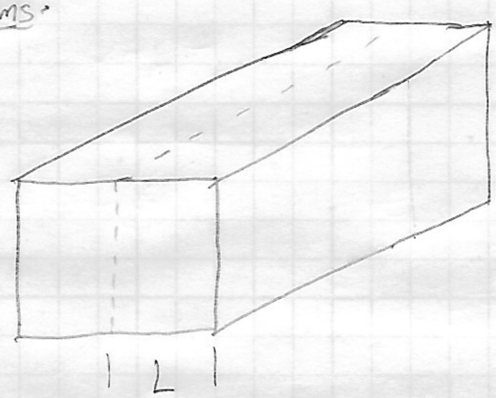
(1) **Purpose:** - To determine the width that the concrete slabs need to be to achieve a temperature of 95°C at the center of the slab after 8hrs of heating with the given Scenario.
- Determine the Temperature at the outside of the slab after 8 hours of heating.

Drawings + Diagrams:



$T_i = 40°C$

$T_{inf} = 200°C$

$T_0 = 95°C$

$h_\infty = 35\ W/m^2K$

Sources:

Bayazitoglu, Y., Ozisik, N., "A textbook for Heat Transfer Fundamentals," Begel House Inc, (2012)

Ayala, O., MET 440 Heat Transfer. Old Dominion University online. 2017. web. Oct 2018.

Design Considerations:

- Time Dependent
- Constant Properties
- 1 Dimensional Heat flow

Data + Variables:

$t = 8\ hrs = 28800\ s$

Per table in book:

$K = 1.37\ W/m\cdot°C$

$\rho = 2000\ kg/m^3$

$C_p = 880\ J/kg\cdot°C$

α w/used variables $= 7.78\times10^{-7}\ m^2/s$

Procedure:

Create spreadsheet using values from table 5.1 and Known values from problem.
Use the spreadsheet to calculate values on right side of equation and compare to left side of equation.
Once it is determined where the solution lies interpolate to get more exact result.
C_1 and ξ_1 must be taken from Table 5.1 and τ can be calculated using Biot, h and K.

Calculations:

once the spreadsheet was made, it was easy to see that the lump capacitance method was not going to work and that the answer lay between Biot numbers of 6 and 7, corresponding to length between 0.234857 and 0.274. (see spreadsheet on following page).

Interpolate to get result:

$$\frac{.711651273 - .595216704}{.65625 - .595216704} = \frac{.274 - .234857}{L - .234857}$$

$$1.9077 = \frac{.039143}{L - .234857}$$

$$(L - .234857)1.9077 = .039143$$

$$1.9077L - .44804 = .039143$$

$$1.9077L = .487183$$

$$L = .2554$$

Since L is ½ the width of the slab,

$$\boxed{w = .511m}$$

The Concrete slabs should be .511m in width so that they reach 95°C at the center after 8 hours of heating at the given conditions.

Temperature at outside of slab:

Calculate Biot using .2554m:

