = 6.67 kN/m^3$ $h_3 = 1.00m$
$h_4 = 0.50m$

Part 2) $\gamma_{Hg} =$ " $h_1 = 381mm$
$\gamma_w =$ " $h_2 = 1.00m$
$\gamma_g =$ " $h_3 = 0.50m$

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Test 1 Problem 1

Procedure and calculations

1) It can be assumed that the pressure at P_1 is $0\ kN/m^2$. Knowing that the specific gravity is 13.54 and the change in depth from P_1 to P_2 is 457mm, the pressure at P_2 can be calculated. First, the specific weight of Mercury must be calculated.

$$sg_{Hg} = \frac{\gamma_{Hg}}{sg \text{ of water}}$$

$$\Rightarrow 13.54 = \frac{\gamma_{Hg}}{9.81\ kN/m^3}$$

$$\gamma_{Hg} = 132.83\ kN/m^3$$

2) Using the specific weight of mercury and the depth, pressure at P_2 can be calculated using the equation $\Delta p = \gamma h$.

$$\Delta p = \gamma h$$

$$\Rightarrow (132.83\ kN/m^3)(0.457m) + P_1$$

$$\Delta p = 60.70\ kN/m^2$$

3) Knowing that when there is no change in height, it can be assumed that P_2 is equal to P_3

$$P_2 = P_3 = 60.70\ kN/m^2$$

4) Using the same equation, $\Delta p = \gamma h$, pressure at P_4 can be calculated. The pressure at P_4 will be subtracted from P_3 as there is a positive change in height. $h_3 = 381mm$

$$P_4 = P_3 - P_4 \qquad P_4 = \gamma_w h_2$$

$$\Rightarrow 60.70\ kN/m^2 - (9.81\ kN/m^3)(0.381m)$$

$$P_4 = 56.96\ kN/m^3$$

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Test 1 Problem 1

5) Continuing to use the equation, $\Delta p = \gamma h$, pressure at P_5 can be found by finding Δp from P_5 to P_4.

$$P_5 = P_4 - \gamma_w h_3$$

$$\Rightarrow 56.96\ kN/m^3 - (9.81\ kN/m^3)(1.00m)$$

$$P_5 = 47.15\ kN/m^2$$

6) Next is to calculate the specific weight of gasoline using the following equation.

$$sg_g = \frac{\gamma_g}{sg\ of\ water}$$

$$\Rightarrow 0.68 = \frac{\gamma_g}{9.81\ kN/m^3}$$

$$\gamma_g = 6.67\ kN/m^3$$

7) Now knowing the specific weight of gasoline, pressure at P_6 can be calculated. P_6 is the pressure of the air above the gasoline, $h_4 = 0.50m$

$$P_6 = P_5 - \gamma_g h_4$$

$$\Rightarrow 47.15\ kN/m^2 - (6.67\ kN/m^3)(0.50m)$$

$$\boxed{P_6 = 43.81\ kPa}$$

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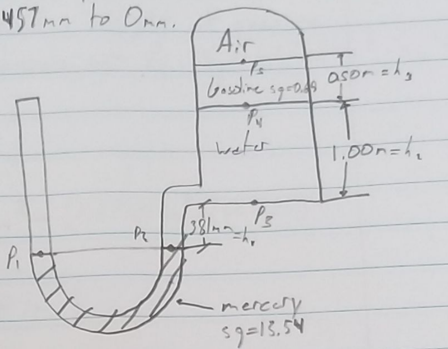
Test 1 Problem 1

1) For the second part of the problem, the height of the mercury falls from 457 mm to 0 mm.



$P_1 = 0 = P_2$

It can be assumed the pressure at P_1 is $0\,kN/m^2$. The pressure at P_2 is also $0\,kN/m^2$ since there is no change in height.

