1. An engineer designed a Brayton cycle with a regenerator (heat exchanger) for an automobile. In the design, air enters the compressor of this engine at 100 kPa and 30°C. The compressor pressure ratio is 10; the maximum cycle temperature is 800°C; and the cold air stream leaves the regenerator 10°C cooler than the hot air stream at the inlet of the regenerator.

   A second engineer looked at the design and noticed that it had a major issue. The regenerator should NOT be used. **Please, state why this other engineer believed that.**

   This other engineer said that in order to properly use the regenerator either the pressure ratio in the compression should be reduced to at least 8.58 or the maximum temperature of the cycle should be increased to at least 895.48 °C.

   Assuming both the compressor and the turbine to be isentropic and constant specific heats at room temperature. Determine: the net work, the heat addition and rejection, the thermal efficiency, and the heat exchanger effectiveness for the following cases:

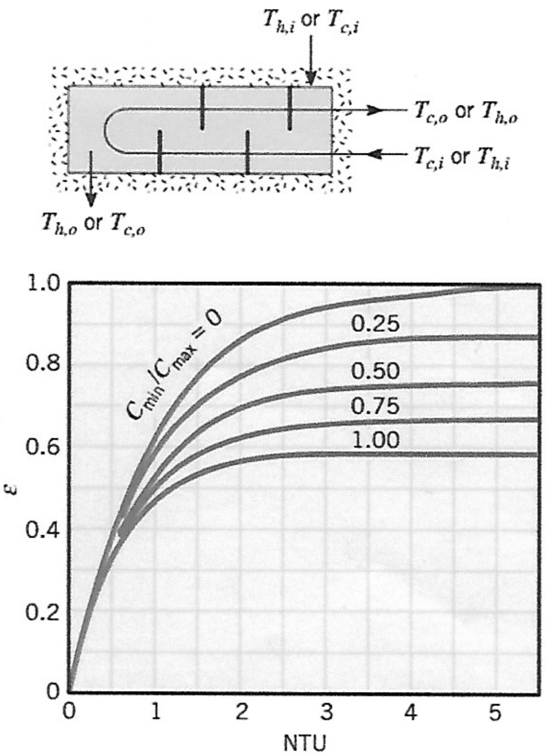



**FIGURE 11.12** Effectiveness of a shell-and-tube heat exchanger with one shell and any multiple of two tube passes (two, four, etc.,

   a. The original design (as stated above)
   b. The original design without the regeneration 8.58
   c. The original design with a pressure ratio of ~~10~~ keeping same other variables
   d. The original design with a maximum cycle temperature of 895.48 °C keeping same other variables
   e. The original design but removing the compressor with a two-stages compressor with intercooling

   Discuss what option is the best. You should consider what might be cheaper to make and whether the heat exchanger (for the regenerator) is feasible. For the heat exchanger feasibility, use the provided figure with $C_{min}/C_{max} = 1.00$, which indeed is the value for this specific problem. Please note that there is maximum achievable effectiveness, anything above is impossible.

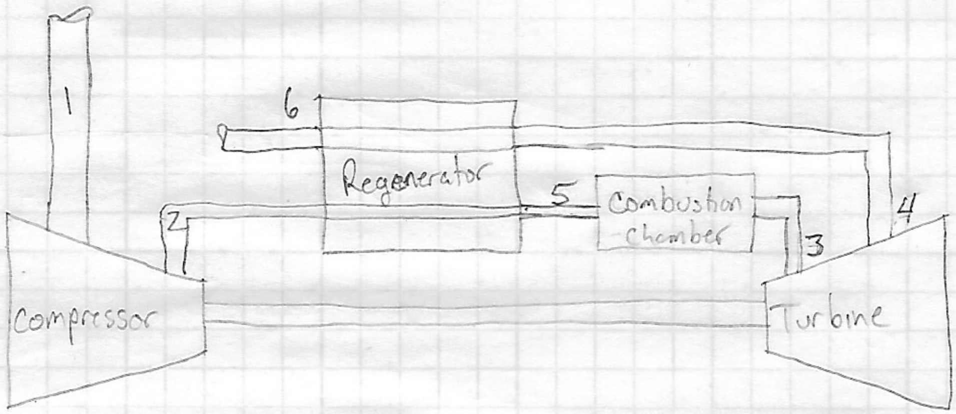
2. A turbojet aircraft flies with a velocity of 900 km/h at an altitude where the air temperature and pressure are -35 °C and 40 kPa. The combustion gases enter the turbine at 950 °C. The turbine produces 500 kJ/kg of work, all of which is used to drive the compressor. Assuming an isentropic efficiency of 80% for the compressor, an isentropic efficiency of 90% for the turbine, and using variable specific heats, determine:

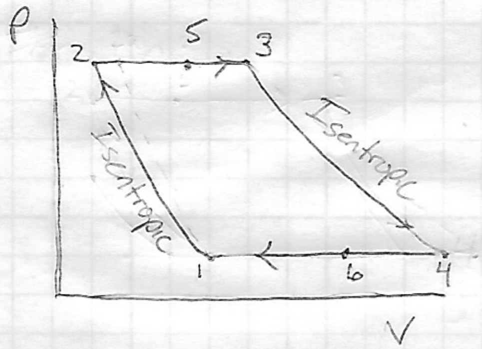
   (a) the pressure of combustion gases at the turbine exit,
   (b) the velocity of the gases at the nozzle exit, and
   (c) the thrust for this engine if the diffuser inlet diameter is 1.6 m.

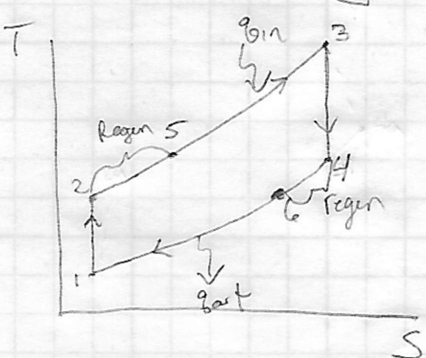
(1) Purpose: Determine the net work, heat addition and rejection, thermal Efficiency and heat exchanger effectiveness for each of the scenarios listed in 1a-1e.