$$B_i = \frac{hL}{k} = \frac{35(.2554)}{1.37} = 6.52$$

Interpolate for ξ at $B_i = 6.52$

$$\frac{7-6}{6.52-6} = \frac{1.3766 - 1.3496}{\xi - 1.3496}$$

$$1.923 = \frac{.027}{\xi - 1.3496}$$

$$1.923\xi - 2.595 = .027$$

$$\xi = 1.363$$

Bi	ζ_1	C_1	L	τ	RHS	%error	Values	
0.1	0.3111	1.0161	0.003914	1463.172	3.20842E-62	100%	k =	1.37
0.15	0.3779	1.0237	0.005871	650.2986	4.76502E-41	100%	ρ =	2000
0.2	0.4328	1.0311	0.007829	365.793	1.80284E-30	100%	c_p =	880
0.25	0.4801	1.0382	0.009786	234.1075	3.81403E-24	100%	α =	7.78E-07
0.3	0.5218	1.045	0.011743	162.5747	6.23804E-20	100%	h =	35
0.4	0.5932	1.058	0.015657	91.44824	1.11985E-14	100%	T_i =	40
0.5	0.6533	1.0701	0.019571	58.52687	1.5172E-11	100%	T_{inf} =	200
0.6	0.7051	1.0814	0.023486	40.64366	1.8128E-09	100%	T_0 =	95
0.7	0.7506	1.0919	0.0274	29.86065	5.39298E-08	100%	θ =	0.65625
0.8	0.791	1.1016	0.031314	22.86206	6.75648E-07	100%	t =	28800
0.9	0.8274	1.1107	0.035229	18.06385	4.73109E-06	100%		
1	0.8603	1.1191	0.039143	14.63172	2.21728E-05	100%		
2	1.0769	1.1785	0.078286	3.65793	0.016942867	97%		
3	1.1925	1.2102	0.117429	1.625747	0.119897585	82%		
4	1.2646	1.2287	0.156571	0.914482	0.284649804	57%		
5	1.3138	1.2402	0.195714	0.585269	0.451607236	31%		
6	1.3496	1.2479	0.234857	0.406437	0.595216704	9%	Interpolate	
7	1.3766	1.2532	0.274	0.298607	0.711651273	-8%		
8	1.3978	1.257	0.313143	0.228621	0.804156596	-23%		
9	1.4149	1.2598	0.352286	0.180639	0.877503068	-34%		
10	1.4289	1.262	0.391429	0.146317	0.936087697	-43%		
20	1.4961	1.2699	0.782857	0.036579	1.170068345	-78%		
30	1.5202	1.2717	1.174286	0.016257	1.224807214	-87%		
40	1.5325	1.2723	1.565714	0.009145	1.245265988	-90%		
50	1.54	1.2727	1.957143	0.005853	1.255156661	-91%		
100	1.5552	1.2731	3.914286	0.001463	1.268602594	-93%		

Temperature at outside:

$$\theta = \theta_o \cos\left(\zeta \frac{x}{L}\right)$$

$$\theta = .65625 \cos\left(1.363 \times \frac{.2554}{.2554}\right)$$

$$\theta = .13539$$

$$\theta = \frac{T_\theta - T_\infty}{T_i - T_\infty}$$

$$.13539 = \frac{T_\theta - 200}{40 - 200}$$

$$\boxed{T_\theta = 178.34°C}$$

While the slab is 95°C at the center, it is 178.34°C at the outside.

The temperature at the surface of the slab is greater than the 100°C that the cement begins to weaken at. The slabs will begin to weaken from the outside-in.

Summary:

The concrete slab will need to be 0.511m wide so that the center of the slab is 95°C after 8 hours of heating with the given criteria. At 0.511m width the outer surface of the slab will be ≈ 178°C at the time that the center is 95°C.

Materials:

Stone mix concrete
Air

Analysis:

To keep the concrete at cooler temperatures the width would need to be increased. This could also be accomplished by decreasing T_∞ or h_∞.

Purpose: Using COMSOL, determine the width that the concrete slab needs to be so that the center of the slab measures 95°C after 8 hours of heating at the given conditions. Once the width is determined, find the temperature at the outer surface of the slab after 8 hours of heating at the same conditions.

Drawings & Diagrams: Same as drawings in problem 1. Also see COMSOL file for geometry.

Sources:

Bayazitoglu, Y., Ozisik, N., "A Textbook for Heat Transfer Fundamentals," Begell House Inc, (2012)

Ayala, O., MET440 Heat Transfer.Old Dominion University Online.2017.Web.Nov2018

Design Considerations:

- Time Dependent
- Constant Properties
- 1 Dimensional Heat Flow

Data & Variables:

