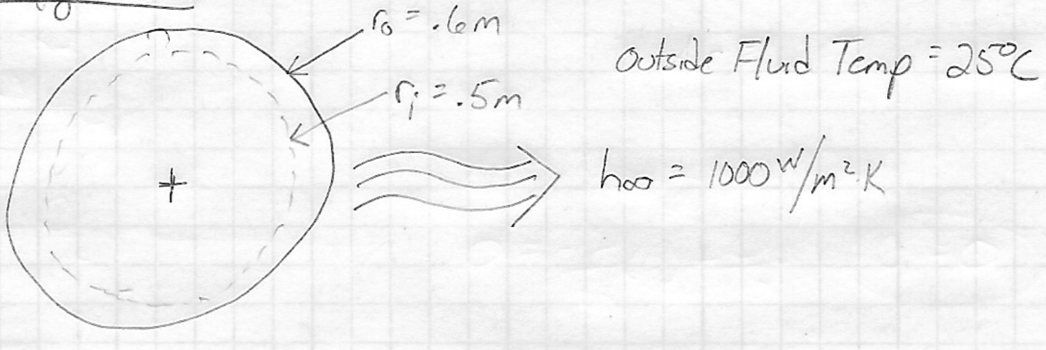
Jason Nathanson	Exam 2	10/14/18

Purpose: Determine the Temperature distribution for the given scenario.
- Determine the expression for the temperature at the center point.
- Determine the expression for the Temperature at T_{si}
- Determine the expression for the Temperature at T_{so}
- use Comsol to display the temperature distribution using numerical method and compare to results of analytically obtained equation.

Drawings + Diagrams:



Sources:

- Bayazitoglu, Y. Ozisik, N, "A Textbook for Heat Transfer Fundamentals." Begel House Inc. (2012)
- Ayala, O. MET 440 Heat Transfer. Old Dominion online. 2018. web. Oct 2018.

Design Considerations:

- Steady state
- Heat generation
- 1D Heat Transfer
- Constant Properties

Data + Variables:

$r_i = .5m$
$r_o = .6m$

$K_{waste} = 20 W/m \cdot K$
$K_{Al} = 204 W/m \cdot K$

$g = 10^5 W/m^3$

H_2O Temp $= 25°C$
$h_\infty = 1000 W/m^2 \cdot K$

Procedure:

- using the equation for a sphere with heat generation + convection along with boundary conditions, determine the Temperature distribution equation for the waste part of the Sphere.
- Determine the equations for the temperatures at given points, Center, T_{si}, T_{so}
- using Comsol, enter given information to solve for temperature within the sphere.
- using Comsol, Compare the temperature determined by Comsole with temp distribution found analytically.

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Calculations:

a) Sphere with Heat Generation + Convection

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{dT(r)}{dr}\right] + \frac{g}{K} = 0$$

Boundary Conditions:

$r = 0 \quad \frac{dT(r)}{dr} = 0$

$r = b \quad -K\frac{dT(r)}{dr} = \bar{h}(T_{r=r_i} - T_\infty)$

$$\frac{d}{dr}\left[r^2\frac{dT(r)}{dr}\right] = -\frac{g}{K}r^2$$

$$\int d\left[r^2\frac{dT(r)}{dr}\right] = -\frac{g}{K}\int r^2\,dr$$

$$r^2\frac{dT(r)}{dr} = -\frac{g}{3K}r^3 + C_1$$

(y') $$\frac{dT(r)}{dr} = -\frac{g}{3K}r + \frac{C_1}{r^2}$$ →

Apply boundary Condition:

$$0 = -\frac{g}{3K}(0) + \frac{C_1}{0} \qquad C_1 = 0$$

$$\int dT(r) = -\frac{g}{3K}\int r\,dr$$

(y'') — $$T(r) = -\frac{g}{6K}r^2 + C_2$$

2nd boundary Condition:

$$+K\left(+\frac{gb}{3K}\right) = \bar{h}\left(-\frac{gb^2}{6K} + C_2 - T_\infty\right)$$

$$\frac{gr}{3\bar{h}} = -\frac{gb^2}{6K} + C_2 - T_\infty$$

$$C_2 = \frac{gr}{3\bar{h}} + \frac{gb^2}{6K} + T_\infty$$

Temp Profile:

$$T(r) = -\frac{gr^2}{6K} + \frac{gb}{3\bar{h}} + \frac{gb^2}{6K} + T_\infty$$

$$T(r) = \frac{bg^2}{6K}\left[1 - \left(\frac{r}{b}\right)^2\right] + \frac{bg}{3\bar{h}} + T_\infty$$

$$T(r) = \frac{bg^2}{6K}\left[1 - \left(\frac{r}{b}\right)^2 + \frac{bg}{3\,\frac{1}{A_n\Sigma R}}\right] + T_\infty$$

$$T(r) = \frac{bg^2}{6K}\left[1 - \left(\frac{r}{b}\right)^2\right] + \frac{bg\,A_n\Sigma R}{3} + T_\infty$$