Drawing + Diagrams:

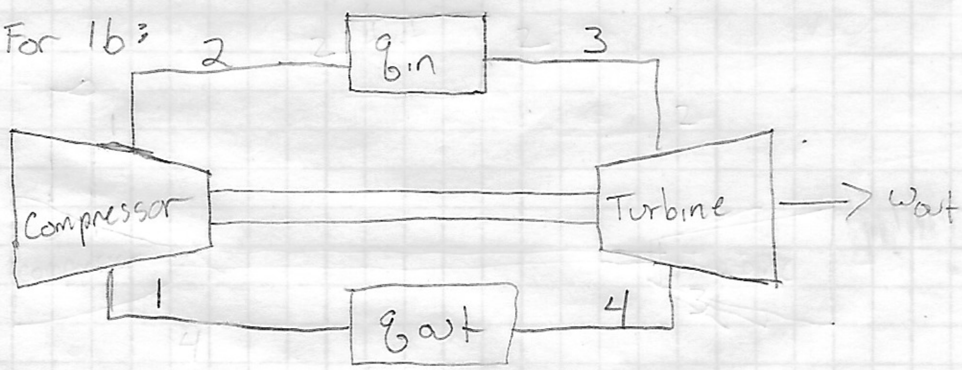
For Problems 1a, 1c, 1d:

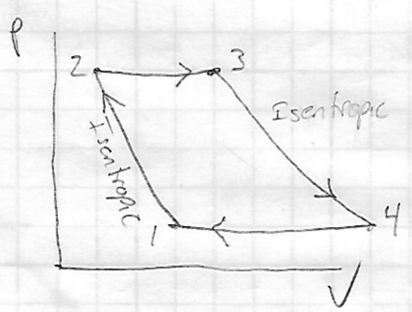


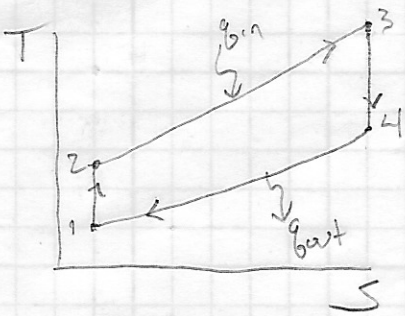




For 1b:

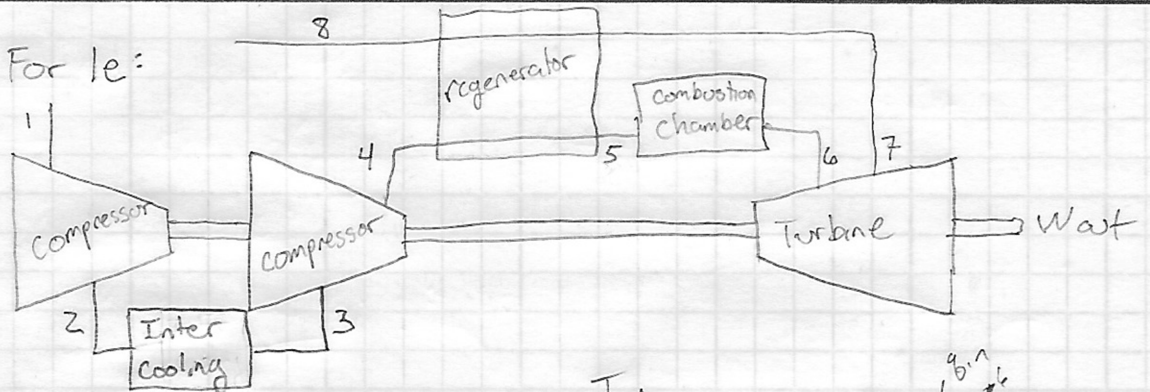


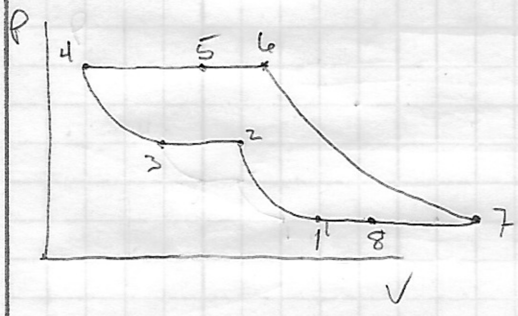



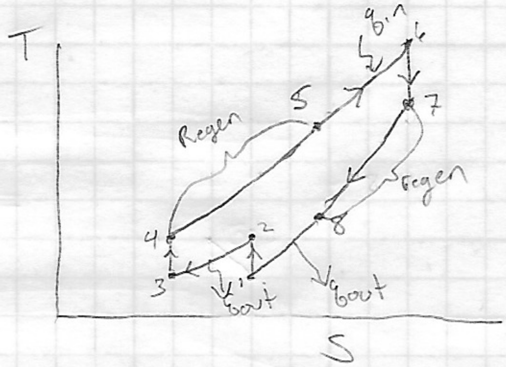



(1 cont)

For 1e:




Sources: Cengel and Boles. Thermodynamics - An Engineering Approach 8th edition. McGraw Hill. 2015

Design Considerations:

1) Compressor + Turbine are isentropic
2) Air as Ideal gas with $R = .287 \, kJ/kg\,K$
3) Constant $C_p$ + $C_v$ at room temp
$C_p = 1.005 \, kJ/kg$ $\quad C_v = .718 \, kJ/kg$ $\quad k = 1.4$
4) No friction or unwanted heat losses

Other assumptions: That I know what I'm doing

Data + Variables:

$P_1 = 100\,kPa \quad T_1 = 30°C = 303K$

for 1a, 1b, 1c, 1e: $T_{max} = 800°C \; (1073K)$ for 1d: $T_{max} = 895.48°C \; (1168.48K)$

for 1a, 1b, 1d, 1e: $r_p = 10$ for 1c: $r_p = 8.58$

$T_{regen\,out\,(c)} = T_{regen\,in\,(h)} - 10$

Exam 1

**Problem 1a**

| Knowns | |
|---|---|
| R (kJ/kg*K) | 0.287 |
| Cp (kJ/kg) | 1.005 |
| Cv (kJ/kg) | 0.718 |
| K | 1.4 |
| Tmax (K) | 1073 |
| Pressure Ratio | 10 |

