Purpose: Specify pump requirements for the figure in the drawing.

The company requires a minimum flow of 275 gpm. Assume pump efficiency of 70%

With the specified pump, calculate the total flow rate if the gate valve on the bypass is 1/4 open, 1/2 open, 3/4 open, and fully open.

Sources: Mott & Untener. Applied Fluid Mechanics 7th Edition. Pearson. 2015

Design Considerations:

1. Incompressible fluid - Water @ 160°F
2. All pipes Schedule 40 steel
3. Tees will have same K-factor as in book
4. Tank and discharge are open to atmosphere.

Data & Variables:

$\gamma_{water} = 61.0 (lb/ft^3)$

$Q = 275 gpm = 0.612 ft^3/s$

$T = 160°F$

$\nu = 4.38 \times 10^{-6} (ft^2/s)$

$L_{suction} = 10 ft$

$L_{discharge} = 40 ft$

$\varepsilon_{steel} = 1.5 \times 10^{-4}$

- 4 in pipe
  - $D_1 = 0.336 ft$
  - $A_1 = 0.0884 ft^2$
  - $f_T = 0.016$
  - $V_1 = 6.923 ft/s$
- 3 in pipe
  - $D_2 = 0.256 ft$
  - $A_2 = 0.05132 ft^2$
  - $f_T = 0.017$
  - $V_2 = 11.925 ft/s$
- 1 in pipe
  - $D_3 = 0.087 ft$
  - $A_3 = 0.006 ft^2$

K values
- gate valve - $K = 8 f_T$
- Check value - $K = 100 f_T$
- Tee - $K = 60 f_T$
- Entrance - $K = 0.5$
- Exit - $K = 1.0$
- Heat exchanger - $K = 12$

$g = 32.2 ft/s^2$

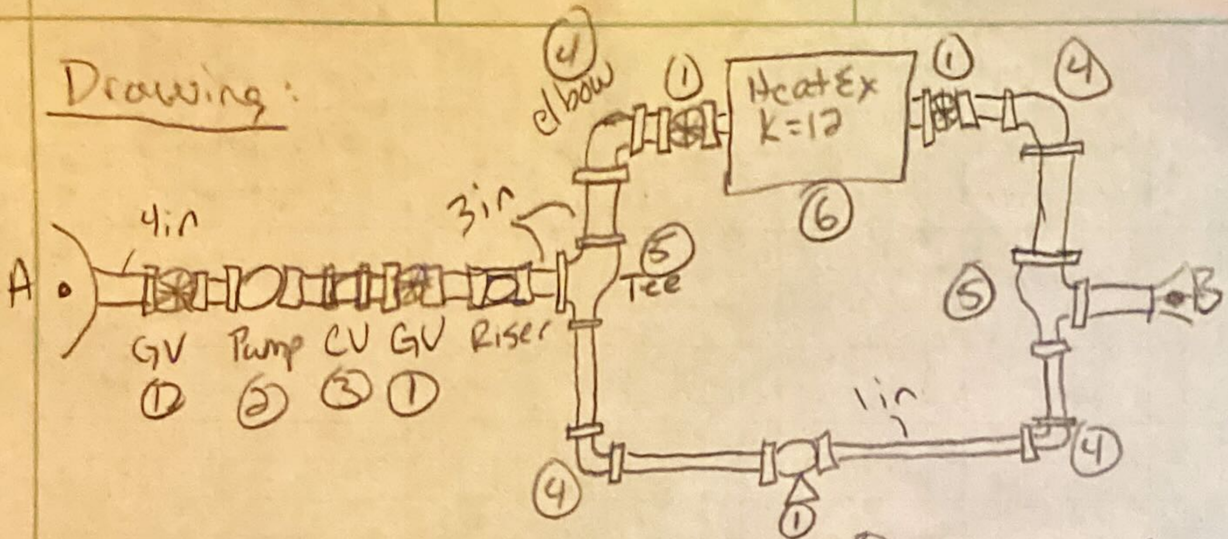
$$\text{Pump Power } P = \frac{\gamma Q h_A}{\eta}$$