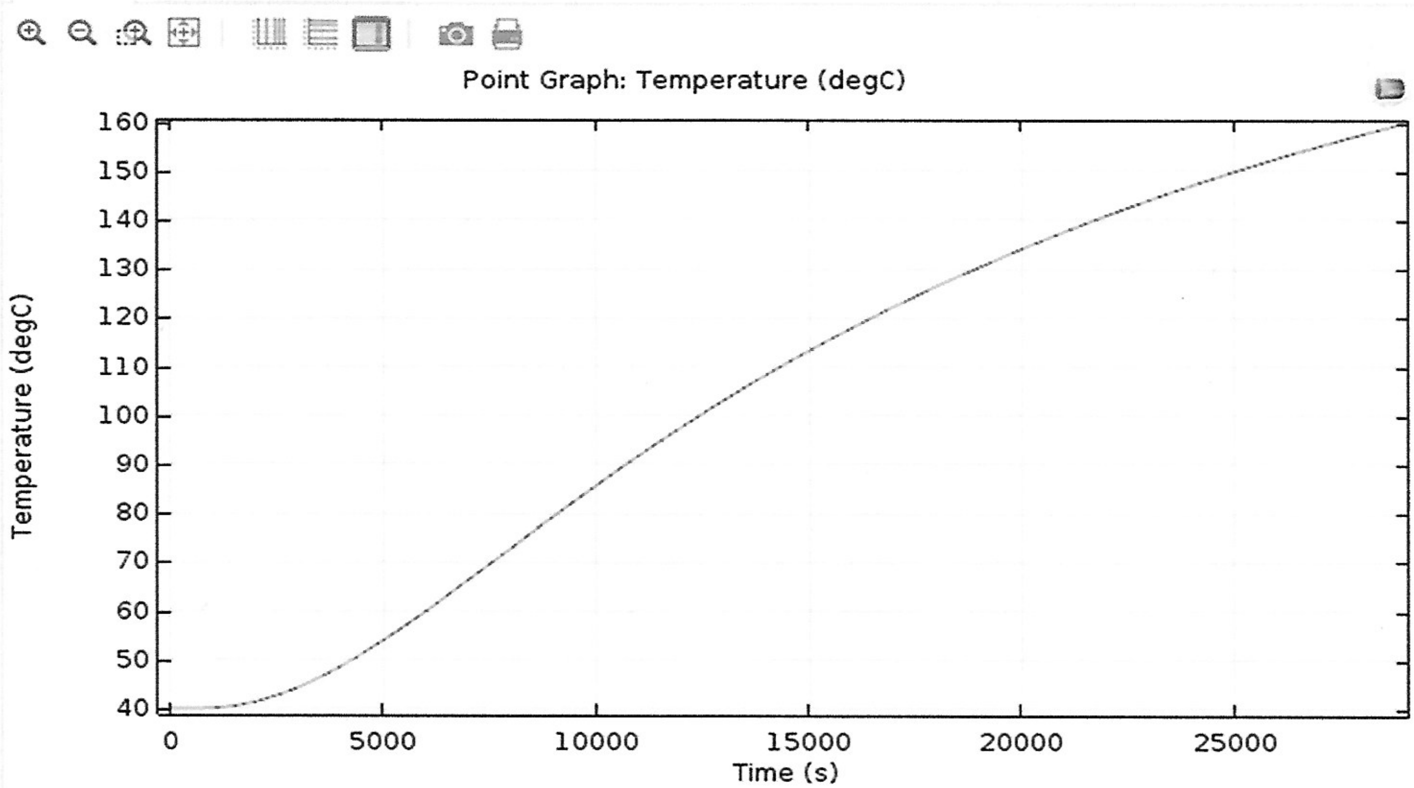
$T_i = 40°C$	$\tau = 8hrs = 28800s$	Using given variables: $\alpha = 7.78x10^{-7} m^2/s$
$T_{inf} = 200°C$	$k = 1.37\ W/m*°C$	
$T_0 = 40°C$	$\rho = 2000kg/m^3$	
$h_{inf} = 35\ W/m^2*K$	$c_p = 880J/kg*°C$	

Procedure:

Iterate using COMSOL by entering given values and running the program with different size concrete slabs. Define a point in the center of each slab and generate a graph showing time and temperature at the given point. Repeat steps until the width that gives the desired results is found. Define a point at the surface of the slab and generate a graph displaying the temperature at the given point after 8 hours.