2) Using the specific weight of mercury found before, pressure at P_3 can be found. $h_1 = 381\,mm$ + $\gamma_w = 132.83\,kN/m^3$

$$P_3 = P_2 - \gamma_w h_1$$
$$\Rightarrow 0 - (132.83\,kN/m^3)(0.381\,m)$$
$$P_3 = -3.738\,kN/m^2$$

3) From P_3 to P_4 there is a 1.00 m change in height. This and the specific weight of water can be used to find pressure at P_4. $\gamma_w = 132.83\,kN/m^3$, $h_2 = 1.00\,m$

$$P_4 = P_3 - \gamma_w h_2$$
$$\Rightarrow -3.738\,kN/m^2 - (132.83\,kN/m^3)(1.00\,m)$$

$$P_4 = -13.548\,kN/m^3$$

Test 1 Problem 1

4) From P_4 to P_5 there is a 0.50 m change in height. This and the specific weight of gasoline can be used to find pressure at P_5. $\gamma_g = 6.67 kN/m^3$, $h_3 = 0.50m$

$$P_5 = P_4 - \gamma_g h_3$$

$$\Rightarrow -13.548 KN/m^2 - (6.67 kN/m^3)(0.50m)$$

$$\boxed{P_5 = -16.88 kN/m^2}$$

P_5 is equal to the pressure of the air above the gasoline when the mercury reading falls to 0mm.

Summary

The pressure of the air above the gasoline when the mercury height is equal to 457mm is 43.81 kN/m^2. The pressure of the air above the gasoline when the mercury height falls from 457mm to 0mm is -16.88 kN/m

Materials

- Tank
- Air
- Gasoline
- Water
- Mercury

Analysis

When the air pressure in the tank decreases, the height of the mercury does as well.

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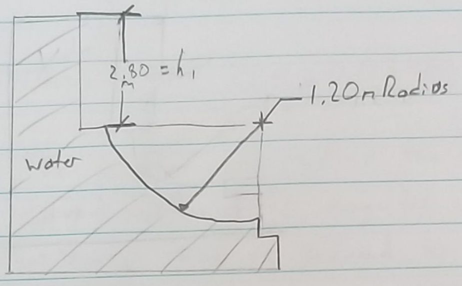
Test I Problem 2

Purpose

- Calculate radius if horizontal force is reduced by half of its original magnitude
- Find Resultant Force in new set up
- Find Resultant force direction and location of the new vertical and horizontal forces

Diagrams + Drawings



Sources

Mott + Untener. Applied Fluid Mechanics, 7th Edition, Pearson 2015
Example problem 4.8

Design Considerations

- The Magnitude and location of the resultant force with the initial conditions
- Water is incompressible
- Isothermal problem
- Stedy state

Materials

- Tank
- Water

Data and Variables

$R = 1.20m$

$h_1 = 2.80m$

length = 1.5 m (assumed from figure 4.52 p. 89)

$\gamma_w = 9.81\ kN/m^3$

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Test 1 Problem 2

Procedure and calculations

1) To find the magnitude of the horizontal force, the equation $F_H = \gamma_w s w h_c$. Where $h_c = h_1 + S_1/2$

$$\Rightarrow F_H = \gamma_w s w \left(h_1 + \frac{S_1}{2}\right)$$

$$\Rightarrow (9.81\,kN/m^3)(1.2m)(1.5)\left(2.8m + \frac{1.2m}{2}\right)$$

Original H force $\longrightarrow$ $F_H = 60.04\,kN$

2) The problem asks to find the radius of the curved surface if the magnitude of the original horizontal force is reduced by half.

$$\frac{60.04\,kN}{2} = \boxed{30.02\,kN = F_H}$$ — New Horizontal Force

3) 30.02 kN is half of the original horizontal magnitude. Working backwards from the original equation the new radius can be calculated.

$$F_H = \gamma_w s w \left(h_1 + \frac{S_1}{2}\right)$$

$$\Rightarrow 30.02\,kN = (9.81\,kN/m^3)(s)(1.5)\left(2.80m + \frac{s}{2}\right)$$

Wolfram Alpha was used to calculate s.