| | Isentropic | | Constant Pressure | | | Isentropic | | Constant Pressure | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **State 1** | | **State 2** | | **State 5** | | **State 3** | | **State 4** | | **State 6** | |
| T (K) | 303 | T (K) | 585 | T (K) | 545.74 | T (K) | 1073 | T (K) | 555.76 | T (K) | 595.02 |
| P (kPa) | 100 | P (kPa) | 1000 | P (kPa) | 1000 | P (kPa) | 1000 | P (kPa) | 100 | P (kPa) | 100 |
| v (m3/kg) | 0.86961 | v (m3/kg) | 0.1679 | v (m3/kg) | 0.1566 | v (m3/kg) | 0.30795 | v (m3/kg) | 1.595 | v (m3/kg) | 1.708 |

| Problem | 1a | 1b | 1c | 1d | 1e |
|---|---|---|---|---|---|
| $W_{net}$ (kJ/kg) | 236.42 | 236.42 | 236.56 | 282.68 | 282.65 |
| $q_{in}$ (kJ/kg) | 529.9 | 490.44 | 504.89 | 576.14 | 529.88 |
| $q_{out}$ (kJ/kg) | 293.48 | 254 | 268.34 | 293.41 | 247.23 |
| Efficiency (%) | 44.62 | 48.21 | 46.85 | 49.06 | 53.3 |
| effectiveness (%) | 134 | NA | 51.5 | 50.52 | 92.58 |

**Problem 1b**

| Knowns | |
|---|---|
| R (kJ/kg*K) | 0.287 |
| Cp (kJ/kg) | 1.005 |
| Cv (kJ/kg) | 0.718 |
| K | 1.4 |
| Tmax (K) | 1073 |
| Pressure Ratio | 10 |

| | Isentropic | | Constant Pressure | Isentropic | | Constant Pressure | |
|---|---|---|---|---|---|---|---|
| State 1 | | State 2 | | State 3 | | State 4 | |
| T (K) | 303 | T (K) | 585 | T (K) | 1073 | T (K) | 555.76 |
| P (kPa) | 100 | P (kPa) | 1000 | P (kPa) | 1000 | P (kPa) | 100 |
| v (m3/kg) | 0.86961 | v (m3/kg) | 0.1679 | v (m3/kg) | 0.30795 | v (m3/kg) | 1.595 |

**Problem 1c**

| Knowns | |
|---|---|
| R (kJ/kg*K) | 0.287 |
| Cp (kJ/kg) | 1.005 |
| Cv (kJ/kg) | 0.718 |
| K | 1.4 |
| Tmax (K) | 1073 |
| Pressure Ratio | 8.58 |

| | Isentropic | | Constant Pressure | | | Isentropic | | Constant Pressure | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **State 1** | | **State 2** | | **State 5** | | **State 3** | | **State 4** | | **State 6** | |
| T (K) | 303 | T (K) | 560 | T (K) | 570.62 | T (K) | 1073 | T (K) | 580.62 | T (K) | 570 |
| P (kPa) | 100 | P (kPa) | 858 | P (kPa) | 858 | P (kPa) | 858 | P (kPa) | 100 | P (kPa) | 100 |
| v (m3/kg) | 0.86961 | v (m3/kg) | 0.18732 | v (m3/kg) | 0.19087 | v (m3/kg) | 0.35892 | v (m3/kg) | 1.6664 | v (m3/kg) | 1.6359 |

**Problem 1d**

**Knowns**

| | |
|---|---|
| R (kJ/kg*K) | 0.287 |
| Cp (kJ/kg) | 1.005 |
| Cv (kJ/kg) | 0.718 |
| K | 1.4 |
| Tmax (K) | 1168.48 |
| Pressure Ratio | 10 |

| | Isentropic | | Constant Pressure | | | | Isentropic | | Constant Pressure | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **State 1** | | **State 2** | | **State 5** | | **State 3** | | **State 4** | | **State 6** | |
| T (K) | 303 | T (K) | 585 | T (K) | 595.21 | T (K) | 1168.48 | T (K) | 605.21 | T (K) | 595 |
| P (kPa) | 100 | P (kPa) | 1000 | P (kPa) | 1000 | P (kPa) | 1000 | P (kPa) | 100 | P (kPa) | 100 |
| v (m3/kg) | 0.86961 | v (m3/kg) | 0.1679 | v (m3/kg) | 0.1708 | v (m3/kg) | 0.3353 | v (m3/kg) | 1.737 | v (m3/kg) | 1.708 |

**Problem 1e**

**Knowns**

| | |
|---|---|
| R (kJ/kg*K) | 0.287 |
| Cp (kJ/kg) | 1.005 |
| Cv (kJ/kg) | 0.718 |
| K | 1.4 |
| Tmax (K) | 1073 |
| Pressure Ratio | 10 |

| | Isentropic | | Constant P | | Isentropic | | Constant Pressure | | | | Isentropic | | Constant Pressure | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| State 1 | | State 2 | | State 3 | | State 4 | | State 5 | | State 6 | | State 7 | | State 8 | |
| T (K) | 303 | T (K) | 421 | T (K) | 303 | T (K) | 421 | T (K) | 545.76 | T (K) | 1073 | T (K) | 555.76 | T (K) | 431 |
| P (kPa) | 100 | P (kPa) | 316.23 | P (kPa) | 316.23 | P (kPa) | 1000 | P (kPa) | 1000 | P (kPa) | 1000 | P (kPa) | 100 | P (kPa) | 100 |
| v (m3/kg) | 0.86961 | v (m3/kg) | 0.3821 | v (m3/kg) | 0.275 | v (m3/kg) | 0.1208 | v (m3/kg) | 0.1566 | v (m3/kg) | 0.308 | v (m3/kg) | 1.595 | v (m3/kg) | 1.237 |

1 cont

## Procedure:

Once all knowns are recorded we must find the fluid properties at each state. This will need to be done for each system scenario as the states will change according to the info given for each system.
We will start with the properties at state 1 and find the properties at the other states using the appropriate equations.
Given that Cp + Cv are constant, depending on whether the process is isentropic or constant P we will use the following equations and interpolation when required:

- $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}} = \left(\frac{V_1}{V_2}\right)^{K-1}$
- $Pv = RT$
- 1st Law
- $\Delta h = C_p \Delta T$

Once all of the states are known, we can solve for the $w$, $q_{in}$, $q_{out}$, $\eta_{th}$ and $\varepsilon$ using the calculated data and appropriate equations.

