

1. a. The elements of $GF(19)$ would be **{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18}** with $p = 19$
- b. The additive inverse of 9 on $Gf(19)$ would be calculated with: **-9+19, with the additive inverse coming out as 10.**

c. For the multiplicative inverse, we keep plugging $9*x \text{ mod } 19$ until we get 1, so:

$9 \text{ mod } 19 = 9 \mid 9*2 \text{ mod } 19 = 1$. **The multiplicative inverse of 9 in $GF(19)$ is 1**

2. a. the Extension field of $GF(2^6)$ with $A(x) = 17$ would be:

$$\mathbf{A = x^4 + 1, \text{ with X being 2 from the GF. } 2^4 = 2*2=4*2=8*2=16 + 1 = 17}$$

(I struggled to understand this one, so I don't know how to properly write it.)

- b. for $B(x) = 8$, it would look like

$$\mathbf{B = x^3 + 0, X \text{ being 2 again. } 2*2 = 4 * 2 = 8}$$

- c. for $C(x) = 1$, it would be

$$\mathbf{C = x^0, X \text{ again being 2, but } 2^0 \text{ comes out as 1}}$$

- d. $A(x) + B(x)$ in poly form would be: **$x^4 + x^3 + 1$, and the integer it comes out to would be 25.**

- e. $B(x) + C(x)$ in poly form would be: **$x^3 + x^0$, it the integer would comes out to 9.**

3. If $GF(2^8)$, determine the multiplicative inverses of $4D_{16}$ and $4E_{16}$ would be:

$$\mathbf{4D: x=2, y=5 \mid 4E: x= E, y= 9}$$

4. If $GF(2^8)$, determine the AES S-box substitutions of $5E_{16}$ and $5F_{16}$:

$$\mathbf{5E: x=9, y=D \mid 5F: x=8, y=4}$$

5.

E5	B0	7B	92
C2	C8	1F	33
D6	F9	8E	74
A8	9C	06	85

After shifting, it becomes:

E5	B0	7B	92
C8	1F	33	C2
8E	74	D6	F9
85	A8	9C	06