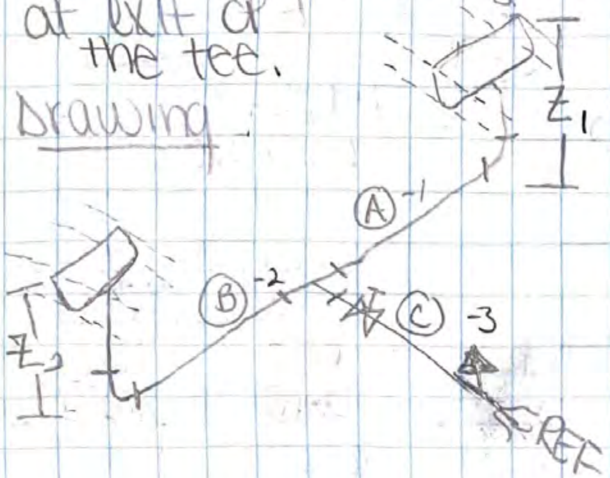
Amber Samm
MET 330
Test 2

① Purpose: Determine the flow out of the system if gate valve is half open. Determine if velocity meets criteria. Compute pressure at exit of the tee.

Sources: MOTT, Robert L.; UNTENER, JOSEPH A., Applied Fluid Mechanics, 7th, -2015.

Drawing



Design Considerations:

① Incompressible
② Isothermal
③ Steady State

Data + Variables

$D = 0.75in \cdot 0.0254m/in = 0.01905m$ Roughness: $D/\varepsilon = 0.01905m / 4.6\times10^{-5}m = 414.13$

$L_A = 10m$ $L_B = 9m$ $L_C = 10m$ $f_T = 0.0246$ (moody chart)

$Z_1 = 4m$ $Z_2 = 3m$ $Z_3 = 0m$ $\gamma_w = 9.79\ kN/m^3$

$\varepsilon = 4.6\times10^{-5}\ m$ (table 8.2)

$K_{valve\,1/2} = 160$ $K_{tee} = 60$ $K_{entrance} = 0.5$ (fig. 10.14) $K_{elb} = 30$

Calculations + Procedures: Materials: water

Apply Bernoulli's Energy equation: A-C $A = \frac{\pi D^2}{4} \rightarrow V = \frac{Q}{A} = \frac{4Q}{\pi D^2}$

$$Z_1 = \frac{V_C^2}{2g} + K_{ent}\frac{V_A^2}{2g} + K_{elb}\frac{V_A^2}{2g} + K_{tee}\frac{V_A^2}{2g} + K_{valve}\frac{V_C^2}{2g} + f\frac{L_A}{D_A}\frac{V_A^2}{2g} + f\frac{L_C}{D_C}\frac{V_C^2}{2g}$$

Substitute:

$$Z_1 = \left(\frac{8}{g\pi^2 D^4} + \frac{8K_{val,1/2}}{g\pi^2 D^4} + \frac{8fL_C}{g\pi^2 D^5}\right)Q_3^2 + \left[\frac{8}{g\pi^2 D^4}\left(K_{ent} + K_{elb} + K_{tee}\right) + \frac{8fL_A}{g\pi^2 D^5}\right]Q_1^2$$

Substitute:

$$4m = \left(\frac{8}{9.81\times\pi^2(0.0190)^4} + \frac{8\times160\times0.0246}{9.81\times\pi^2(0.01905)^4} + \frac{8\times0.0246\times10}{9.81\times\pi^2(0.01905)^5}\right)Q_3^2$$

$$+\left[\frac{8}{9.81\times\pi^2(0.01905)^4}(0.5+30\times0.0246+60\times0.0246) + \frac{8\times0.0246\times10}{9.81\times\pi^2(0.0190)^5}\right]Q_1^2$$

EQ ① $\boxed{4m = 11198622.15\,Q_3^2 + 9804549.62\,Q_1^2}$

Apply Bernoulli's Energy Equation B-C:

$$Z_2 = \frac{V_C^2}{2g} + K_{ent}\frac{V_B^2}{2g} + K_{elb}\frac{V_B^2}{2g} + K_{tee}\frac{V_B^2}{2g} + K_{1/2}\frac{V_C^2}{2g} + f\frac{L_B}{D_B}\frac{V_B^2}{2g} + f\frac{L_C}{D_C}\frac{V_C^2}{2g}$$

$$Z_2 = 11198622.5\,Q_3^2 + \left[627395.38\times2.7457 + \frac{(8\cdot0.0246\cdot9)}{g\pi^2(0.0190)^5}\right]Q_2^2$$

$$\boxed{3 = 11198622.5\,Q_3^2 + 9013819.03\,Q_2^2} \quad \text{EQ 2}$$

$$\boxed{Q_3 = Q_2 + Q_1} \quad \text{EQ3}$$

Solve for flow rate:

$$Q_1 = \sqrt{\frac{4 - 11198622.5\,Q_3^2}{9804549.62}} \qquad Q_2 = \sqrt{\frac{3 - 11198622.5\,Q_3^2}{9013819.03}}$$