## Calculations:

1a)

Find $v_1$:

$$v_1 = \frac{RT_1}{P_1} = \frac{(.287)(303)}{100} = .86961 \ m^3/kg$$

Find $h_1$:

$$h_1 = \frac{(303-300)(305.22-300.19)}{(305-300)} + 300.19$$

$$h_1 = 303.21 \ kJ/kg$$

Find $T_2$:

$$\frac{T_2}{T_1} = r_p^{\frac{K-1}{K}} =$$

$$T_2 = 303(10)^{\frac{1.4-1}{1.4}} = 585 K$$

1a cont

- Find $P_2$:

$$r_p = \frac{P_2}{P_1} \qquad P_2 = r_p P_1 = (10)(100) = 1000 \text{ KPa}$$

- Find $V_2$:

$$V_2 = \frac{RT_2}{P_2} = \frac{(.287)(585)}{1000} = .1679$$

- ~~Find $h_2$:~~

$$h_2 = \frac{(585-580)(596.52-586.04)}{(590-580)} + 586.04$$

$$h_2 = 591.28 \text{ KJ/kg}$$

- Find $V_3$:

$$V_3 = \frac{RT_3}{P_3} = \frac{(.287)(1073)}{1000} = .30795 \text{ m}^3/\text{kg}$$

- ~~Find $h_3$:~~

$$h_3 = \frac{(1073-1060)(1137.89-1114.86)}{(1080-1060)} + 1114.86$$

$$h_3 = 1129.83 \text{ KJ/kg}$$

- Find $T_4$:

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{K-1}{K}}$$

$$T_4 = 1073\left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} = 555.76 \text{K}$$

- Find $V_4$:

$$V_4 = \frac{RT_4}{P_4} = \frac{(.287)(555.76)}{100} = 1.595 \text{ m}^3/\text{kg}$$

- ~~Find $h_4$:~~

$$h_4 = \frac{(555.76-550)(565.17-555.74)}{(560-550)} + 555.74$$

$$h_4 = 561.17 \text{ KJ/kg}$$

- Find $T_5$:

$$T_5 = T_4 - 10 = 555.76\text{K} - 10 = 545.76 \text{ K}$$

1a cont

- Find $V_5$:

$$V_5 = \frac{RT_5}{P_5} = \frac{(.287)(545.74)}{1000} = .1566$$

- Find $h_5$:

~~$$h_5 = \frac{(545.74-540)(555.74-544.35)}{(550-540)} + 544.35$$~~

~~$h_5 = 550.89$~~

- Find $T_6$:

$$T_6 = T_4 + (T_2 - T_5)$$

$$T_6 = 555.76 + (585 - 545.74) = 595.02\ K$$

- Find $V_6$:

$$V_6 = \frac{RT_6}{P_6} = \frac{(.287)(595.02)}{100} = 1.708\ m^3/kg$$

- Find $h_6$:

~~$$h_6 = \frac{(595.02-590)(607.02-596.52)}{(600-590)} + 596.52$$~~

~~$h_6 = 601.8\ kJ/kg$~~

- ~~Find $w_{net}$:~~

~~$$w_{net} = w_{out} - w_{in} = (h_3 - h_4) - (h_2 - h_1)$$~~

~~$$w_{net} = (1129.83 - 561.17) - (591.28 - 303.21)$$~~

~~$w_{net} = 280.59\ kJ/kg$~~

I calculated all of the enthalpy values using table A-17 in the book. Since $C_p$ and $C_v$ are constant $\Delta h = C_p \Delta T$ should be used instead of the tables.

- Find $w_{net}$:

$$w_{net} = C_p(T_3 - T_4) - C_p(T_2 - T_1)$$

$$w_{net} = 1.005(1073 - 555.76) - 1.005(585 - 303)$$

**$w_{net} = 236.42\ kJ/kg$**

- Find $q_{in}$:

$$q_{in} = c_p(T_3 - T_5)$$

$$q_{in} = 1.005(1073 - 545.74)$$

$$\boxed{q_{in} = 529.9 \ kJ/kg}$$

- Find $q_{out}$:

$$q_{out} = C_p(T_6 - T_1)$$

$$q_{out} = 1.005(595.02 - 303)$$

$$\boxed{q_{out} = 293.48 \ kJ/kg}$$

- Find $\eta_{th}$:

$$\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{236.42}{529.9} = .4462$$

$$\boxed{\eta_{th} = 44.62\%}$$

- Heat exchanger Effectiveness:

$$\varepsilon = \frac{C_p(T_5 - T_2)}{C_p(T_4 - T_2)} = \frac{1.005(545.74 - 585)}{1.005(555.76 - 585)} = \frac{-39.46}{-29.39} = \boxed{1.34}$$

- Effectiveness cannot be greater than 1. This regenerator should not be used because the temperature of the air leaving the turbine is less than the temperature of the air leaving the compressor. Because of this, the heat would flow from the "cold side" to the "hot side", cooling the air before combustion instead of heating it.

(1b)

- Find $T_2$:

$$\frac{T_2}{T_1} = r_p^{\frac{k-1}{k}}$$

$$T_2 = 303(10)^{\frac{1.4-1}{1.4}} = 585\,K$$

- Find $P_2$:

$$r_p = \frac{P_2}{P_1} \qquad P_2 = r_p P_1 = (10)(100) = 1000\,kPa$$

- state 3:

$$T_3 = T_{max} = 1073\,K$$

$$P_3 = P_2 = 1000\,kPa$$

- Find $v_3$:

$$v_3 = \frac{RT_3}{P_3} = \frac{(.287)(1073)}{1000} = .30795\ m^3/kg$$

- Find $T_4$:

$$\frac{T_4}{T_3} = \left(\frac{1}{r_p}\right)^{\frac{k-1}{k}}$$

$$T_4 = 1073\left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}}$$

$$T_4 = 555.76\,K$$

- Find $v_4$:

$$v_4 = \frac{RT_4}{P_4} = \frac{(.287)(555.76)}{100} = 1.595\ m^3/kg$$

- Find $w_{net}$:

$$w_{net} = c_p(T_3 - T_4) - c_p(T_2 - T_1)$$

$$w_{net} = 1.005(1073 - 555.76) - 1.005(585 - 303)$$

$$\boxed{w_{net} = 236.42\ kJ/kg}$$

1b cont

- Find $q_{in}$:

$$q_{in} = C_p(T_3 - T_2) = 1.005(1073 - 585)$$

$$\boxed{q_{in} = 490.44 \text{ kJ/kg}}$$

- Find $q_{out}$:

$$q_{out} = C_p(T_4 - T_1) = 1.005(555.76 - 303)$$

$$\boxed{q_{out} = 254 \text{ kJ/kg}}$$

- Find $\eta_{th}$:

$$\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{236.42}{490.44} = .4821$$

$$\boxed{\eta_{th} = 48.21\%}$$

- There is no regenerator to find effectiveness.

