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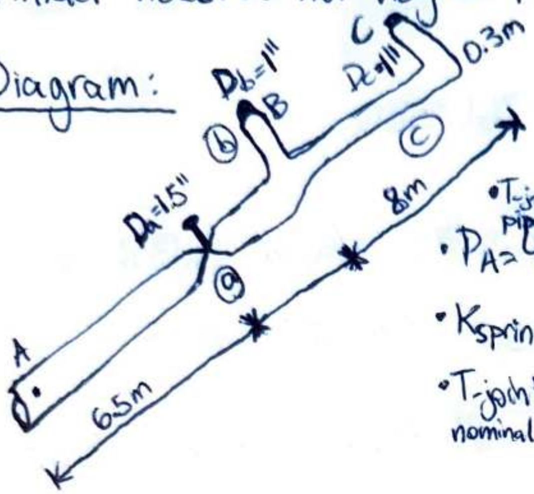
MET 330 - Test 3

*Please grade Question 2 towards this test.

1) Purpose: For a wide-open ball valve, determine the flow rate delivered to each sprinkler head. Do not neglect minor losses.

Diagram:



Data & Variables

- A to T-joint: $1\frac{1}{2}$ in nominal pipe of 6.5m
- T-joint to 1st sprinkler head: 1in nominal pipe of 0.3m
- $P_A = 400$ kPa (gauge)
- $K_{sprinkler} = 50$
- T-joint to 2nd sprinkler head: 1in nominal pipe of 8.3m

Sources:

Mott & Unlener. "Applied Fluid Mechanics", 7th edition, Pearson Education Inc (2015)

Design Considerations:

- Incompressible Fluid
- Steady state
- Isothermal

Materials:

Water flowing in Schedule-40 steel pipe

Procedure & Calculations:

Applying Bernoulli between A & B:

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + \cancel{Z_A} = \cancel{\frac{P_B}{\gamma}}^{(gauge)} + \frac{V_B^2}{2g} + Z_B + (h_L)_{A-B}$$

$$\frac{P_A}{\gamma} - Z_B = \frac{V_B^2 - V_A^2}{2g} + (h_L)_{A-B} \rightarrow (1)$$

$(h_L)_{A-B} = (h_L)_{pipe-a} + (h_L)_{pipe-b} + (h_L)_{valve} +$

$(h_L)_{T-joint} + (h_L)_{sprinkler} + (h_L)_{reduction}$

$$(h_L)_{pipe-a} = f_a \frac{L_a}{D_a} \frac{V_a^2}{2g}$$

$$= f_a\left(\frac{L_a}{D_a}\right)\frac{1}{2g}\left(\frac{Q_a}{A_a}\right)^2$$

$$= \frac{8 f_a L_a Q_a^2}{\pi^2 g D_a^5}$$

$$= \frac{8 f_a (6.5) Q_a^2}{\pi^2 (9.81)(40.9\times10^{-3})^5}$$

$$= 4692648.781 f_a Q_a^2$$

$$(h_L)_{pipe-b} = \frac{8 f_b L_b Q_b^2}{\pi^2 g D_b^5}$$

$$= \frac{8 f_b (0.3) Q_b^2}{\pi^2 (9.81)(26.6\times10^{-3})^5}$$

$$= 1861376.839 f_b Q_b^2$$

$$(h_L)_{valve} = K_{valve}\left(\frac{V_a^2}{2g}\right) = \frac{K_{valve}}{2g}\left(\frac{4Q_a}{\pi D_a^2}\right)^2$$

$$= 340 \times 0.02 \frac{8}{\pi^2 \times 9.81} \times \frac{1}{(40.9\times10^{-3})^4} Q_a^2$$

$$= 200787.612 Q_a^2$$

$$(h_L)_{T-joint} = K_T \times \frac{V_a^2}{2g} = 35433.108 Q_a^2$$

$$(h_L)_{reduction-b} = K_{elb} \times \frac{V_b^2}{2g}$$

$$= 0.14 \times \frac{8}{\pi^2 \times 9.81} \times \frac{Q_b^2}{(26.6\times10^{-4})^4}$$