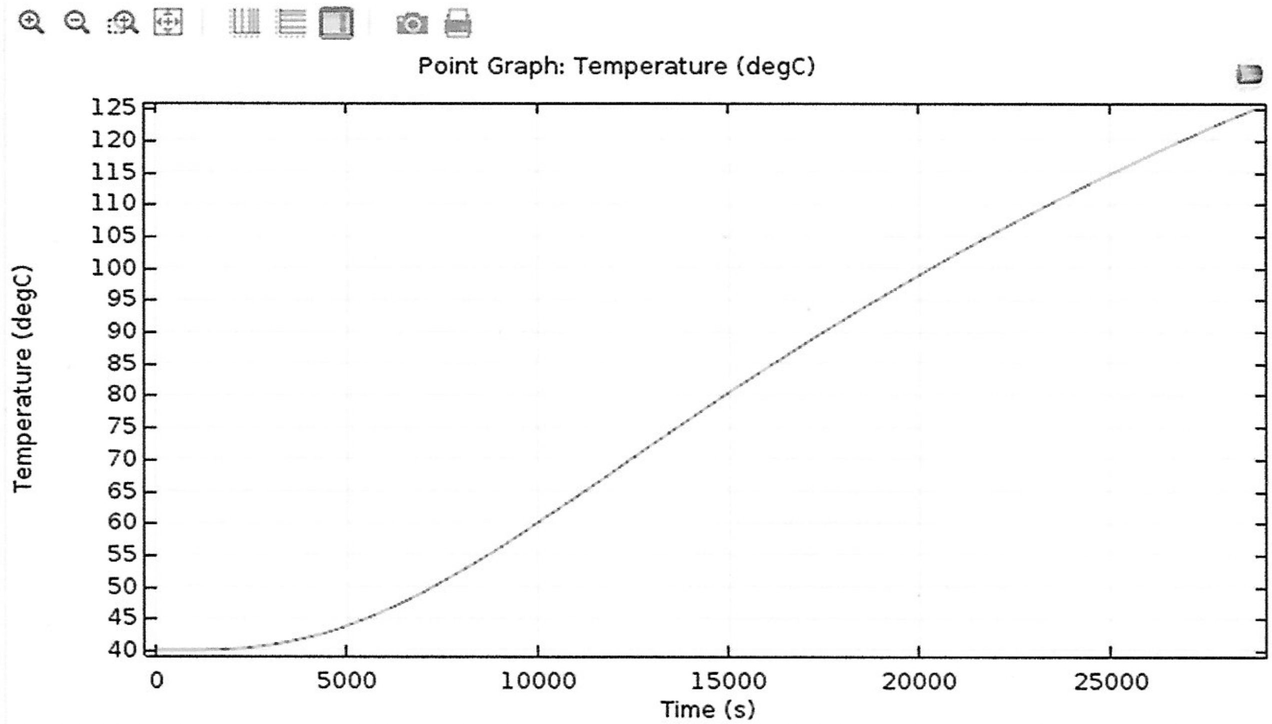
Calculations:

The first slab built had a width of 0.3m and generated a central temperature of 160°C.