1c

- Find $T_2$:

$$\frac{T_2}{T_1} = r_p^{\frac{k-1}{k}}$$

$$T_2 = 303(8.58)^{\frac{1.4-1}{1.4}}$$

$$T_2 = 560\text{K}$$

- Find $P_2$:

$$r_p = \frac{P_2}{P_1} \qquad P_2 = r_p P_1 = (8.58)(100) = 858 \text{ KPa}$$

- Find $v_2$:

$$v_2 = \frac{RT_2}{P_2} = \frac{(.287)(560)}{858} = .18732 \text{ m}^3/\text{kg}$$

- Find $v_3$:

$$v_3 = \frac{RT_3}{P_3} = \frac{(.287)(1073)}{858} = .35892 \text{ m}^3/\text{kg}$$

- Find $T_4$:

$$\frac{T_4}{T_3} = \left(\frac{1}{r_p}\right)^{\frac{k-1}{k}}$$

$$T_4 = 1073\left(\frac{1}{8.58}\right)^{\frac{1.4-1}{1.4}}$$

$$T_4 = 580.62\text{K}$$

- Find $v_4$:

$$v_4 = \frac{RT_4}{P_4} = \frac{(.287)(580.62)}{100} = 1.6664$$

- Find $T_5$:

$$T_5 = T_4 - 20$$

$$T_5 = 580.62 - 10 = 570.62\text{K}$$

- Find $v_5$:

$$v_5 = \frac{RT_5}{P_5} = \frac{(.287)(570.62)}{858} = .19087 \text{ m}^3/\text{kg}$$

1e cont

- Find $T_6$:

$$T_6 = T_4 - (T_5 - T_2)$$

$$T_6 = 580.62 - (570.62 - 560)$$

$$T_6 = 570\,K$$

- Find $v_6$:

$$v_6 = \frac{RT_6}{P_6} = \frac{(.287)(570)}{100} = 1.6359$$

- Find $W_{net}$:

$$W_{net} = C_p(T_3 - T_4) - C_p(T_2 - T_1)$$

$$W_{net} = 1.005(1073 - 580.62) - 1.005(560 - 303)$$

$$\boxed{W_{net} = 236.56\ \text{kJ/kg}}$$

- Find $q_{in}$:

$$q_{in} = C_p(T_3 - T_5)$$

$$q_{in} = 1.005(1073 - 570.62)$$

$$\boxed{q_{in} = 504.89\ \text{kJ/kg}}$$

- Find $q_{out}$:

$$q_{out} = C_p(T_6 - T_1)$$

$$q_{out} = 1.005(570 - 303)$$

$$\boxed{q_{out} = 268.34\ \text{kJ/kg}}$$

- Find $\eta_{th}$:

$$\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{236.56}{504.89} = .4685$$

$$\boxed{\eta_{th} = 46.85\%}$$

- Find effectiveness

$$\varepsilon = \frac{C_p(T_5 - T_2)}{C_p(T_4 - T_2)} = \frac{1.005(570.62 - 560)}{1.005(580.62 - 560)} = .5150$$

$$\boxed{\varepsilon = .5150 \text{ or } 51.5\%}$$

(1d)

- Find $T_2$:

$$\frac{T_2}{T_1} = r_p^{\frac{k-1}{k}}$$

$$T_2 = 303(10)^{\frac{1.4-1}{1.4}}$$

$$T_2 = 585\,K$$

- Find $V_3$:

$$V_3 = \frac{RT_3}{P_3} = \frac{(287)(1168.48)}{1000} = .3353\ m^3/kg$$

- Find $T_4$:

$$\frac{T_4}{T_3} = \left(\frac{1}{r_p}\right)^{\frac{k-1}{k}}$$

$$T_4 = 1168.48\left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}}$$

$$T_4 = 605.21\,K$$

- Find $V_4$:

$$V_4 = \frac{RT_4}{P_4} = \frac{(.287)(605.21)}{100} = 1.737\ m^3/kg$$

- Find $T_5$:

$$T_5 = T_4 - 10$$

$$T_5 = 605.21 - 10 = 595.21\,K$$

- Find $V_5$:

$$V_5 = \frac{RT_5}{P_5} = \frac{(.287)(595.21)}{1000} = .1708\ m^3/kg$$

- Find $T_6$:

$$T_6 = T_4 - (T_5 - T_2)$$

$$T_6 = 605.21 - (595.21 - 585)$$

$$T_6 = 595\,K$$

- Find $V_6$:

$$V_6 = \frac{RT_6}{P_6} = \frac{(.287)(595)}{100} = 1.708\ m^3/kg$$

1d cont

- Find $W_{net}$:

$$W_{net} = c_p(T_3 - T_4) - c_p(T_2 - T_1)$$

$$W_{net} = 1.005(1168.48 - 605.21) - 1.005(585 - 303)$$

$$\boxed{W_{net} = 282.68 \ \text{kJ/kg}}$$

- Find $q_{in}$:

$$q_{in} = c_p(T_3 - T_5)$$

$$q_{in} = 1.005(1168.48 - 595.21) =$$

$$\boxed{q_{in} = 576.14 \ \text{kJ/kg}}$$

- Find $q_{out}$:

$$q_{out} = c_p(T_6 - T_1)$$

$$q_{out} = 1.005(595 - 303)$$

$$\boxed{q_{out} = 293.46 \ \text{kJ/kg}}$$

- Find $\eta_{th}$:

$$\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{282.68}{576.14} = .4906$$

$$\boxed{\eta_{th} = 49.06\%}$$

- Find Effectiveness:

$$\varepsilon = \frac{c_p(T_5 - T_2)}{c_p(T_4 - T_2)} = \frac{1.005(595.21 - 585)}{1.005(605.21 - 585} = .5052$$

$$\boxed{\varepsilon = 50.52\%}$$

(1e)