$$= 23105.891 Q_b^2$$

$$(h_L)_{sprinkler} = K_{spr} \times \frac{8Q_b^2}{\pi^2 g D_b^4}$$

$$= 50 \times \frac{8Q_b^2}{\pi^2 \times 9.81 \times (26.6\times10^{-3})^4}$$

$$= 8252103.988\, Q_b^2$$

$$(h_L)_{pipe\text{-}c} = \frac{8 f_c L_c Q_c^2}{\pi^2 g D_c^5} = \frac{0.029 \times 8 \times 8.3 \times Q_c^2}{(26.6\times10^{-3})^5 \pi^2 \times 9.81}$$

$$= 1493444.684\, Q_c^2$$

$$(h_L)_{To\ bend} = K_{rec} \times \frac{8Q_a^2}{\pi^2 g D_a^4}$$

$$= \frac{20 \times 0.02 \times 8Q_a^2}{\pi^2 \times 9.81 \times (40.9\times10^{-3})^4}$$

$$= 11811.036\, Q_a^2$$

$$(h_L)_{red\ c} = K_{red} \times \frac{8Q_c^2}{\pi^2 g D_c^4}$$

$$= \frac{0.14 \times 8 Q_c^2}{\pi^2 \times 9.81 \times (26.6\times10^{-3})^4}$$

$$= 23105.891\, Q_c^2$$

$$(h_L)_{elbow} = \frac{30 \times 0.022 \times 8Q_c^2}{\pi^2 \times 9.81 \times (26.6\times10^{-3})^4}$$

$$= 108927.773\, Q_c^2$$

$$(h_L)_{Spri} = 8252103.988\, Q_c^2$$

$$40.599 = \frac{V_c^2 - V_a^2}{2g} + (h_L) \quad \Bigg| \quad V_c = \frac{4Q_c}{\pi D_c^2}, \quad V_A = \frac{4Q_a}{\pi D_a^2}$$

Substitute in Bernoulli's equ. 1

$$\frac{P_A}{\gamma} - Z_B = \frac{V_B^2 - V_A^2}{2g} + (h_L)_{A\text{-}B}$$

$$V_B = \frac{4Q_b}{\pi D_b^2} = 1799.479\, Q_b$$

$$V_A = \frac{4Q_a}{\pi D_a^2} = 761.138\, Q_a$$

$$\frac{P_A}{\gamma} - Z_B = 4692648.781 f_a Q_a^2 + 1861376.839 f_b Q_b^2 + 206693.144 Q_a^2 + 8440251.958 Q_b^2$$

$$40.599 = 8510984.278 Q_b^2 + 324009.3635 Q_a^2 \rightarrow (2)$$

Bernoulli's between A-C:

$$\frac{P_A}{\gamma} - Z_C = \frac{V_C^2 - V_A^2}{2g} + (h_L)_{pipe\text{-}a} + (h_L)_{pipe\text{-}c} + (h_L)_{valve} + (h_L)_{T\text{-}bend} + (h_L)_{red\ c} + (h_L)_{elb} + (h_L)_{spri\text{-}c} \rightarrow (3)$$

$$40.599 = 165042.079 Q_c^2 - 29527.576 Q_a^2 + 1493444.684 Q_c^2 + 200787.612 Q_a^2 + 11811.036 Q_a^2 + 23105.891 Q_c^2 + 108927.773 Q_c^2 + 8252103.988 Q_c^2$$