$$\boxed{s = 0.6525\,m}$$

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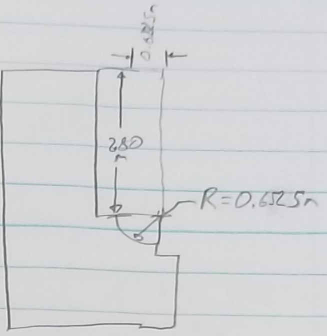
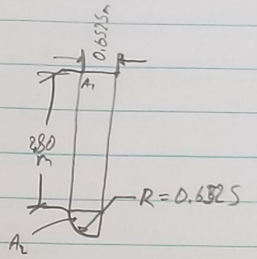
Test 1 Problem 2

4) Now that a new radius is acquired, a new drawing must be made.



5) There is now an imaginary area above the curved surface. The volume of this area must be calculated using the new radius.



$A_1 = b \cdot h$

$\Rightarrow (0.6525m)(2.80m)$

$A_1 = 1.827 m^2$

$A_2 = \dfrac{\pi (r)^2}{4}$

$\Rightarrow \dfrac{\pi (0.6525m)^2}{4}$

$A_2 = 0.3344 m^2$ $\qquad \Sigma A = 2.1614 m^2$

Volume of imaginary fluid above curved surface.

$V = A w$

$\Rightarrow (2.1614 m^2)(1.5m)$

$V = 3.2421 m^3$

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Test 1 Problem 2

6) Using the volume and specific weight of the water, the new vertical force can be calculated.

$$W = \gamma_w V$$

$$\Rightarrow (9.81 kN/m^3)(3.2421 m^3)$$

$\boxed{W = 31.81 kN = F_v}$ - New Vertical Force

7) In order to find the location of the new forces, the centroid must be calculated.

$$\bar{X} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

x_1 + x_2 can be found in Appendix L

$A_1 = 1.827 m^2$

$A_2 = 0.3344 m^2$

$x_1 = \frac{b}{2}$

$x_2 = 0.424 r$

$$x_1 = \frac{b}{2}$$

$$\Rightarrow 0.6525m / 2 = 0.32625 m$$

$$x_2 = 0.424 r$$

$$\Rightarrow (0.424)(0.6525m) = .27666 m$$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$\Rightarrow \frac{(1.827m^2)(0.32625m) + (0.3344m^2)(0.27666m)}{1.827m^2 + 0.3344m^2}$$

$\bar{x} = 0.3186 m$

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Test 1 Problem 2

8) Using the equation $h_c = h_1 + \frac{s}{2}$ the depth of the centroid can be calculated.

$$h_c = h_1 + \frac{s}{2}$$

$$\Rightarrow (2.80m) + \frac{0.6525m}{2}$$

$$h_c = 3.12625\,m$$

9) Now knowing the depth of the centroid, the depth of the line of action of the horizontal component from the equation $h_p = h_c + \frac{s^2}{12 h_c}$

$$\Rightarrow 3.12625m + \frac{0.6525m^2}{12(3.12625m)}$$

$$h_p = 3.1376\,m$$

10) Now knowing everything needed for the problem, the resultant force and its angle can be calculated using trig and the pythagorean theorem.

$$F_R = \sqrt{F_V^2 + F_H^2}$$

$$\Rightarrow \sqrt{31.81kN^2 + 30.02kN^2}$$

$$F_R = \boxed{43.74\,kN}$$

$$\phi = \tan^{-1}\left(\frac{F_V}{F_H}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{31.81kN}{30.02kN}\right) = 46.47^\circ$$

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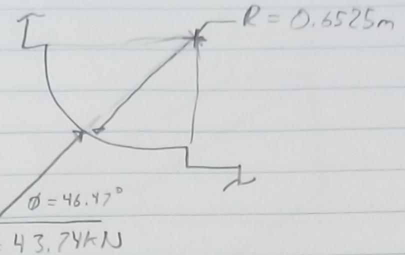
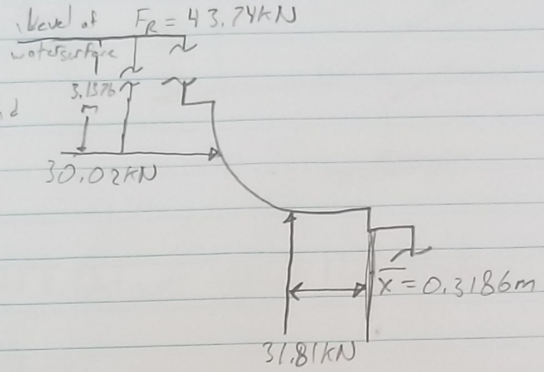
Test 1 Problem 2