Temp Profile

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b) Expression for Temp at center of sphere:

$$T_{(0)} = \frac{bg^2}{6K}\left[1-\left(\frac{0}{b}\right)^2\right] + \frac{bg\,A_{ri}\,\Sigma \dot{R}}{3} + T_\infty$$

$$T_{(0)} = \frac{bg^2}{6K}(1) + \frac{bg\,A_{ri}\,\Sigma R}{3} + T_\infty$$

$$\boxed{T_{(0)} = \frac{bg^2}{6K} + \frac{bg\,A_{ri}\,\Sigma R}{3} + T_\infty}$$

c) $$T_{(b)} = \frac{bg^2}{6K}\left[1-\left(\frac{b}{b}\right)^2\right] + \frac{bg\,A_{ri}\,\Sigma R}{3} + T_\infty$$

$$T_{(b)} = \frac{bg^2}{6K}\left[1-1^2\right]^{\to 0} + \frac{bg\,A_{ri}\,\Sigma R}{3} + T_\infty$$

$$\boxed{T_{(b)} = \frac{bg\,A_{ri}\,\Sigma R}{3} + T_\infty}$$

d) $$T_{so} = \frac{\left(\frac{bg\,A_{ri}\,\Sigma R}{3} + T_\infty\right) - T_\infty}{\left(\frac{r_o - r_i}{4\pi k_a (r_o)(r_i)}\right) + \left(\frac{1}{A_{ro}\,h_\infty}\right)}$$

Report date	Oct 14, 2018 9:23:41 PM

Contents

1. Global Definitions

Date	Oct 14, 2018 9:11:24 PM

Global settings

Name	Exam2 Comsol.mph
Path	F:\Exam2_Comsol.mph
COMSOL version	COMSOL 5.1 (Build: 145)
Unit system	SI

1.1. Parameters 1

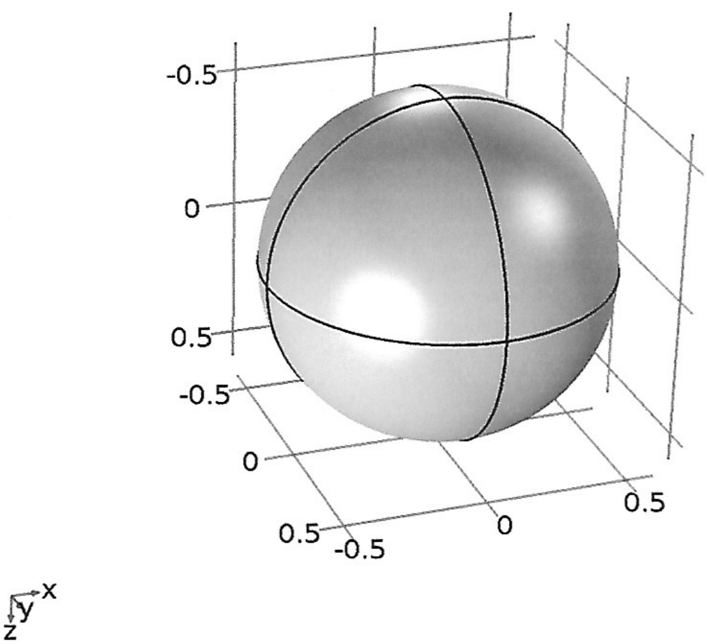
Parameters

Name	Expression	Value	Description
ri	.5 [m]	0.5 m	Inner Radius
ro	.6 [m]	0.6 m	Outer Radius
g	10^5 [W/m^3]	1E5 W/m³	Heat Generation
hinf	1000 [W/m^2*C]	1000 kg·A/s²	Convection Coefficient
Tinf	25 [degC]	298.15 K	Ambient Temperature
krw	20 [W/m*C]	20 kg·m·A/s²	Waste Conduction

ka	204 [W/m*C]	204 kg·m·A/s²	Al Conduction
Ari	3.142 [m^2]	3.142 m²	Surface Area at ri
Rro	.00013 [degC/W]	1.3E−4 K/W	Thermal Resistance for Sphere
Rinf	.000318 [degC/W]	3.18E−4 K/W	Thermal Resistance for conv.

2. Component 1

2.2. Geometry 1



Geometry 1

Units	
Length unit	m
Angular unit	deg

Geometry statistics

Description	**Value**
Space dimension	3
Number of domains	2
Number of boundaries	16
Number of edges	24

Number of vertices	12

2.2.1. Sphere 1 (Sph1)

Position

Description	Value
Position	{0, 0, 0}

Axis

Description	Value
Axis type	z - axis
Layers	

Size

Description	Value
Radius	.5

2.2.2. Sphere 2 (Sph2)

Position

Description	Value
Position	{0, 0, 0}

Axis

Description	Value
Axis type	z - axis
Layers	

Size

Description	Value
Radius	.6

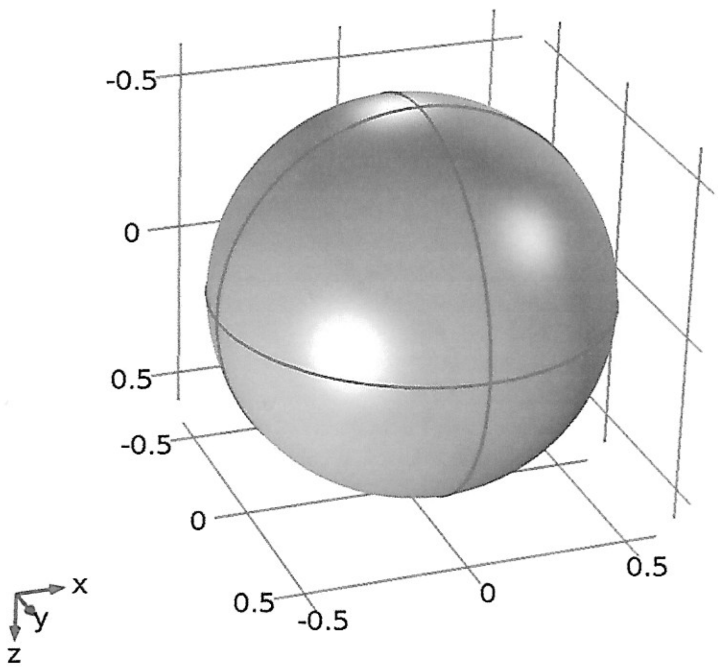
2.2.3. Difference 1 (Dif1)