$$40.599 = 300387.2915 Q_a^2 + 10042624.42 Q_c^2 \rightarrow (4)$$

From 2, 4 we have 2 equations & 3 unknowns and $Q_a = Q_b + Q_c \rightarrow (3)$

2, 3, 4

$\rightarrow Q_b = 0.002\ m^3/s$

$Q_c = 0.0019\ m^3/s$

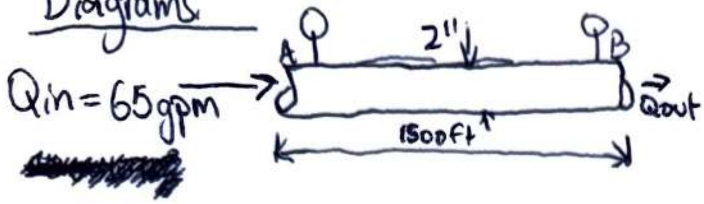
$Q_a = 0.0039\ m^3/s$

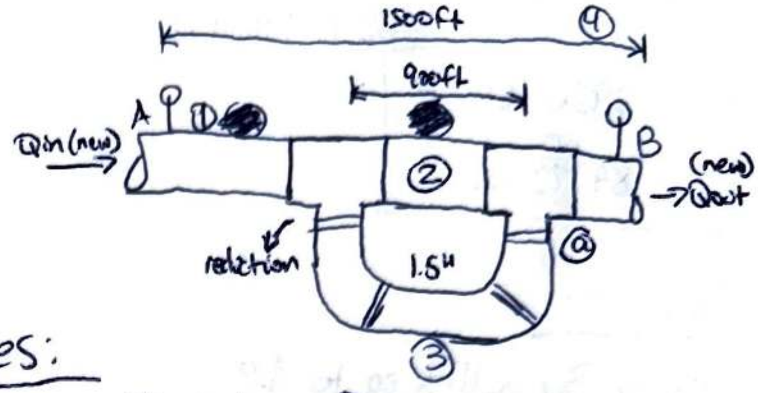
Analysis:

The flows through sprinklers are very similar. There is a difference of 5.26%, so no need to modify the system. The flow velocity varies from 3.02 m/s to 3.69 m/s (for 1st sprinkler). We should use a larger pipe in path (b) but that will increase the flow rate towards 1st sprinkler. We could either leave pipe (b) as it is a increase in size, but use a valve in pipe (b) to control flow.

2) Purpose: Determine the pressure drop & expected increase in flow rate after modification

Diagrams:





Data & Variables:

- Length of standard steel tubing = 1500ft
- D (inner diameter of 2in tube) = 0.1558ft
- g (acceleration due to gravity) = 32.2ft/s
- Q (flow rate) = 65 gpm = 0.144831 cubic ft/s

Sources:

Mott & Untener. "Applied Fluid Mechanics", 7th edition Pearson Education Inc (2015)

Materials:

water flowing through standard steel tubing

Design Considerations:

- Incompressible state
- Steady state
- Isothermal

Procedure & Calculations:

For case 1:

Applying Bernoulli's equation between A&B:

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B + h_L \quad (1)$$

As, flow rate is same throughout tube & diameter is also the same. $V_A = V_B$, $Z_A = Z_B$ (elevation change)

$$\frac{P_A - P_B}{\rho g} = h_L \rightarrow \boxed{\Delta P = \rho g h_L} \quad (2)$$

Cross sectional area of tube:

$$A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 0.1558^2$$

$$A = 0.01906\ ft^2$$

By continuity equation:

Flow rate, $Q = A \times V$

$$V = \frac{Q}{A} = \frac{0.144831}{0.01906}$$

$$\boxed{V = 7.5147 ft/sec}$$

Reynolds number:

$$Re = \frac{VD}{\nu} = \frac{7.5947 \times 0.1558}{1.89 \times 10^{-5}}$$

(Kinematic viscosity $= 1.89 \times 10^{-5}\ ft^2/sec$)

$$\boxed{Re = 62,606.1}$$

$$\text{Relative roughness} = \frac{D}{\varepsilon} = \frac{0.1558}{1.5 \times 10^{-4}} = 1038.67$$

(For steel pipe, $\varepsilon = 1.5 \times 10^{-4}$ ft)

$$\boxed{\text{Relative Roughness} = 1038.67}$$

From Moody Chart, Friction Factor, $f = 0.024$

$$\text{Head loss, } h_L = f\left(\frac{L}{D}\right)\frac{V^2}{2g}$$

$$= 0.024\left(\frac{1500}{0.1558}\right)\left(\frac{7.5947^2}{2 \times 32.2}\right)$$

$$\boxed{h_L = 206.952 ft}$$

From eq. (2):

Pressure drop (ΔP) $= sg\, h_L$

$= 62.43 \times 206.952$

$$\boxed{\Delta P = 12920\ lbf/ft^2 \text{ or } 89.72\ psi}$$

Analysis:

Rate of flow through sections 1 & 4 will remain unchanged as initial.