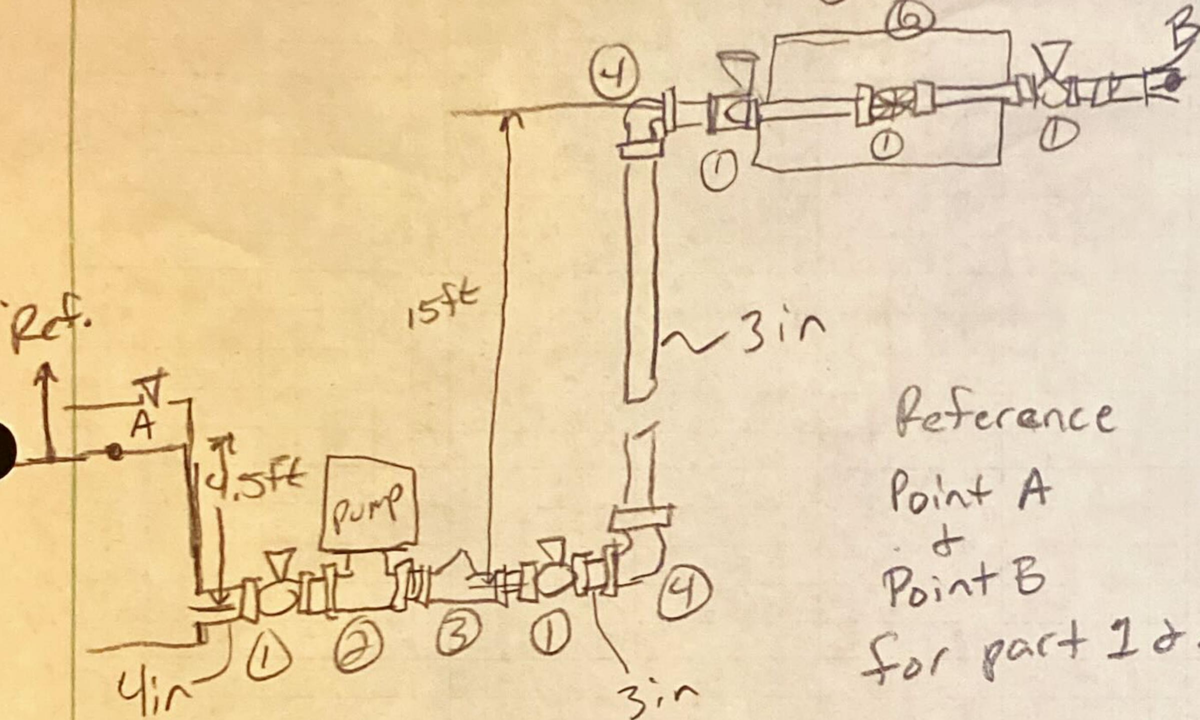
$\eta = 0.70$

$$\frac{L}{D}\Big|_{suction} = \frac{10}{0.336} = 29.76 ft$$

$$\frac{L}{D}\Big|_{discharge} = \frac{40}{0.256} = 156.25 ft$$

Drawing:





Referance
Point A
&
Point B
for part 1 & 2

1. Gate valve
2. Pump
3. Check valve
4. Elbow
5. Tee
6. Heat exchanger

Procedure: Use Bernoulli's to determine h_A.
Once h_A has been determined, calculate pump power required to pump 275 gpm through heat exchanger with gate valve on 1 in bypass pipe closed.