11) Diagrams of New forces acting on curved surface.

Resultant Force



Vertical and Horizontal Forces



Summary

- The vertical Force acts on the curved surface with a force of 31.81kN 0.3186m from the farthest right side of the drawing.
- The Horizontal forces acts on the curved surface with a force of 30.02kN 3.1376m from the surface of the water.
- The resultant of these two forces acts on the curved surface with a force of 43.74kN at an angle of 46.47°.

Analysis

It was found that when the radius of the circle decreased, the volume of imaginary volume of water decreased, Therefore, lowering the forces acting on the curved surface.

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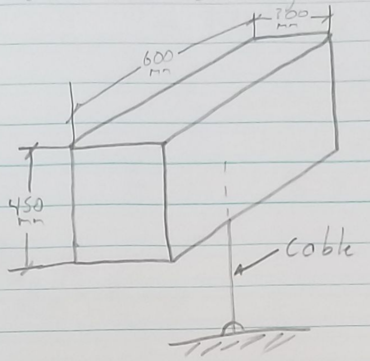
Test 1 Problem 3

Purpose

- prove that one orientation of the package is more stable than the one given

Drawings & Diagrams



Sources

Mott & Untener, Applied Fluid Mechanics 7th edition, pearson 2015

Design Considerations

- Steady State
- Isothermal problem

Materials

- Water
- Box

Data and Variables

$W = 258\ N$

$\gamma_w = 10.05\ kN/m^3$

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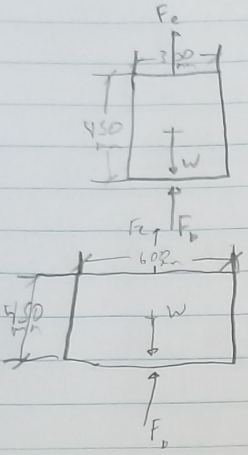
Test 1 Problem 3

Procedure and calculations

1) FBD of Package in original Orientation



The first step of this problem is to draw a FBD of the assembly. For this setup, there are only forces in the y-direction.

$$\Sigma F_y = 0$$

$$0 = F_e + F_b - W$$

2) When calculating buoyancy, the first step is to calculate the moment of inertia. Since the problem asks to prove that another position will be more stable than the given position, two sets of variables will be used for comparison.

Original	Rotated 90° about long axis
$I = \frac{ab^3}{12}$	
$\Rightarrow \frac{(.6)(.3)^3}{12}$	$\Rightarrow \frac{(.6)(.45)^3}{12}$
$I_o = 0.00135\ m^4$	$I_R = 0.004556\ m^4$

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Test 1 Problem 3

3) Using the formula $X = \frac{W}{B \times L \times \gamma_f}$ the distance that the package is underwater can be calculated

Original

$X = \frac{W}{B \times L \times \gamma_f}$

$\Rightarrow \frac{.258\,kN}{(0.6)(0.3)} \times \frac{m^3}{10.05\,kN}$

$X_o = 0.1426\,m$

Rotated

$\Rightarrow \frac{.258\,kN}{(0.6)(0.45)} \times \frac{m^3}{10.05\,kN}$

$X_R = 0.0951\,m$

4) Now that MOI and the amount of the object that is submerged is known, MB can be calculated using $MB = \frac{I}{V_d}$ where $V_d = L \times B \times X$

Original

$V_d = L \times B \times X$

$\Rightarrow (.6)(.3)(.1426)$

$= 0.025668\,m^3$

$MB = \frac{I}{V_d}$

$\Rightarrow \frac{0.00135\,m^4}{0.025668\,m^3}$

$= 0.05259\,m$

$= 52.59\,mm$

Rotated

$\Rightarrow (.6)(.45)(.0951)$

$= 0.025677\,m^3$

$\Rightarrow \frac{0.004556\,m^4}{0.025677\,m^3}$

$= 0.17744\,m$

$= 177.44\,mm$

Foster [illegible]

Test 1 Problem 3

5) The distance from the metacenter can be acquired by adding the center of buoyancy and MB.