Case 2:

Applying Bernoulli's eq. to A-B:

$$\frac{P_A}{sg} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{sg} + \frac{V_B^2}{2g} + Z_B + h_L$$

$(\because Z_A = Z_B \ \&\ V_A = V_B)$

$$\frac{P_A - P_B}{sg} = h_L \longrightarrow (3)$$

Energy loss @ branch 2:

$$\frac{\Delta P}{sg} = f_2\left(\frac{L}{D}\right)\left(\frac{V_2^2}{2g}\right) + 2\left[20 f_T \frac{V_2^3}{2g}\right] \longrightarrow (4)$$

As, $Q_2 = A_2 V_2 \rightarrow V_2 = \frac{Q_2}{A_2}$

$$\frac{\Delta P}{sg} = \left[f_2\left(\frac{1500}{0.1558}\right) + 2(20)(0.019)\right] \times \frac{1}{2(32.2)} \frac{1}{(0.01906)^2} \times Q_2^2$$

$$Q_2 = \sqrt{\left(\frac{0.0234\,\Delta P}{sg}\right)\left(\frac{1}{f_2(9627.7)+0.76}\right)}$$

Let us assume $f_2 = 0.01$

$$Q_2 = \sqrt{\frac{(0.0234)(12920)}{62.43}\left(\frac{1}{0.01 \times (9627.7)+0.76}\right)}$$

$$\boxed{Q_2 = 0.223395\ ft^3/s \text{ or } 100.266\ gpm}$$

Increased flow rate for section 2:

Similarily for section 3,

$$\frac{\Delta P}{sg} = h_{L3} + 2(h_L)_{TEE} + (h_L)_{reduction} + (2 h_{L\,elbow}) + h_{L\,(enlargement)}$$

$$\Rightarrow \frac{\Delta P}{sg} = f_3\left(\frac{L}{D}\right)\frac{V_3^2}{2g} + 2(60 f_T)\frac{V_3^2}{2g} + 0.045\left(\frac{V_3^3}{2g}\right) + 2(30 f_T)\frac{V_3^2}{2g} + 0.36\left[\frac{V_3^2}{2g}\right]$$

$$\Rightarrow \frac{\Delta P}{sg} = \frac{V_3^2}{2g}\left[f_3\left(\frac{L}{D}\right) + 2(60(0.022)) + 0.045 + 2(30)(0.022) + 0.36\right]$$

$$\Rightarrow \frac{\Delta P}{sg} = \frac{1}{2g A_3^2} Q_3^2 \left[f_3\left(\frac{900}{0.1142}\right) + 4.365\right]$$

$$= \frac{1}{2(32.2)(0.0123)^2} Q_3^2 \left[f_3(7880.9) + 4.365\right]$$

$\left[\because A_3 = \frac{\pi}{4} \times d_3^2 \ \&\ d_3 = 1.5\,in = 0.125\,ft \right.$

$\left. A_3 = \frac{\pi}{4} \times (0.125)^2 = 0.0123\,ft^2 \right]$

$$\Rightarrow Q_3 = \sqrt{\left(\frac{0.00675\,\Delta P}{sg}\right)\left(\frac{1}{f_3(7880.9)+4.365}\right)}$$

Let us assume $f_3 = 0.01$

$$Q_3 = \sqrt{\frac{0.00675 \times 12920}{62.4 \times [0.01 \times 7880.9 + 4.365]}}$$

$$\boxed{Q_3 = 0.129596\,ft^3/s \text{ or } 58.166724\ gpm}$$

Increased flow rate through section 3