Settings

Description	Value
Keep input objects	On

2.3. Materials

2.3.1. Material 1



Material 1

Selection	
Geometric entity level	Domain
Selection	Domain 1

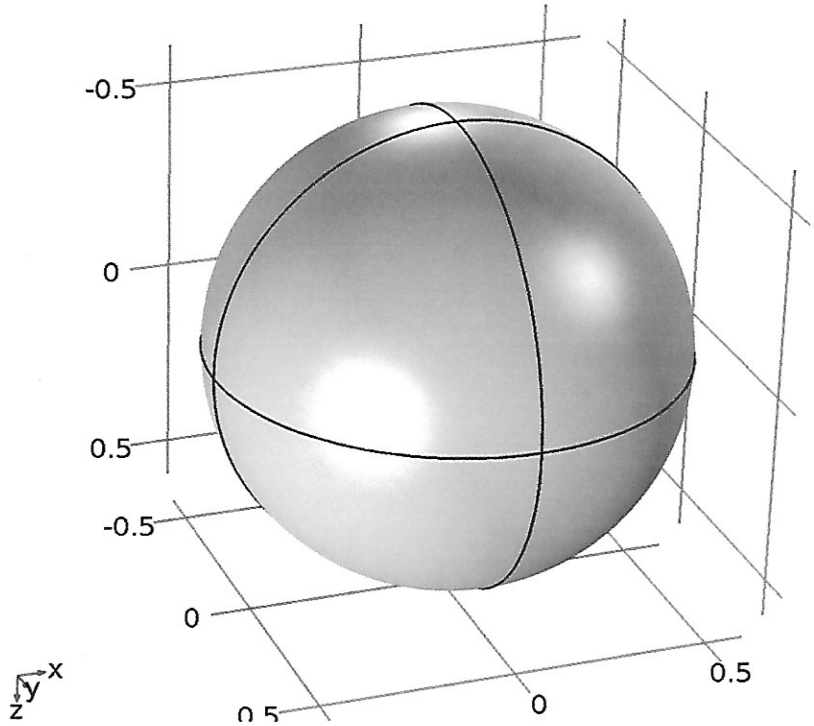
Material parameters

Name	Value	Unit
Thermal conductivity	20	W/(m*K)
Density	1	kg/m^3
Heat capacity at constant pressure	1	J/(kg*K)

Basic Settings

Description	Value
Thermal conductivity	{{20, 0, 0}, {0, 20, 0}, {0, 0, 20}}
Density	1
Heat capacity at constant pressure	1

2.3.2. Material 2



Material 2

Selection	
Geometric entity level	Domain
Selection	Domain 2

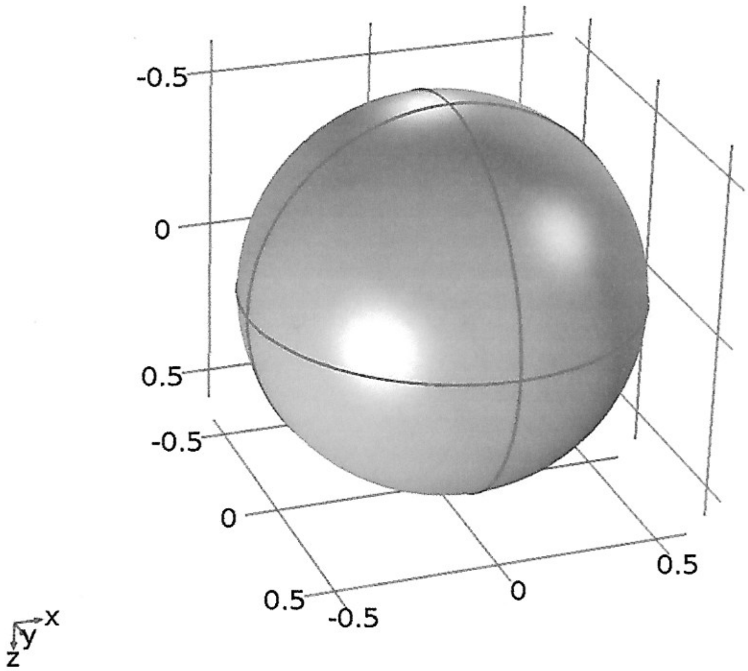
Material parameters

Name	Value	Unit
Thermal conductivity	204	W/(m*K)
Density	1	kg/m^3
Heat capacity at constant pressure	1	J/(kg*K)

Basic Settings

Description	Value
Thermal conductivity	{{204, 0, 0}, {0, 204, 0}, {0, 0, 204}}
Density	1
Heat capacity at constant pressure	1