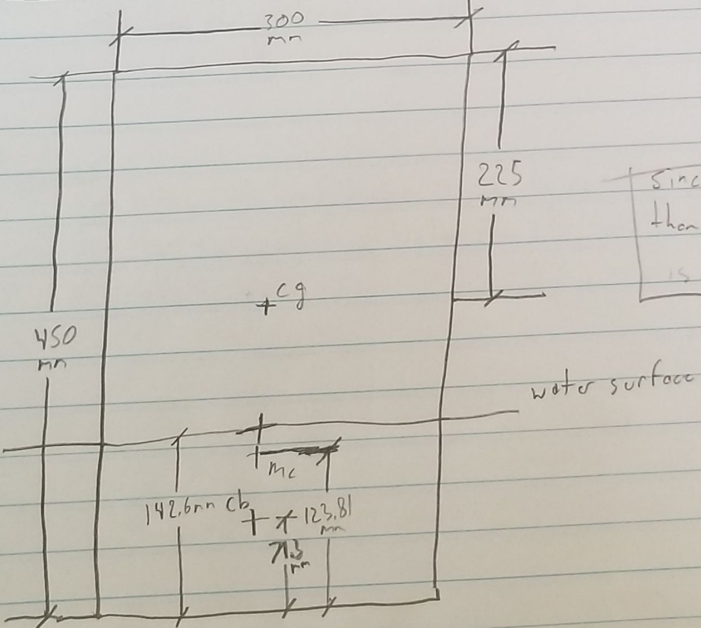
$$y_{mc} = y_{cb} + MB$$

$$y_{cb} = \frac{X}{2}$$

Original	Rotated
$y_{mc} = \frac{142.6mm}{2} + 52.59mm$	$y_{mc} = \frac{95.1mm}{2} + 177.44mm$
$y_{mc} = 123.81\ mm$	$y_{mc} = 224.99\ mm$

6) Using all previously acquired information, the stability of each orientation.

Original Orientation



Since y_{mc} is less than cg, the package is unstable

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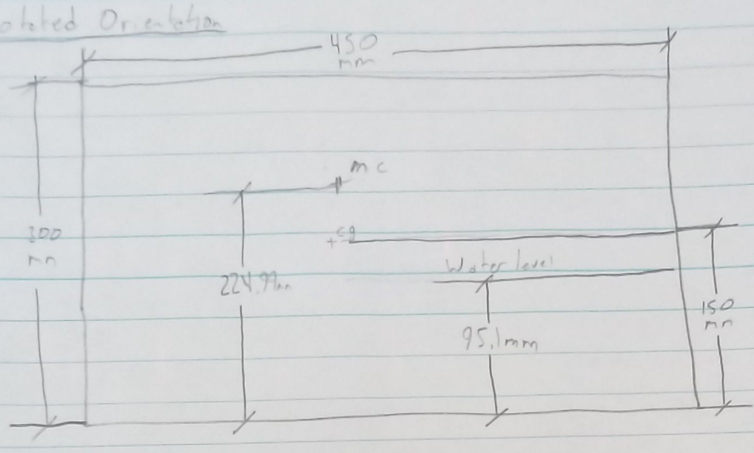
Test 1 Problem 3

Rotated Orientation



Since mc is > cg, this posisition is stable

Summary

- When the package is rotated 90° on its long axis it is stable.

Analysis

- When the package is in its rotated position, the metacenter is higher than its cg ∴ it is stable.