Bernoulli's

$$h_A + \cancel{\frac{p_1}{\gamma}}^{0} + \cancel{\frac{v_1^2}{2g}}^{0} + z_1 = \cancel{\frac{p_2}{\gamma}}^{0} + \frac{v_2^2}{2g} + z_2 + h_L$$

Tank and exit are open to atmosphere

$p_1 = p_2 = 0$.

$$h_A = \frac{v_2^2}{2g} + (z_2 - z_1) + h_L$$

$v_1 = \frac{Q}{A_1}$ $v_2 = \frac{Q}{A_2}$ $z_2 = 15\text{ ft}$ $z_1 = 4.5\text{ ft}$

Calculations:

$h_L = h_1 + h_2 + h_3 + h_4 + h_5 + h_6 + h_7 + h_8 + h_9 + h_{10}$

Branch 1 $+ h_{11} + h_{12} + h_{13}$

h_1 = Entrance => $K = 0.5$
h_2 = gate valve => $K = 8f_T$
(4 in pipe → $f_{T_1} = 0.016$)

h_3 = Check valve => $K = 100 f_T$
h_4 = gate valve => $K = 8f_T$
h_5 = elbow$_1$ => $K = 30 f_T$
h_6 = elbow$_2$ => $K = 30 f_T$
h_7 = tee$_1$ => $K = 60 f_T$
h_8 = gate valve => $K = 8 f_T$
h_9 = heat exchanger => $K = 12$
h_{10} = gate valve => $K = 8 f_T$
h_{11} = elbow$_4$ => $K = 30 f_T$
h_{12} = Tee$_2$ => $K = 60 f_T$
h_{13} = Exit => $K = 1.0$
(3 in pipe → $f_{T_2} = 0.017$)

h_{14} = Elbow$_3$ => $K = 30 f_T$
$L_{suction} = 10\text{ ft}$
$L_{discharge} = 40\text{ ft}$

1 in pipe → $f_{T_3} = 0.022$
Elbow $= 30 f_T = 0.66$
Gate $= 8 f_T = 0.176$
Elbow $= 30 f_T = 0.66$
Tee $= 60 f_T = 1.32$
$\frac{L}{D} = \frac{30}{0.087} = 344.83$

$h_2 = 0.128$
$h_4 = h_8 = h_{10} = 8(0.017) = 0.136$
$h_5 = h_6 = h_{14} = h_{11} = 30(0.017) = 0.51$
$h_7 = h_{12} = 60(0.017) = 1.02$
$h_3 = 1.7$

$$h_L = f_1\left(h_1 + h_2 + \frac{L}{D}\Big|_{suction}\right)\left(\frac{V_1^2}{2g}\right) +$$
$$+ f_2\left(h_3 + 3h_4 + 4h_5 + 2h_7 + h_9 + h_{13} + \frac{L}{D}\Big|_{discharge}\right)\left(\frac{V_2^2}{2g}\right)$$

$$h_L = f_1(0.5 + 0.128 + 29.76)\frac{6.923^2}{2(32.2)}$$
$$+ f_2(1.7 + 3(0.136) + 4(0.51) + 2(1.02) + 12 + 1$$
$$+ 156.25)\left(\frac{11.925^2}{2(32.2)}\right)$$
$+31.25$
206.688
387.40

$$h_L = f_1(23.47) + f_2(456.40)$$

Reynold's + $\frac{D}{\varepsilon}$

4in pipe

$$Re = \frac{VD}{\nu} = \frac{(6.923\ ft/s)(0.336ft)}{4.38\times10^{-6}\ ft^2/s} = 531079 = 5.31\times10^5$$

$$\frac{D}{\varepsilon} = \frac{0.336}{1.50\times10^{-4}} = 2240$$

from Moody $f_1 = 0.0177$

3in pipe

$$R_e = \frac{VD}{\nu} = \frac{(11.925)(0.256)}{4.38\times10^{-6}} = 696986$$
$$6.969\times10^5$$