2.4. Heat Transfer in Solids



Heat Transfer in Solids

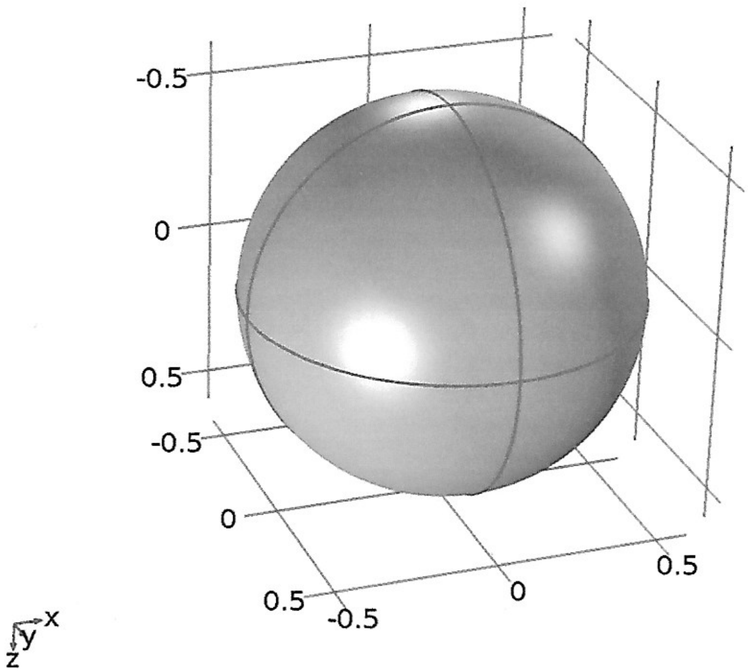
Selection	
Geometric entity level	Domain
Selection	Domains 1–2

Equations

$$\rho C_p \mathbf{u} \cdot \nabla T + \nabla \cdot \mathbf{q} = Q + Q_{\mathrm{ted}}$$

$$\mathbf{q} = -k \nabla T$$

2.4.1. Heat Transfer in Solids 1



Heat Transfer in Solids 1

Selection	
Geometric entity level	Domain
Selection	Domains 1–2

Equations

$$\rho C_p \mathbf{u} \cdot \nabla T + \nabla \cdot \mathbf{q} = Q + Q_{\mathrm{ted}}$$

$$\mathbf{q} = -k\nabla T$$

Settings

Description	Value
Thermal conductivity	From material
Density	From material
Heat capacity at constant pressure	From material

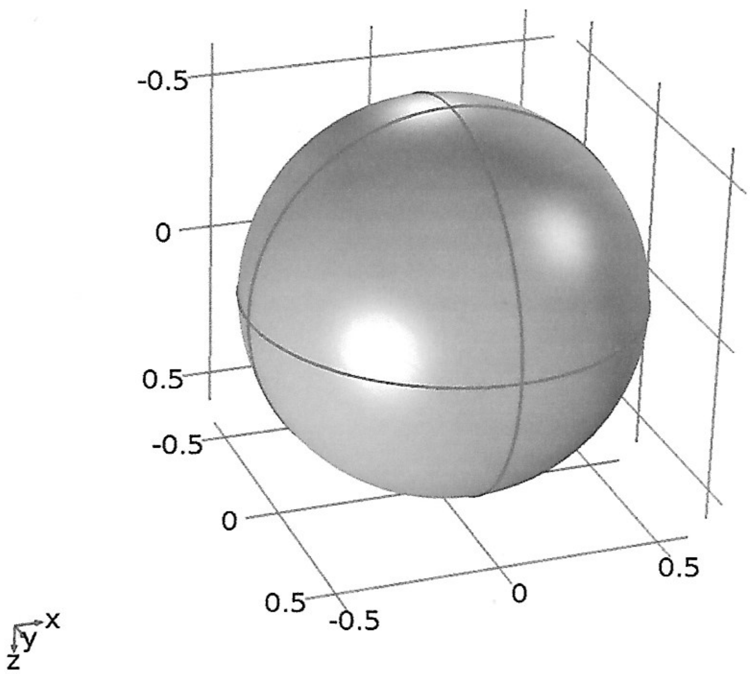
Properties from material

Property	Material	Property group
Thermal conductivity	Material 1	Basic
Density	Material 1	Basic

Heat capacity at constant pressure	Material 1	Basic
Thermal conductivity	Material 2	Basic
Density	Material 2	Basic
Heat capacity at constant pressure	Material 2	Basic

Variables

2.4.2. Initial Values 1



Initial Values 1

Selection	
Geometric entity level	Domain
Selection	Domains 1–2

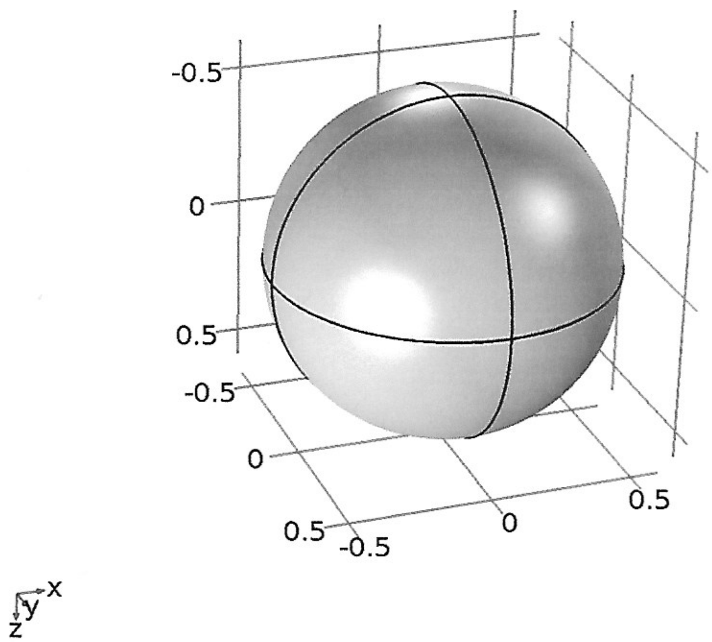
Settings

Description	**Value**
Temperature	293.15[K]

