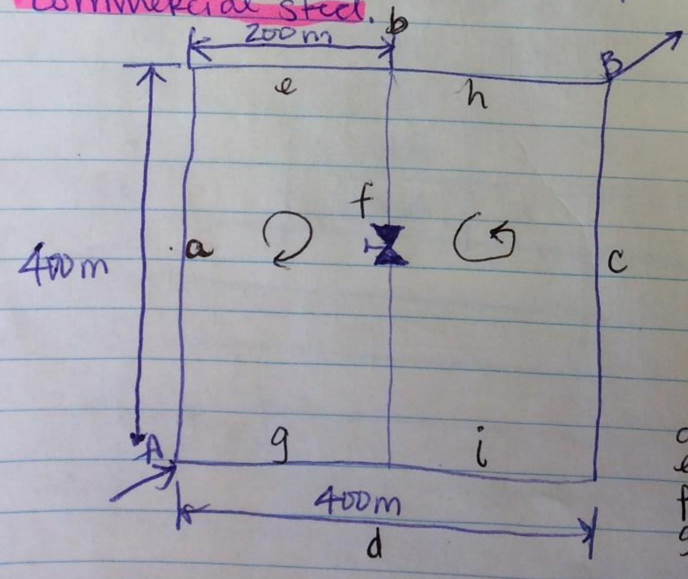
1. In the system shown in the figure below the water flow entering & leaving the system is 100 gal/min. The temperature of the water is 20°C. All pipes are of 1 inch inner diameter. As you see, the piping system forms a square 400m on each side with a pipe with a closed valve that connects the system as shown. Calculate the pressure drop between points A & B. Check the maximum velocity criterion ($V_{max} = 3 m/s$). Is it violated? If so, what would you do?

Now, it was decided to completely open the valve shown. Calculate the new pressure drop between points A & B if the flow in & out of the system remains 100 gal/min. Does the criterion of speed in the pipes get violated in this configuration?

Assume that the friction factor is only a function of the Relative Roughness. Neglect minor losses. The material of the pipes is commercial steel.



Round 1

1 circuit

a
b
c
d

Round 2

2 circuits

a
e
f
g

f
h
c
i

$100\ gal/min \cdot \frac{1\ ft^3}{7.48\ gal} \cdot \frac{1\ min}{60\ sec.}$ $\frac{100\ gal}{min} \cdot \frac{1\ min}{60s} \cdot \frac{1\ m^3}{264.2\ gal}$

$Q = 0.006308\ m^3/s$

$Q = 0.2228\ ft^3/s$ $Q = Av$

Water $\frac{Q}{A} = v$ $v = 40.851\ ft/s$

Temp = 20°C

1" diameter pipes $A = \frac{(0.0833)^2 \pi}{4} = 0.005454\ ft^2$

$1\ in. \cdot \frac{1\ m}{39.37\ in} = \boxed{0.0254\ m}$ — D $A = \frac{(0.0254)^2(\pi)}{4} = 0.0005067$

A.

L Circuit

pipe A → 400m $\frac{0.006308}{0.0005067} = v = 12.45\ m/s$

All pipes have the same diameter

Circuit 1

Pipe	length (m)	D (m)	e (m)
a	400	0.0254	4.6×10^{-5}
b	400	0.0254	"
c	400	0.0254	"
d	400	0.0254	"

water @ 20°C

$\eta = 1.02 \times 10^{-3}\ Pa \cdot s$

$\nu = 1.02 \times 10^{-6}\ m^2/s$

$\rho = 998\ kg/m^3$

$\gamma = 9.79\ kN/m^3$

$h = kQ^2$

$$k_i = \frac{8(L_i + \sum L_{e_i}) f_i}{\pi^2 g D_i^5}$$

First, only one circuit was set up (see Part 1 spreadsheet).
This was named circuit 1.

10, 25, 10, & 15 gal/min were set up (a, b, c, d respectively)
These were just guesses. Then, I subtracted & added ΔQ as needed based on % error.

Second, two circuits were set up (see Part 2 spreadsheet).
These were named circuit 2 & circuit 3 to differentiate from part 1.

$a \rightarrow 10$ gal/min
$e \rightarrow 25$
$f \rightarrow 10$
$g \rightarrow 15$

$f \rightarrow 10$
$h \rightarrow 20$
$c \rightarrow 50$
$i \rightarrow 5$

I knew they'd need to have numbers smaller than 100 gal/min, but they were guesses. I again added & subtracted ΔQ as needed based on % error.