- Find $T_2$:

$$\frac{T_2}{T_1} = \left(\sqrt{r_p}\right)^{\frac{K-1}{K}}$$

$$T_2 = 303\left(\sqrt{10}\right)^{\frac{1.4-1}{1.4}}$$

$$T_2 = 421\,K$$

- Find $P_2$:

$$\sqrt{r_p} = \frac{P_2}{P_1}$$

$$P_2 = (\sqrt{10})(100) = 316.23\ kPa$$

- Find $V_2$: $V_2 = \frac{RT_2}{P_2} = \frac{(.287)(421)}{316.23} = .3821\ m^3/kg$

- $P_3 = P_2$, $T_3 = T_1$

- Find $V_3$: $V_3 = \frac{RT_3}{P_3} = \frac{(.287)(303)}{316.23} = .275\ m^3/kg$

- Find $T_4$:

$$\frac{T_4}{T_3} = \left(\sqrt{r_p}\right)^{\frac{K-1}{K}}$$

$$T_4 = (303)(\sqrt{10})^{\frac{1.4-1}{1.4}}$$

$$T_4 = 421\,K$$

- Find $P_4$:

$$\frac{P_4}{P_3} = \sqrt{r_p}$$

$$P_4 = (316.23)(\sqrt{10})$$

$$P_4 = 1000\ KPa$$

- Find $V_4$: $V_4 = \frac{RT_4}{P_4} = \frac{(.287)(421)}{1000} = .1208$

(1e cont)

State 6 = $T_{max}$ = 1073K

$P_6 = P_5 = P_4$

Find $v_6$:

$$v_6 = \frac{RT_6}{P_6} = \frac{(.287)(1073)}{1000} = .3080$$

Find $T_7$:

$$\frac{T_7}{T_6} = \left(\frac{1}{r_p}\right)^{\frac{k-1}{k}}$$

$$T_7 = (1073)\left(\frac{1}{16}\right)^{\frac{1.4-1}{1.4}}$$

$$T_7 = 555.76\text{ K}$$

Find $v_7$:

$$v_7 = \frac{RT_7}{P_7} = \frac{(.287)(555.76)}{100} = 1.595$$

Find $T_5$:

$$T_5 = T_7 - 10 = 555.76 - 10$$

$$T_5 = 545.76\text{ K}$$

Find $v_5$:

$$v_5 = \frac{RT_5}{P_5} = \frac{(.287)(545.76)}{1000} = .1566\ m^3/kg$$

Find $T_8$:

$$T_8 = T_7 - (T_5 - T_4)$$

$$T_8 = 555.76 - (545.76 - 421)$$

$$T_8 = 431\text{ K}$$

Find $v_8$:

$$v_8 = \frac{RT_8}{P_8} = \frac{(.287)(431)}{100} = 1.237$$

Find $W_{net}$:

$$W_{net} = W_{out} - W_{in}$$

$$W_{net} = C_p(T_6 - T_7) - [C_p(T_4 - T_3) + C_p(T_2 - T_1)]$$

$$W_{net} = 1.005(1073 - 555.76) - [1.005(421-303) + 1.005(421-303)]$$

$$= 519.83 - 237.18$$

$$\boxed{W_{net} = 282.65\ kJ/kg}$$

(1e cont)

- find $q_{in}$:

$$q_{in} = C_p(T_6 - T_5)$$

$$q_{in} = 1.005(1073 - 545.76)$$

$$\boxed{q_{in} = 529.88 \ kJ/kg}$$

- Find $q_{out}$:

$$q_{out} = C_p(T_8 - T_1) + C_p(T_2 - T_3)$$

$$q_{out} = 1.005(431 - 303) + 1.005(421 - 303)$$

$$\boxed{q_{out} = 247.23 \ kJ/kg}$$

- Find $\eta_{th}$:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{282.65}{529.88} = .5334$$

$$\boxed{\eta_{th} = 53.34\%}$$

- Find regenerator effectiveness:

$$\varepsilon = \frac{h_5 - h_4}{h_7 - h_4} = \frac{C_p(T_5 - T_4)}{C_p(T_7 - T_4)} = \frac{1.005(545.76 - 421)}{1.005(555.76 - 421)} = .9258$$

$$\boxed{\varepsilon = 92.58\%}$$

P1 cont

## Summary:

As previously stated, the regenerator should not be used in the original design because the temperature of the air leaving the turbine is cooler than the air leaving the compressor. In this situation the heat would be transferred from the "cool" side to the "hot" side, cooling the air before the combustion chamber and heating the exhaust which is wasting the heat and causing more heat to be added at the combustion process.

The design with 2 compressors clearly yields the highest efficiency at 53.3% and effectiveness at 92.6% but is it worth it. Figure 11.12 shows that the maximum performance at Cmin/Cmax = 1 is approximately 59% so anything more than that is a waste. In addition, to achieve this high of an effectiveness, a second compressor was required to increase the pressure while keeping the temperature low. An additional compressor, plus a means of intercooling would be expesive and not worth the added cost.

I believe that the design with the increased max cycle temperature is the best option, while it requires additional heat in to reach the max temperature, it yields the highest net work of 282.68 kJ/kg while having an efficiency of 49% and comparable effectiveness at 50.52%.

## Materials:

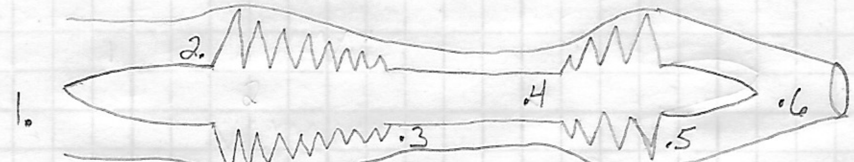
Air as an ideal gas.

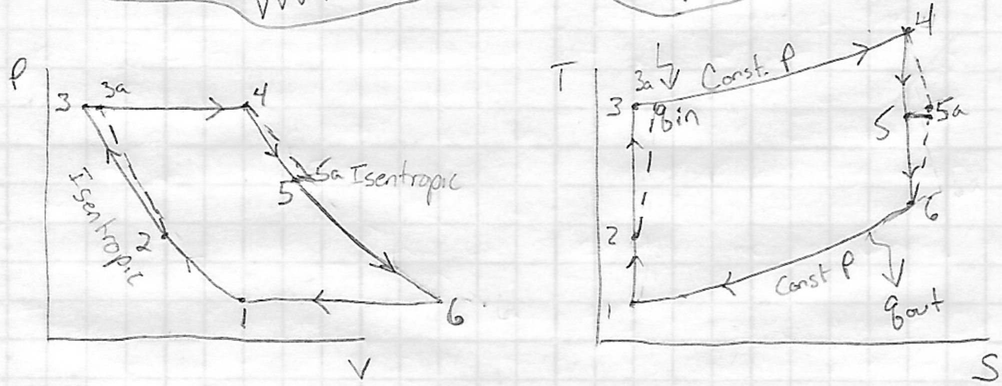
## Analysis:

Using a regenerator is an effective way of increasing a system's efficiency and reduce required input, however, the system design must be carefully chosen with the regenerator for ideal efficiency and effectiveness.
Characteristics, such as pressure ratio and operating temperatures can have a substantial effect on a systems output. Too high of a pressure ratio may cause an excessive increase in temperature, while too low of a pressure ratio may require an excessive input.