$$\frac{D}{\varepsilon} = \frac{0.256}{1.50\times10^{-4}} = 1706$$

Moody $\rightarrow$ $f_2 = 0.0180$

$$h_L = (0.0177)(23.47) + (0.018)(456.40)$$

$$h_L = 8.63\ ft$$

Solve for h_A

$$h_A = \frac{V_2^2 - V_1^2}{2g} + (z_2 - z_1) + h_L$$

$$= \frac{11.925^2}{2(32.2)} + (15 - 4.5) + 8.63\ \text{ft}$$

$h_A = 22.34$ ft

For pump power

$$P = \frac{\gamma Q h_A}{\eta} = \frac{(61.0\ \text{lb/ft}^3)(0.612\ \text{ft}^3/\text{s})(22.34)}{0.7}$$

$$P = 1191.33\ \text{lb}\cdot\text{ft/s} \times \frac{\text{hp}}{550\ \text{lb}\cdot\text{ft/s}}$$

$$\boxed{P = 2.166\ \text{HP}}$$

Part 2 procedure: Starting with each branch from point A in the drawing as in part 1, start with Bernoulli's for each Branch.

Branch 1

$$h_A + \cancel{\frac{P_1}{\gamma}}^{0} + \cancel{\frac{V_1^2}{2g}}^{0} + \cancel{z_1}^{0} = \cancel{\frac{P_2}{\gamma}}^{0} + \frac{V_2^2}{2g} + z_2 + h_L$$

Since starting from point A, the surface of the tank is open to the atmosphere. Therefore P_A and V_A are equal to zero. Also starting there would make $z_1 = 0$. This leaves us with the following

$$h_A = \frac{V_2^2}{2g} + z_2 + h_L$$

$h_A \rightarrow h_A$ will depend on Q as the gate in Branch 2 is open. Pump power will remain the same.

$$P = \frac{\gamma Q h_A}{\eta} = 1310.79 \ lb \cdot ft/s$$

Solve for h_A and replace h_A in Bernoulli's with $h_A = \frac{1310.79 \cdot \eta}{\gamma Q}$

Replace $\frac{V^2}{2g}$ with $\frac{8Q^2}{g\pi^2 D^4}$

Derive equation for h_L

Part 2 calculations:

Branch 1

$$h_A = \frac{V_2^2}{2g} + z_2 + h_L$$

$$\frac{(1310.79)(0.7)}{(61.0)\,Q_1} = \frac{8Q_1^2}{g\pi^2 D_2^4} + 11.5 + h_L$$

4in 3in

$$h_L = f_1\left(K_{enter} + K_{gate} + \frac{L}{D}\Big|_{suct}\right)\frac{V_1^2}{2g} + f_2\left(K_{check} + K_{gate} + 2K_{elbow} + \frac{L}{D}\Big|_{dis}\right)\frac{V_2^2}{2g} + f_3\left(K_{Tee} + 2K_{elbow} + 2K_{gate} + \frac{L}{D}\Big|_{pipe} + K_{heat} + K_{Tee} + K_{exit}\right)\frac{V_3^2}{2g}$$

$$= f_1(0.5 + 0.128 + 29.76)\frac{8Q_1^2}{g\pi^2 D_1^4} + f_2(1.7 + 0.136 + 2(0.51) + 156.25)\frac{8Q_1^2}{g\pi^2 D_2^4} + f_3(1.02 + 2(0.51) + 2(0.136) + 12 + 1.02 + 1 + 31.25)\frac{8Q_2^2}{g\pi^2 D_2^4}$$

47.58

$$h_{L_1} = f_1(31.0)\,Q_1^2 + f_2(925.75)\,Q_1^2 + f_3(278.88)\,Q_2^2$$

$$\frac{V_2^2}{2g} = \frac{8Q_2^2}{g\pi^2 D_2^4}$$

$$\frac{(1310.79)(0.7)}{(61.0)\,Q_1} = (64.47)Q_2^2 + f_1(31.0)Q_1^2 + f_2(925.75)Q_1^2 + f_3(98.76)\,Q_2^2$$

$$Q_2 = \sqrt{\frac{\frac{15.04}{Q_1} - 11.5 - f_2(925.75)Q_1^2 - f_1(31.0)Q_1^2}{(64.4 - f_3(278.88))}}$$