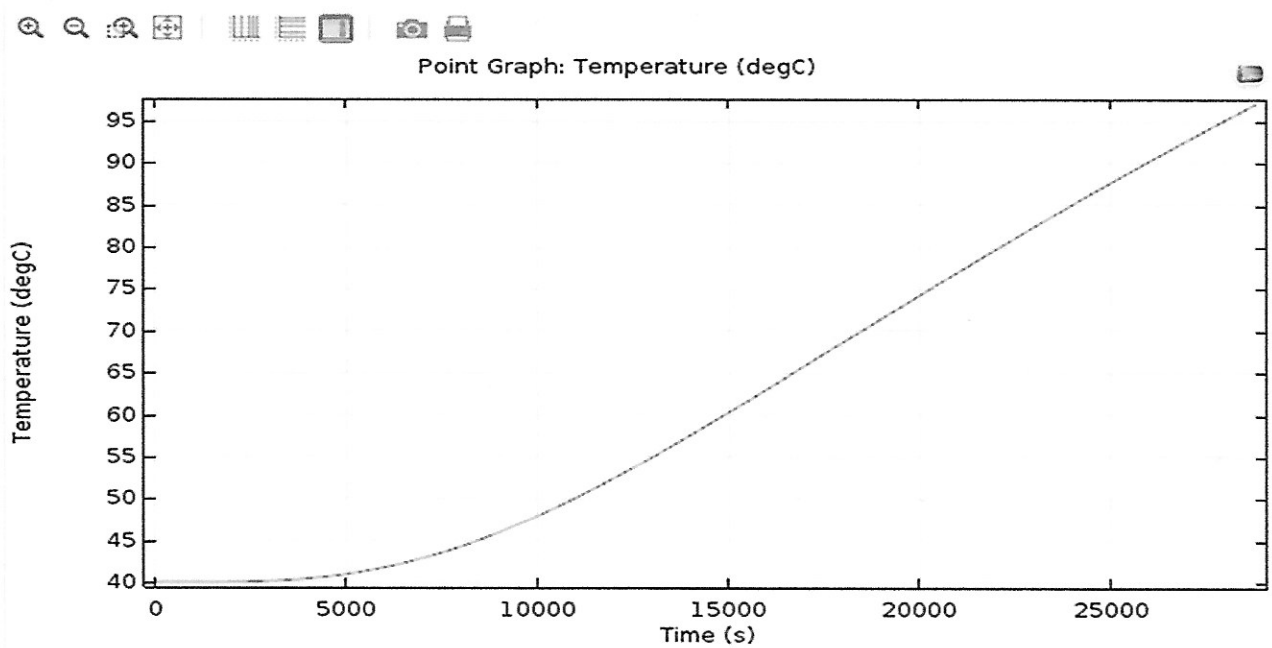
Graph 1: 0.3m Slab Central Temperature

The second slab had a width of 0.4m and generated a central temperature of approximately 125°C