Variables

Name	**Expression**	**Unit**	**Description**	**Selection**
ht.Tinit	293.15[K]	K	Temperature	Domains 1–2

2.4.3. Thermal Insulation 1



Thermal Insulation 1

Selection	
Geometric entity level	Boundary
Selection	No boundaries

Equations

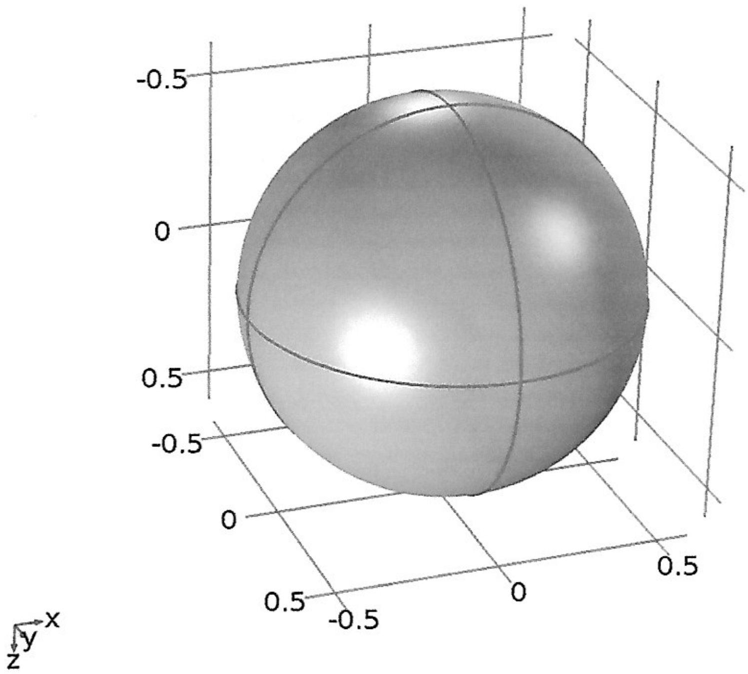
$$-\mathbf{n}\cdot\mathbf{q}=0$$

2.4.4. Heat Source 1

Weak Expressions

Weak expression	Integration frame	Selection
ht.hs1.Q*test(T)*ht.d	Material	Domain 2

2.4.5. Heat Flux 1



Heat Flux 1

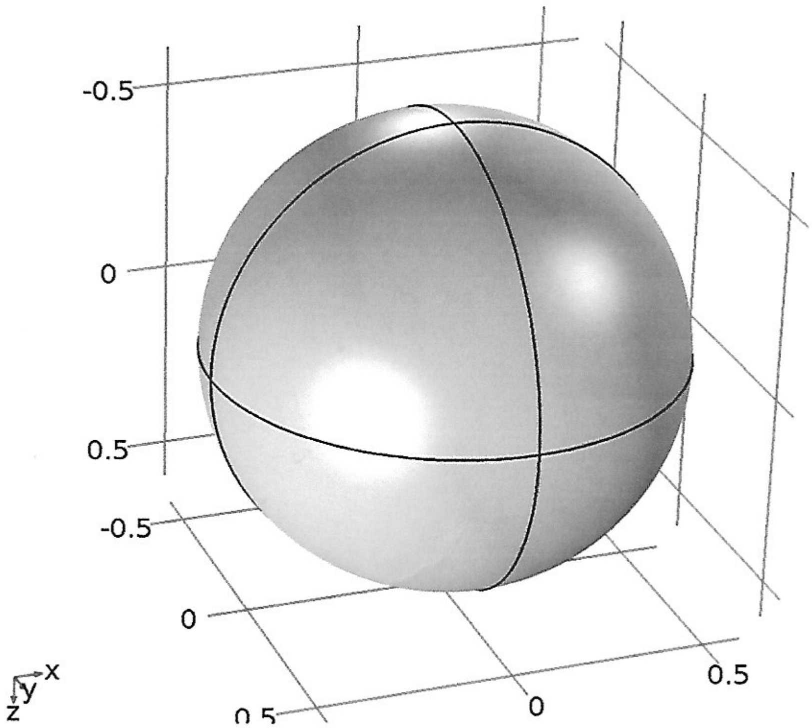
Selection	
Geometric entity level	Boundary
Selection	Boundaries 1–4, 9–10, 13, 16

Equations

$$-\mathbf{n}\cdot\mathbf{q} = q_0$$

Settings

Description	Value
Heat flux	Convective heat flux
Heat transfer coefficient	User defined
Heat transfer coefficient	1000
External temperature	25 [degC]



Heat Source 1

Selection	
Geometric entity level	Domain
Selection	Domain 2

Equations

$$Q = Q_0$$

Settings	
Description	**Value**
Heat source	General source
Heat source	User defined
Heat source	g