To get Q into m^3/s, I divided by 264.2 and again by 60.

Using iterations, the Q values for the one-circuit system (see pipe "names" on diagrams) are as follows:

~~actual velocities~~

$Q_a = 0.00184\ m^3/s$ $\quad V_a = \frac{0.00184}{0.005454} = \underline{0.337\ m/s}$

$Q_b = {}^-0.00226\ m^3/s$ $\quad V_b = {}^-0.414\ m/s$

$Q_c = 0.00184\ m^3/s$ $\quad V_c = 0.337\ m/s$

$Q_d = {}^-0.00131\ m^3/s$ $\quad V_d = {}^-0.240\ m/s$

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

$$\frac{P_1}{9.79\,kN/m^3} + \phi + \frac{(V_a+V_b)^2}{2(9.81\,m/s^2)} = \frac{P_2}{9.79\,kN/m^3} + 400m + \frac{(V_c+V_d)^2}{2(9.81)}$$

$$\frac{P_1 - P_2}{9.79\,kN/m^3} = 400m + \frac{(-0.077)^2}{19.62\,m/s^2} - \frac{(0.097)^2}{19.62\,m/s^2}$$

$0.0003022 - 0.004796$

$$P_1 - P_2 = (400)(9.79)$$

$$\boxed{P_1 - P_2 = 3916\ Pa}$$

No, velocity criterion is NOT violated

Using iterations, the Q values for the two circuit system (see pipe "names" on diagram) are as follows:

$Q_a = {}^-0.000327\ m^3/s$ $\quad V = {}^-0.060\ m/s$ $\quad Q_f = {}^-0.00123$ $\quad V = {}^-0.226\ m/s$

$Q_e = {}^+0.000619\ m^3/s$ $\quad V = 0.113\ m/s$ $\quad Q_h = {}^-0.000596$ $\quad V = {}^-0.109\ m/s$

$Q_f = {}^-0.000327\ m^3/s$ $\quad V = {}^-0.060\ m/s$ $\quad Q_c = 0.00130$ $\quad V = 0.238\ m/s$

$Q_g = 0.000211\ m^3/s$ $\quad V = 0.0387\ m/s$ $\quad Q_i = {}^-0.00154$ $\quad V = {}^-0.282\ m/s$

Velocity criterion is NOT violated.

$$\frac{P_1 - P_2}{9.79} = 400 + \frac{-(V_f + V_i + V_c + V_h)^2}{19.62} - \frac{(V_a + V_e + V_f + V_g)^2}{19.62}$$

$$= 400 - 0.000000218 - 0.00005128$$

$$P_1 - P_2 = (399.9999)(9.79)$$

$$P_1 - P_2 = \boxed{3915.999\ Pa}$$

[redacted]

2. One day you decided to jump out of an airplane with a parachute. When you reach stability, your weight will be equal to the drag force of the air against the parachute. If you are falling at 5m/s, calculate the parachute diameter. Assume the air temperature to be 40°C.

$$F_D = C_D \frac{1}{2} \rho V^2 \underbrace{A}_{\frac{\pi D^2}{4}}$$

Assuming

One circular disk

$C_D = 1.11$

Hemispherical cup, open front

$C_D = 1.35$

$\rho_{air} @ 40°C = 1.127\ kg/m^3$

$V = 5\ m/s$

$\gamma_{air} @ 40°C = 11.05\ N/m^3$

$\eta_{air} @ 40°C = 1.91 \times 10^{-5}\ Pa \cdot s$

$\nu_{air} @ 40°C = 1.69 \times 10^{-5}\ m^2/s$

$122\ lb = 55.338\ kg$

$55.338\ kg = (1.11)\frac{1}{2}(1.127\ kg/m^3)(5\ m/s)^2 \frac{\pi D^2}{4}$

$55.338 = 12.281\ D^2$

$4.506 = D^2$

$\boxed{D = 2.12\ m}$

4. At the end of a steel pipe ($E = 2 \times 10^7\ N/cm^2$) of internal diameter $D = 600mm$, & thickness $\delta = 10mm$, there is a valve. The water velocity in the pipe is $V = 2.5\ m/s$. Calculate the pressure increment when the valve closes all of a sudden. Water compressibility module is $E_0 = 2.03 \times 10^5\ N/cm^2$.

$$C = \frac{\sqrt{\frac{E_0}{\rho}}}{\sqrt{1 + \frac{E_0 D}{E \delta}}}$$

$\Delta P = \rho C V$

$D = 6m$

$\delta = 0.01m$

$\rho = 997\ kg/m^3$ for 25°C (room temp)