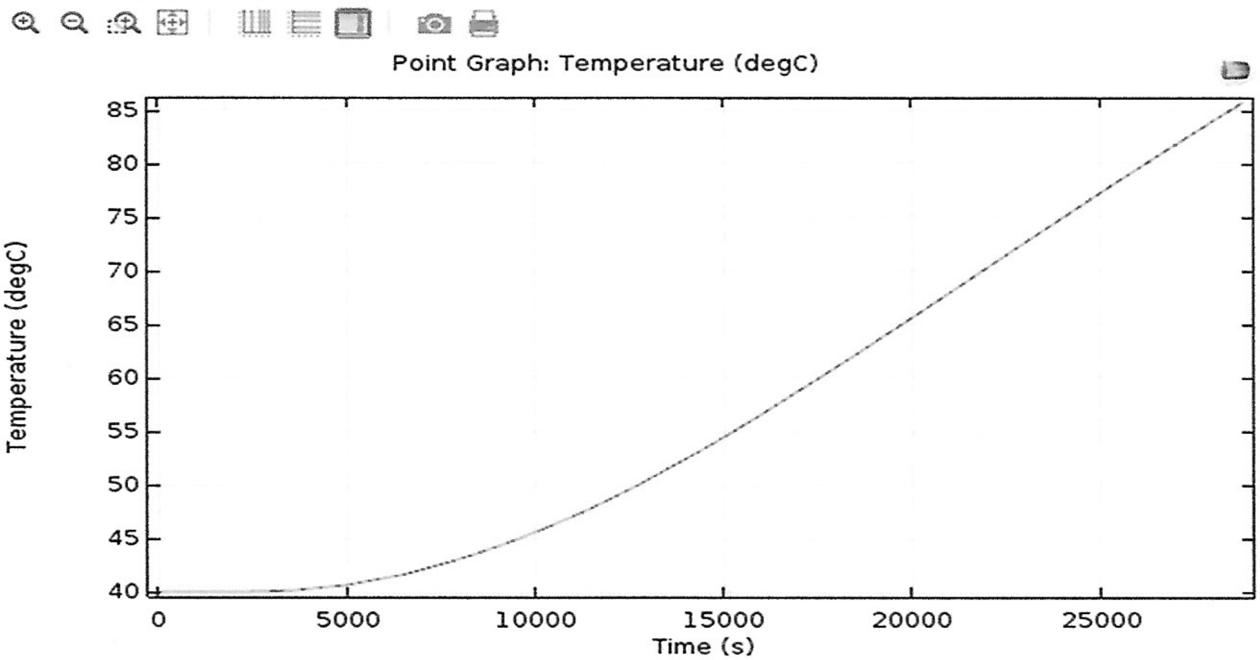
Graph 2: 0.4m Slab Central Temperature

The third slab had a width of 0.5m and generated a central temperature of approximately 97°C.



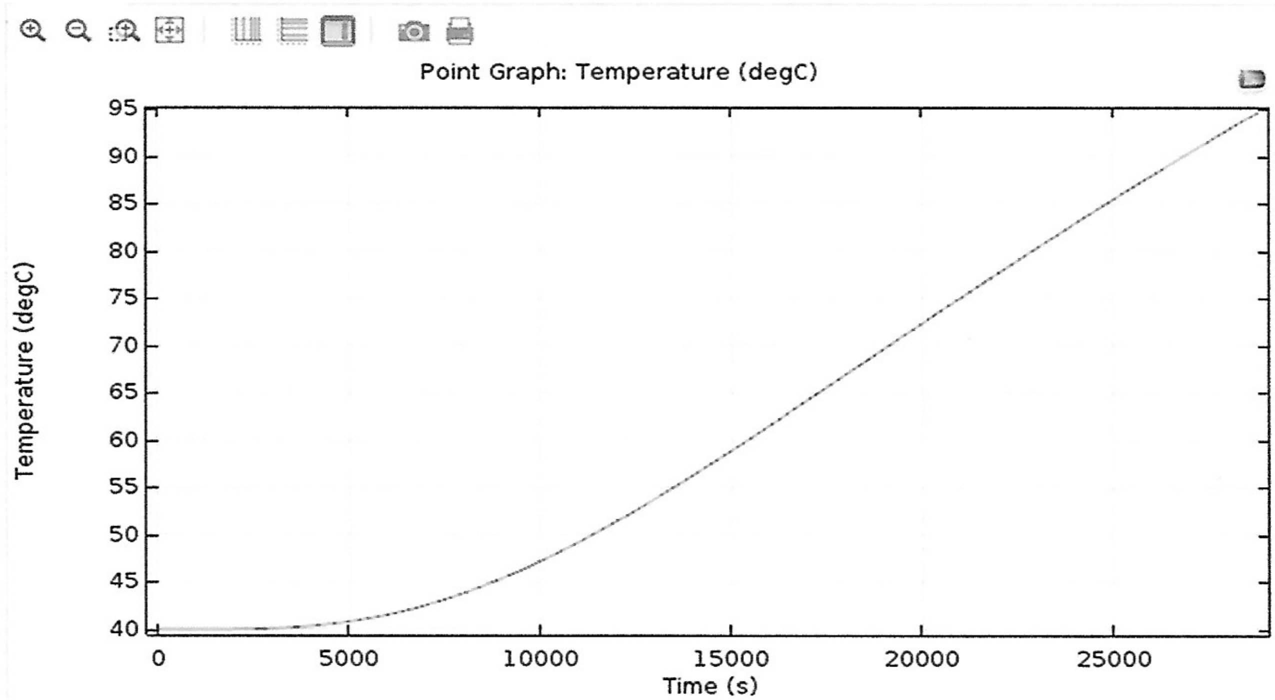
Graph 3: 0.5m Slab Central Temperature

Since the 0.5m slab was close to the desired temperature, the width of the fourth slab was increased 0.05m to 0.55m and yielded a central temperature of approximately 86°C.