(2) **Purpose:** To determine the pressure of combustion gases at the turbine exit, the velocity of the gases at the nozzle exit and the thrust of the engine given the described scenario.

**Drawings + Diagrams:**




- **Sources:** Cengel and Boles. Thermodynamics - An Engineering approach 8th Edition McGraw Hill. 2015

- **Design Considerations:**

  - Variable $C_p$ + $C_v$
  - Air as an ideal gas
  - $\eta_C = 80\%$
  - $\eta_T = 90\%$
  - No friction or unwanted heat losses

- **Data + Variables:**

$V = 900 km/h = 250 m/s$
$T_1 = -35°C = 238K$
$P_1 = 40 kPa$
$T_4 = 950°C = 1223K$
$W_{net} = 500 kJ/kg$
diffuser dia = 1.6 m

- **Procedure:**

Evaluate cycle at each state and record fluid properties at each state using equations for the given processes and obtaining values from table A-17.
Once all states are defined, answer questions using equations for Jet propulsion and data obtained in previous step.

**Problem 2**

**Knowns**

| | |
|---|---|
| V (m/s) | 250 |
| T1 (K) | 238 |
| P1 (kPa) | 40 |
| T4 (K) | 1223 |
| W turbine (kJ/kg) | 500 |
| Comp Eff (%) | 80 |
| Turbine Eff (%) | 90 |
| Specific Heats | Variable |

| Pressure | | Isentropic | | Constant Pressure | | | | Isentropic | | | | | | Constant |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **State 1** | | **State 2** | | **State 3** | | **State 3a** | | **State 4** | | **State 5** | | **State 5a** | | **State 6** | |
| T (K) | 238 | T (K) | 269.16 | T (K) | 751.82 | T (K) | 865.38 | T (K) | 1223 | T (K) | 784.4 | T (K) | 829.84 | T (K) | 411.76 |
| P (kPa) | 40 | P (kPa) | 61.43 | P (kPa) | 2440.19 | P (kPa) | 2440.19 | P (kPa) | 2440.19 | P (kPa) | 420.29 | P (kPa) | 420.29 | P (kPa) | 40 |
| v (m3/kg) | 1.708 | v (m3/kg) | 1.258 | v (m3/kg) | 0.08842 | v (m3/kg) | 0.10178 | v (m3/kg) | 0.14384 | v (m3/kg) | 0.5356 | v (m3/kg) | 0.5667 | v (m3/kg) | 2.954 |
| h (kJ/kg) | 238.02 | h (kJ/kg) | 269.27 | h (kJ/kg) | 769.27 | h (kJ/kg) | 894.27 | h (kJ/kg) | 1304.85 | h (kJ/kg) | 804.85 | h (kJ/kg) | 854.85 | h (kJ/kg) | 412.91 |
| Pr | 0.61794 | Pr | 0.94907 | Pr | 37.7 | Pr | 64.94 | Pr | 257.34 | Pr | 44.32 | Pr | 55.05 | Pr | 4.218 |

p2 cont

- Find $v_1$:

$$v_1 = \frac{RT_1}{P_1} = \frac{(.287)(238)}{40} = 1.708 \ m^3/kg$$

- Find $h_1$:

$$h_1 = \frac{(238-230)(240.02-230.02)}{(240-230)} + 230.02$$

$$h_1 = 238.02 \ kJ/kg$$

- Find $P_{r_1}$:

$$P_{r_1} = \frac{(238-230)(.6355-.5477)}{(240-230)} + .5477$$

$$P_{r_1} = .61794$$

- Find $h_2$ neglecting $V_2$:

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \rightarrow 0$$

$$h_2 = h_1 + \frac{V_1^2}{2}$$

$$h_2 = 238.02 + \frac{250^2 \ m/s}{2} \times \frac{kJ/kg}{1000 \ m^2/s^2}$$

$$h_2 = 269.27 \ kJ/kg$$

- Find $T_2$ using interpolation:

$$T_2 = \frac{(269.27-260.09)(270-260)}{(270.11-260.09)} + 260$$

$$T_2 = 269.16 \ K$$

- Find $P_{r_2}$ using interpolation:

$$P_{r_2} = \frac{(269.27-260.09)(.9590-.8405)}{(270.11-260.09} + .8405$$

$$P_{r_2} = .94907$$

- Find $P_2$:

$$\frac{P_2}{P_1} = \frac{P_{r_2}}{P_{r_1}}$$

$$P_2 = P_1\left(\frac{P_{r_2}}{P_{r_1}}\right) = 40\left(\frac{.94907}{.61794}\right)$$

$$P_2 = 61.43 \ kPa$$

- Find $v_2$:

$$v_2 = \frac{RT_2}{P_2} = \frac{(.287)(269.16)}{61.43} = 1.258$$

(P2 cont)