$20000000 \frac{N}{cm^2} \cdot \frac{100cm}{1m} \cdot \frac{100}{1}$

$E = 2 \times 10^{11}\ N/m^2$

$E_0 = 2.03 \times 10^9\ N/m^2$

$V = 2.5 m/s$

$$C = \frac{\sqrt{\frac{2030000000}{997}}}{\sqrt{1 + \frac{(2030000000)(6)}{(200000000000)(0.01)}}}$$

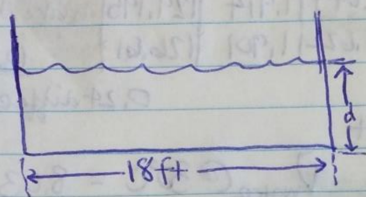
$$\frac{1.218 \times 10^{10}}{2000000000} = 6.09$$

$$C = \frac{1426.92}{\sqrt{1 + 6.09}} = \frac{1426.92}{2.66}$$

$$\boxed{C = 536.436}$$

$$\Delta P = \rho C v = (997)(536.436)(2.5) = \boxed{1{,}337{,}066.73\ Pa}$$

5. A rectangular unfinished concrete channel with a width of 18ft conveys water at 350 ft³/s. The temperature of the water is at 30°C. The channel has a slope of 0.001. Determine the liquid depth for uniform flow. Is the flow supercritical or subcritical?



350 ft³/s

slope = 0.001

$$Q = Av = \left(\frac{1.49}{n}\right) A R^{2/3} S^{1/2}$$

$S \to 0.001$

$$R = \frac{A}{WP} = \frac{(18)d}{18+2d}$$

$n = 0.017$

$$350 = \left(\frac{1.49}{0.017}\right)(18d)\left(\frac{18d}{18+2d}\right)^{2/3}(0.001)^{1/2}$$

$$350 = (87.647)(18d)\left(\frac{18d}{18+2d}\right)(0.0316)$$

$$350 = (2.7696)(18d)\left(\frac{18d}{18+2d}\right)^{2/3}$$

$$\frac{126.37}{\cancel{18}} = \frac{18d\left(\frac{18d}{18+2d}\right)^{2/3}}{\cancel{18}}$$

$$\cancel{7.0206 = d\left(\frac{18d}{18+2d}\right)^{2/3}}$$

$$\cancel{18.602 = d^{3/2}\,\frac{18d}{18+2d}} \rightarrow \frac{18d^{5/2}}{18+2d}$$

$$\cancel{334.836 + 37.204d = 18d^{5/2}}$$

$$126.37 = 18d\left(\frac{18d}{18+2d}\right)^{2/3}$$

d (~~ft~~)	A (~~ft~~^{2})	WP (~~ft~~)	R (~~ft~~)	$R^{2/3}$	$AR^{2/3}$	Required change in d.
2.0	36.0	22.0	1.636	~~1.636~~ 1.388	49.968	make d higher
4.0	72.0	26.0	2.769	1.972	141.98	make d lower
3.5	63.0	25.0	2.52	1.852	116.676	make d higher
3.75	67.5	25.5	2.647	1.914	129.195	make d lower 2.2%
3.7	66.6	25.4	2.622	1.901	126.61	0.19%

0.24 difference

the water is about
3.7 ~~ft~~ deep

ν_{water} @ 30°C = 8.03×10^{-7}

$N_F = \dfrac{v}{\sqrt{g y_h}}$ $\quad y_h = \dfrac{A}{T}$ $\quad T = 18ft \quad y_h = 3.7$

$A = 66.6 ft^2$

$N_R = \dfrac{vR}{\nu} =$

$N_F = \dfrac{\cancel{2.25}\ 5.255 ft/s}{\sqrt{(32.2)(3.7)}}$

$N_F = \dfrac{5.255}{\sqrt{119.14}} = \dfrac{5.255}{10.915} = \boxed{0.481}$

$Q = Av$
$350 \frac{ft^3}{s} = 66.6 ft^2\ v$
$v = 5.255$ ft/s

$R_e\# = \dfrac{(5.255)(2.622)}{0.000000803} = \underline{17158916.56}$

$N_F < 1.0 \rightarrow$ subcritical
$R_e\# > 2000 \rightarrow$ turbulent

65.7 25.3 2.597 1.889

3.69 66.42 25.38 2.617 1.899 126.13