Branch 2

$$h_A = \frac{V_3^2}{2g} + z_2 + h_L$$

$$h_A = \frac{(1310.79)(0.7)}{(161.0)\,Q_1} \qquad \frac{V_3^2}{2g} = \frac{8Q_3^2}{g\pi^2 D_3^4}$$

$$h_L = f_1(31.0)Q_1^2 + f_2(925.75)Q_1^2 + f_4\left(K_{Tee} + K_{gate} + 2K_{elbows} + K_{Tee} + \frac{L}{D}\Big|_{pipe}\right)\frac{8Q_3^2}{g\pi^2 D_3^4}$$

$$= f_1(31.0)Q_1^2 + f_2(925.75)Q_1^2 + f_4(347.55)\,Q_3^2$$

$$\frac{(1310.79)(0.7)}{(161.0)Q_1} = (439.40)Q_3^2 + f_1(1.24)Q_1^2 + f_2(925.75)Q_1 + f_4(1.707)\,Q_3^2 + 11.5$$

$$\frac{15.04}{Q_1} - 11.5 - f_1(1.24)Q_1^2 - f_2(925.75)Q_1^2) = (439.40 + f_4(347.55))\,Q_3^2$$

$$Q_3 = \sqrt{\frac{\frac{15.04}{Q_1} - 11.5 - f_1(31.0)Q_1^2 - f_2(925.75)Q_1^2}{(439.4 + f_4(347.55))}}$$

Summary: For part I, with the bypass valve closed, the pump head h_A was calculated by finding the head losses and using Reynolds number + $\frac{D}{\varepsilon}$ to find friction factor for each section. Total head loss was 8.631 ft.
This was used to calculate $h_A = 22.34$ ft
Pump power at 70% effiency was calculated with $P = \frac{\gamma Q h_A}{\eta}$. The total pump power was 1191.33 lb·ft/s, or 2.17 HP.

For part 2, the same reference points were used. Branch 1 & 2 were calculated By using the pump power equation to replace h_A, and $\frac{v^2}{2g} = \frac{8Q^2}{g\pi^2 D^4}$
Bernoulli's for each branch was

$$h_A = \frac{v_2^2}{2g} + z_2 + h_L$$

h_L formula was derived for each branch

Branch 1 was $Q_2 \sqrt{\dfrac{\frac{15.04}{Q_1} - 11.5 - f_2(925.75)Q_1^2 - f_1(31.0)Q_1^2}{(64.4 - f_3(278.88))}}$

Branch 2 was $Q_3 = \sqrt{\dfrac{\frac{15.04}{Q_1} - 11.5 + f_1(310)Q_1^2 - f_2(925.75)Q_1^2}{(439.4 + f_4(347.55))}}$

These formudas were to be taken to excel to use iterations to solve for $Q_1 = Q_2 + Q_3$ and f_1, f_2, f_3, and f_4. The K value of the gate on the bypass can now be adjusted to change the h_L for Branch 2 and then adjust Q and f values.

The formulas derived for Q did not give an accurate value for flow rate and could not be used in excel.

The excel spread is attached with the progress made.

Analysis: With the bypass gate closed, the pump was able to deliver 275 gpm at 2.16 hp with a power efficieny at 70%. As the bypass valve opened the total flow rate increased. The flow rate divided between both pipes Q_2 and Q_3. The flow then combined at the tee to equal the total flow rate again with $Q_1 = Q_2 + Q_3$

The derived formulas were not accurate due to miscalculations. This didnot allow iterations as the formulas at the initial 275gpm flow rate gave an invalid value.

My overall analysis is incomplete due to this error.