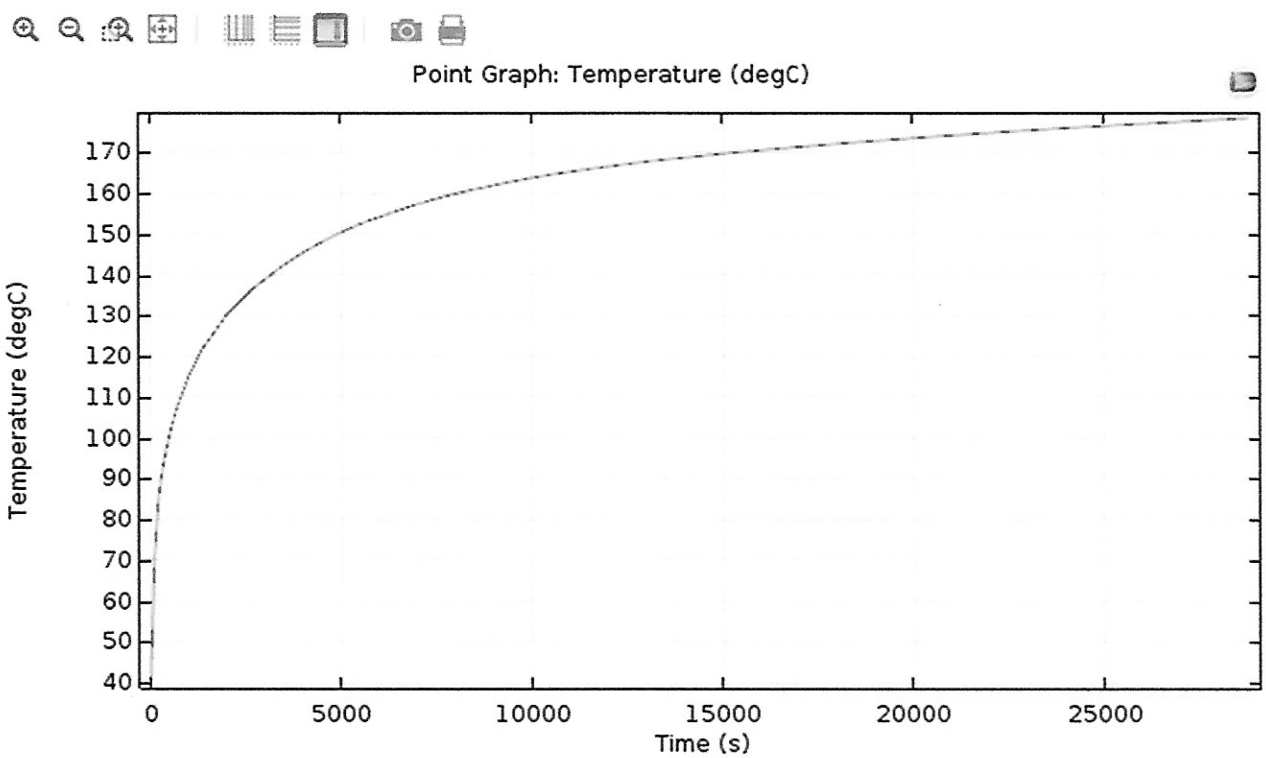
Graph 4: 0.55m Slab Central Temperature

The 0.55m slab yielded to low of a temperature after 8 hours so the next slab was decreased to 0.51m. The central temperature of this slab was approximately 95°C, the desired temperature.



Graph 5: 0.51m Slab Central Temperature

Since the 0.51m slab yielded the desired temperature of 95°C after 8 hours of heating, a point was defined on the outer surface of the slab to determine the surface temperature at the point. The temperature on the outer surface of the slab was approximately 178°C.



Graph 6: 0.51m Slab Surface Temp

Summary:

The COMSOL results were consistent with the results determined using the analytical method. They both resulted in a .51m slab to obtain a 95°C central temperature and 178°C surface temperature after 8 hours of heating at the given conditions.

Materials:

Stone mix concrete

Air

Analysis:

As stated in the previous problem, if we desired a lower central temperature to be sure the concrete stayed well below the weakening point of 100°C, we could make the slabs thicker or decrease the T_{inf} or h_{inf}. It did not take much adjustment to the thickness of the slab to increase or decrease the central temperature a couple of degrees.