- Find $h_3$ using $W_{in} = 500\ kJ/kg$

$$W_{in} = h_3 - h_2$$

$$h_3 = W_{in} + h_2 = 500 + 269.27$$

$$h_3 = 769.27\ kJ/kg$$

- Find $T_3$ using interpolation:

$$T_3 = \frac{(769.27 - 767.29)(760 - 750)}{(778.18 - 767.29)} + 750$$

$$T_3 = 751.82K$$

- Find $P_{r_3}$ using interpolation:

$$P_{r_3} = \frac{(769.27 - 767.29)(39.27 - 37.35)}{(778.18 - 767.29)} + 37.35$$

$$P_{r_3} = 37.70$$

- Find $P_3$

$$\frac{P_3}{P_2} = \frac{P_{r_3}}{P_{r_2}}$$

$$P_3 = P_2\left(\frac{P_{r_3}}{P_{r_2}}\right) = 61.43\left(\frac{37.70}{.94907}\right)$$

$$P_3 = 2440.19\ KPa$$

- Find $v_3$:

$$V_3 = \frac{RT_3}{P_3} = \frac{(.287)(751.82)}{2440.19} = .08842$$

- Find $h_{3a}$:

$$\eta_c = \frac{h_3 - h_2}{h_{3a} - h_2}$$

$$h_{3a} = \frac{h_3 - h_2}{\eta_c} + h_2$$

$$h_{3a} = \frac{769.27 - 269.27}{.80} + 269.27$$

$$h_{3a} = 894.27$$

P2 cont

- find $T_{3a}$ using interpolation

$$T_{3a} = \frac{(894.27-888.27)(880-860)}{(910.56-888.27)} + 860$$

$$T_{3a} = 865.38\ K$$

- find $v_{3a}$:

$$v_{3a} = \frac{RT_{3a}}{P_{3a}} = \frac{(.287)(865.38)}{2440.19} = .10178$$

- find $P_{r3a}$ using interpolation:

$$P_{r3a} = \frac{(894.27-888.27)(68.98-63.09)}{(910.56-888.27)} + 63.09$$

$$P_{r3a} = 64.94$$

- Find $v_4$:

$$v_4 = \frac{RT_4}{P_4} = \frac{(.287)(1223)}{2440.19} = .14384$$

- Find $h_4$ using interpolation:

$$h_4 = \frac{(1223-1220)(1324.93-1301.31)}{(1240-1220)} + 1301.31$$

$$h_4 = 1304.85\ kJ/kg$$

- Find $P_{r4}$ using interpolation:

$$P_{r4} = \frac{(1223-1220)(272.3-254.7)}{(1240-1220)} + 254.7$$

$$P_{r4} = 257.34$$

- Find $h_5$ using $W_{Turbine}$:

$$W_T = h_4 - h_5$$

$$h_5 = h_4 - W_T = 1304.85 - 500$$

$$h_5 = 804.85\ kJ/kg$$

- Find $T_5$ using interpolation:

$$T_5 = \frac{(804.85-800.03)(800-780)}{(821.95-800.03)} + 780$$

$$T_5 = 784.4$$

- Find $P_{r5}$ using interpolation:

$$P_{r5} = \frac{(804.85-800.03)(47.75-43.35)}{(821.95-800.03)} + 43.35$$

$$P_{r5} = 44.32$$

(P2 cont)

- Find $P_5$:

$$\frac{P_5}{P_4} = \frac{P_{r5}}{P_{r4}}$$

$$P_5 = P_4\left(\frac{P_{r5}}{P_{r4}}\right) = 2440.19\left(\frac{44.32}{257.34}\right)$$

$$P_5 = 420.29 \text{ kPa}$$

- Find $v_5$:

$$v_5 = \frac{RT_5}{P_5} = \frac{(.287)(784.4)}{420.29} = .5356$$

- Find $h_{5a}$

$$\eta_T = \frac{h_4 - h_{5a}}{h_4 - h_5}$$

$$h_{5a} = h_4 - \eta(h_4 - h_5)$$

$$h_{5a} = 1304.85 - .9(1304.85 - 804.85)$$

$$h_{5a} = 854.85 \text{ kJ/kg}$$

- Find $T_{5a}$ using interpolation:

$$T_{5a} = \frac{(854.85 - 843.98)(840 - 820)}{(866.08 - 843.98)} + 820$$

$$T_{5a} = 829.84 \text{ K}$$

- Find $P_{r5a}$ using interpolation:

$$P_{r5a} = \frac{(854.85 - 843.98)(57.60 - 52.59)}{(866.08 - 843.98)} + 52.59$$

$$P_{r5a} = 55.05$$

- Find $v_{5a}$:

$$v_{5a} = \frac{RT_{5a}}{P_{5a}} = \frac{(.287)(829.84)}{420.29} = .5667$$

- Find $P_{r6}$:

$$\frac{P_6}{P_5} = \frac{P_{r6}}{P_{r5}} \qquad P_{r6} = P_{r5}\left(\frac{P_6}{P_5}\right) = 44.32\left(\frac{40}{420.29}\right)$$

$$P_{r6} = 4.218$$

P2 Cont

- Find $T_6$ using interpolation:

$$T_6 = \frac{(4.218-4.153)(420-410)}{(4.522-4.153)} + 410$$

$T_6 = 411.76\ K$

- Find $h_6$ using interpolation:

$$h_6 = \frac{(4.218-4.153)(421.26-411.12)}{(4.522-4.153)} + 411.12$$

$h_6 = 412.91\ kJ/kg$

- Find $v_6$: $v_6 = \frac{RT_6}{P_6} = \frac{(.287)(411.76)}{40} = 2.954$

a) Pressure at turbine exit $P_5 = 420.29\ kPa$

b) Find velocity of gases at nozzle exit.

$$h_5 + \frac{V_5^2}{2} \approx 0 = h_6 + \frac{V_6^2}{2}$$

$$h_5 = h_6 + \frac{V_6^2}{2}$$

$$\sqrt{2000(h_5 - h_6)} = \sqrt{2000(804.85-412.91}$$

$$\boxed{V_6 = 885.37\ m/s}$$

c) Find Thrust:

$$Thrust = \dot{m}(V_{exit} - V_{inlet})$$

$$\rho_{air} = \frac{P}{RT} = \frac{40\ kPa}{(.287)(238K)} = .5856\ kg/m^3$$

$$\dot{m} = \rho V A = (.5856\ \tfrac{kg}{m^3})(250\ \tfrac{m}{s})(1.257m^2) = 184\ \tfrac{kg}{s}$$

$$Thrust = 184\ \tfrac{kg}{s}(885.37\ \tfrac{m}{s} - 250\ \tfrac{m}{s})$$

$$\boxed{Thrust = 116908\ kg\,m/s^2}$$

or

116.9 kN

P2 cont

Summary:

Using the given characteristics of a jet engine, it was found that the pressure at the turbine exit was 420.29 kPa, the Velocity of the gases at the nozzle exit was 885.37 m/s and it produced 116.9 kN of thrust.
The compressor produced a large amount of pressure and the temperature was increased to 1223 K before being expanded through the turbine.

Materials:

Air as an ideal gas

Analysis:

A Jet engine is an effective way to create thrust just using the air entering the engine and internal components. The engine's efficiency would depend on how much the air was compressed and the temperature of combustion.