Variables

Name	Expression	Unit	Description	Selection
ht.Q	ht.hs1.Q	W/m^3	Heat source	Domain 2
ht.Qtot	ht.hs1.Q	W/m^3	Total heat source	Domain 2
ht.hs1.Q	g	W/m^3	Heat source	Domain 2

Variables

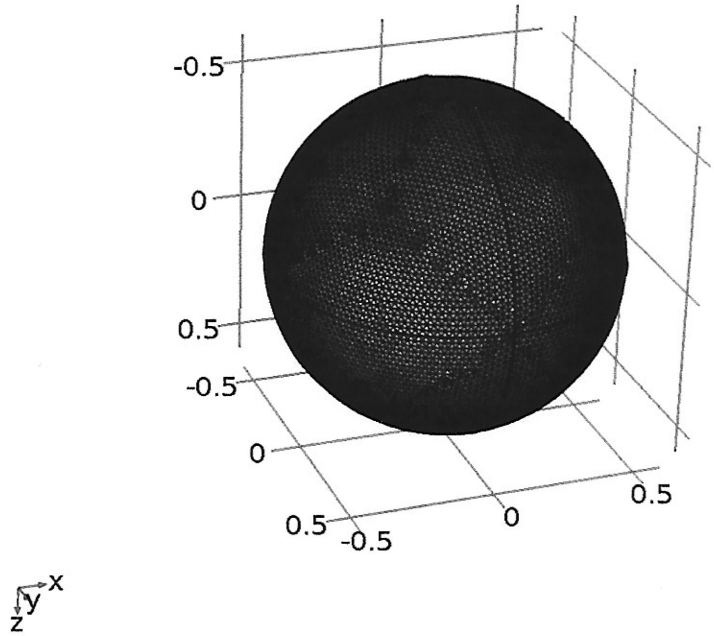
Name	Expression	Unit	Description	Selection
ht.q0	ht.hf1.q0	W/m^2	Inward heat flux	Boundaries 1–4, 9–10, 13, 16
ht.hf1.h	1000	W/(m^2*K)	Heat transfer coefficient	Boundaries 1–4, 9–10, 13, 16
ht.hf1.Text	25[degC]	K	External temperature	Boundaries 1–4, 9–10, 13, 16
ht.hf1.q0	ht.hf1.h*(ht.hf1.Text-ht.hf1.Tvar)	W/m^2	Boundary convective heat flux	Boundaries 1–4, 9–10, 13, 16
ht.hf1.Tvar	ht.Tu	K	Temperature	Boundaries 1–4, 9–10, 13, 16

Weak Expressions

2.5. Mesh 1

Mesh statistics

Description	Value
Minimum element quality	0.191
Average element quality	0.7718
Tetrahedral elements	1115595
Triangular elements	33504
Edge elements	876
Vertex elements	12



Mesh 1

2.5.1. Size (Size)

Settings	
Description	**Value**
Maximum element size	0.024
Minimum element size	2.4E-4
Curvature factor	0.2
Maximum element growth rate	1.3
Predefined size	Extremely fine

2.5.2. Free Tetrahedral 1 (Ftet1)

Selection	
Geometric entity level	Remaining

3. Study 1

Computation information	
Computation time	1 min 27 s

CPU	Intel(R) Xeon(R) CPU E5-2670 0 @ 2.60GHz, 3 cores
Operating system	Windows 7

3.2.1. Solution 1

Compile Equations: Stationary (St1)

Study and step

Description	Value
Use study	Study 1
Use study step	Stationary

Dependent Variables 1 (V1)

General

Description	Value
Defined by study step	Stationary

Initial values of variables solved for

Description	Value
Solution	Zero

Values of variables not solved for

Description	Value
Solution	Zero

Fully Coupled 1 (Fc1)

General

Description	Value
Linear solver	Iterative 1

Method and termination

Description	Value
Initial damping factor	0.01
Minimum damping factor	1.0E-6
Maximum number of iterations	50

Postsmoother (Po)

SOR Line 1 (Sl1)

Main	
Description	**Value**
Relaxation factor	0.2

Secondary	
Description	**Value**
Number of secondary iterations	2
Relaxation factor	0.5

Coarse Solver (Cs)

Direct 1 (D1)

General	
Description	**Value**
Solver	PARDISO

Information 1 (Prob1)

Warnings 1 (Warning1)

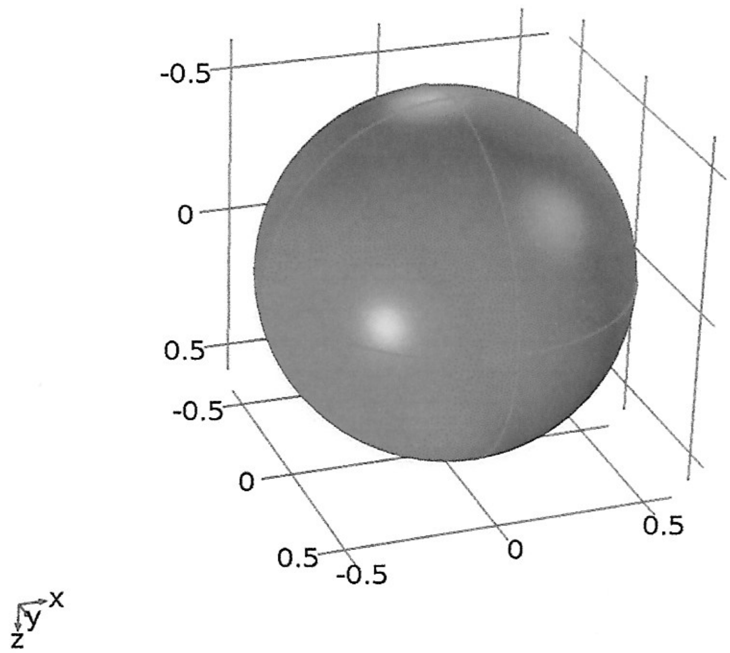
Log
There was a warning message from the linear solver.
Ill-conditioned preconditioner. Increase factor in error estimate.

4. Results

4.1. Data Sets

4.1.1. Study 1/Solution 1

Solution	
Description	**Value**
Solution	Solution 1
Component	Save Point Geometry 1



Data set: Study 1/Solution 1

4.1.2. Cut Line 3D 1

Data

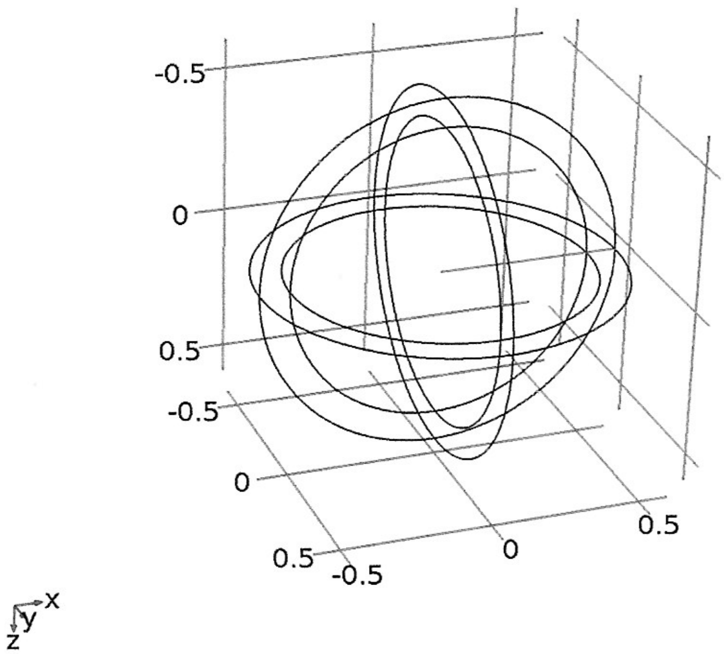
Description	Value
Data set	Study 1/Solution 1

Line data

Description	Value
Line entry method	Two points
Points	{{0, 0, 0}, {.6, 0, 0}}

Advanced

Description	Value
Space variable	cln1x



Data set: Cut Line 3D 1

4.2. Tables

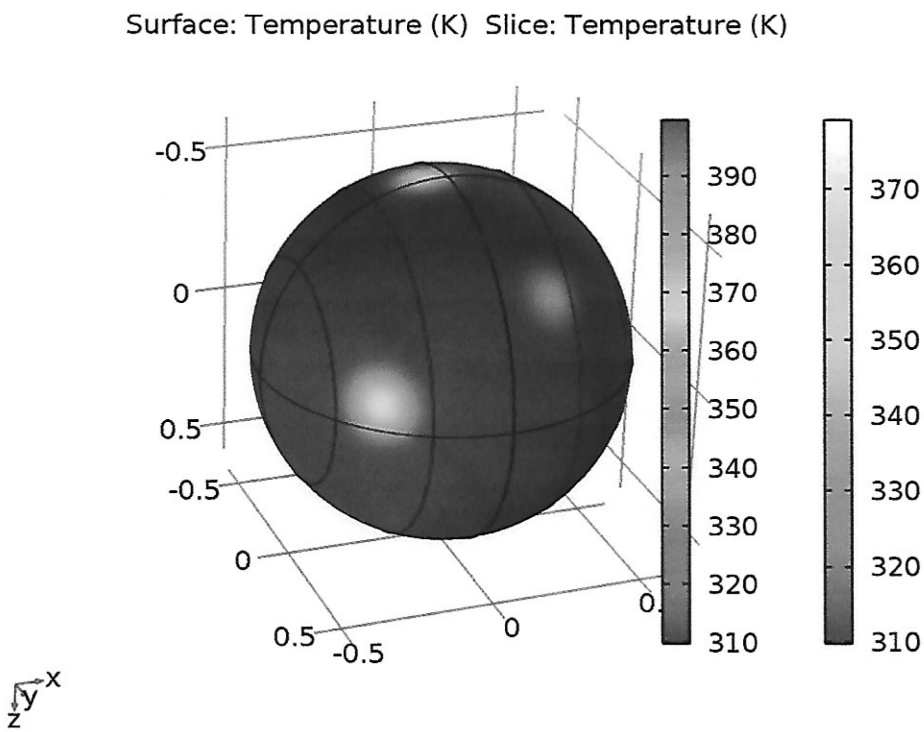
4.2.1. Evaluation 3D

Interactive 3D values

Evaluation 3D			
x	**y**	**z**	**Value**
-0.42485	0.12728	0.40392	309.71

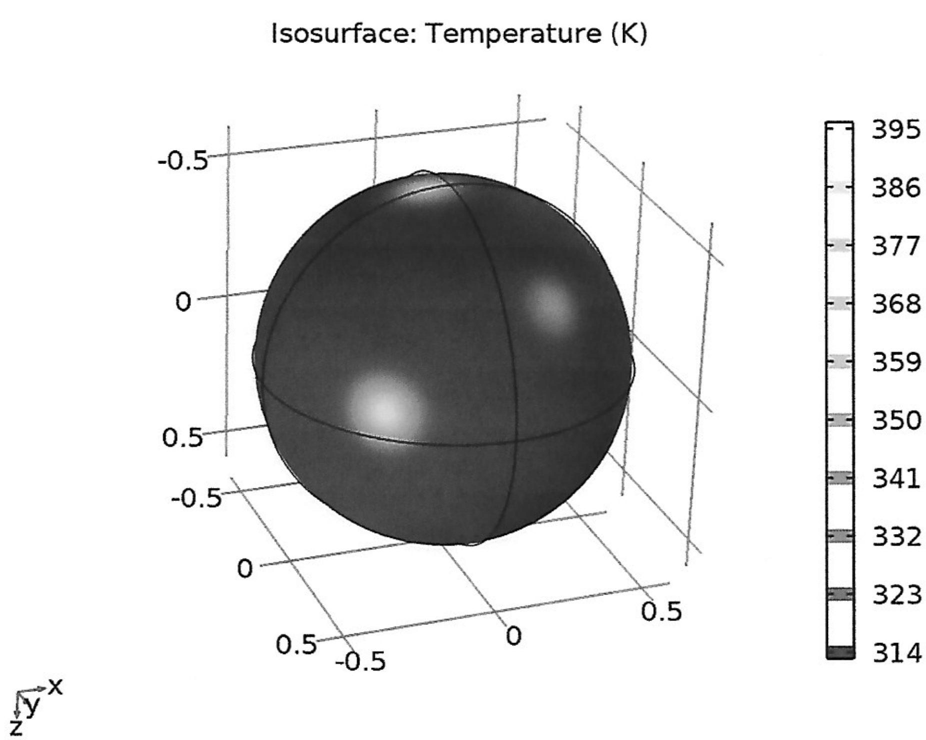
4.3. Plot Groups

4.3.1. Temperature (Ht)



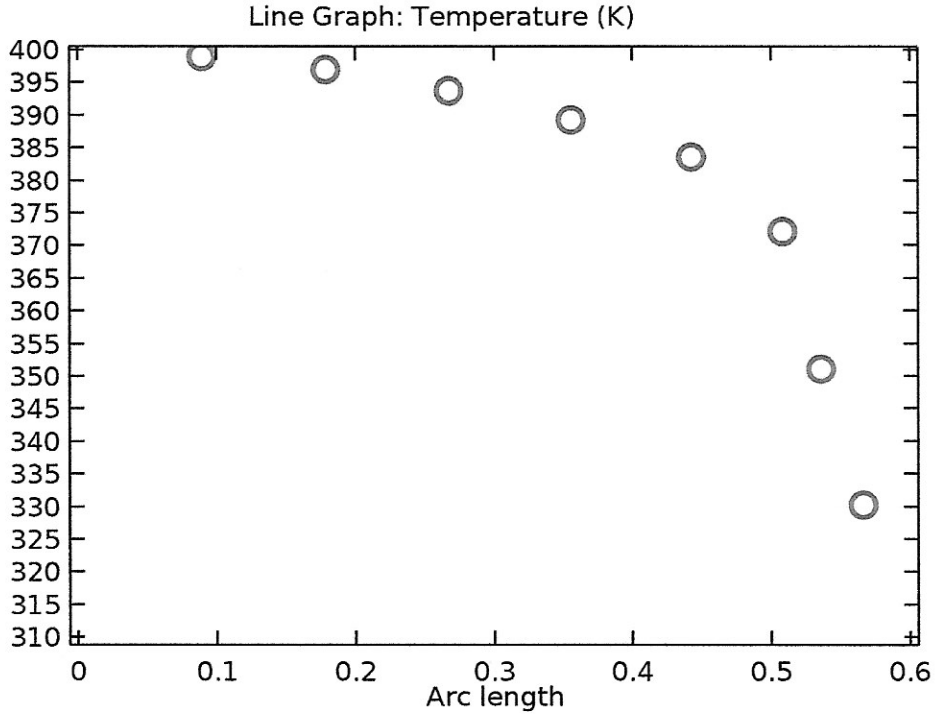
Surface: Temperature (K) Slice: Temperature (K)

4.3.2. Isothermal Contours (Ht)



Isosurface: Temperature (K)

4.3.3. 1D Plot Group 3



*Line Graph: Temperature (K) Line Graph: -(ro*g^2/6*krw)*(1-((y^2+z^2)/ro))+((ro*g*Ari*(Rro+Rinf))/3)+Tinf*

Results:

I was not able to get my temperature profile equation to work in Comsol so I was not able to compare the numerical and analytical results. I do not know where the issue is, it must be either in my equation itself or the way it was being entered into Comsol.

By the looks of the graph created by Comsol, the temperature gradually decreased from the center of the sphere to the edge of the radioactive waste. From that point, through the aluminum to the outside fluid, the temperature decreased much more rapidly. This makes sense since the thermal conductivity of the aluminum is 10 times the thermal conductivity of the waste and the smaller the thermal conductivity, the larger the thermal resistance.