Solve Q_3 by iterations: Q_3 guess: 0.5×10^{-3} m³/s

Iteration	f3	Q3 (m³/s)	V1 (m/s)	Re	NEW f3	% diff
5	0.026026	4.05×10^{-4}	1.15	2.16	0.02626	0.00%

Plug in Q_3 to solve Q_2:

$$Q_2 = \sqrt{\frac{3 - 11198622.5\,(4.05 \cdot 10^{-4})^2}{9013819.03}} = 1.73 \cdot 10^{-4}$$

Solve Q_1: $Q_3 = Q_2 + Q_1$

$4.05 \times 10^{-4} = 1.73 \cdot 10^{-4} + Q_1$

$Q_1 = 2.32 \cdot 10^{-4}$

$$\boxed{Q_3 = 4.05 \times 10^{-4}\ \text{m}^3/\text{s} \quad Q_1 = 2.32 \times 10^{-4}\ \text{m}^3/\text{s} \quad Q_2 = 1.73 \times 10^{-4}\ \text{m}^3/\text{s}}$$

Check velocities to meet criteria:

$$V_A = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}(D)^2} = \frac{2.32 \times 10^{-4}}{2.85 \times 10^{-4}} = 0.814\ \text{m/s}\ ✓$$

$$V_B = \frac{1.73 \times 10^{-4}}{2.85 \times 10^{-4}} = 0.607\ \text{m/s}\ ✓$$

$$V_C = \frac{4.05 \times 10^{-4}}{2.85 \times 10^{-4}} = 1.42\ \text{m/s}\ ✓$$

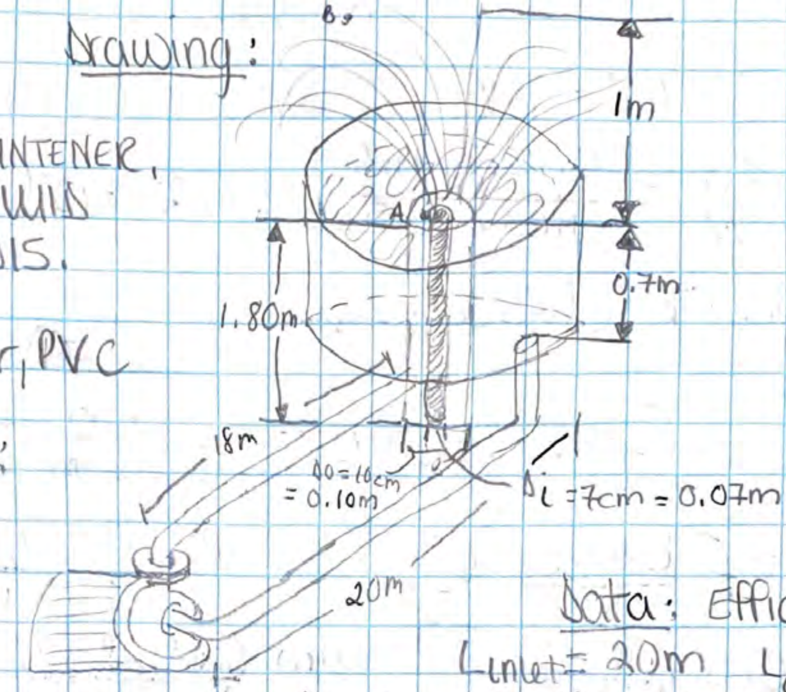
$\boxed{\text{All velocities are below the Vmax of 3 m/s}}$

Summary:
The flow rate for pipe (a) is 2.32×10^{-4} m³/s, (b) is 1.73×10^{-4} m³/s, and (c) is 4.05×10^{-4} m³/s, which is the sum of $Q_2 + Q_1$. The velocities are all well below Vmax which means that they meet the criteria.

Analysis: The lower the elevation, the slower the flow rate as is the case with pipe b. Pipe (A) has a higher flow rate due to higher elevation. I have no suggestions, as the velocities meet criteria.

(2) Purpose: Find the pump power required for flow configuration. Determine electrical power requirements.

Drawing:



Sources:
MOTT, Robert L., UNTENER, JOSEPH A., APPLIED FLUID MECHANICS, 7th, 2015.

Materials: water, PVC

Design Considerations:
1. incompressible
2. isothermal
3. steady state
4. considering all minor losses

Data: Efficiency 92% = 0.92

$L_{inlet} = 20m$ $L_{outlet} = 18m$

$K_{ent} = 0.5$ $L = 1.80m$ $D_O = 10cm = 0.10m$

$K_{elb} = 30$ $K_{exp} = 2$ $D_i = 7cm = 0.07m$

$h_{water} = 1m$ $t = 3/2 = 1.5cm = 15mm$

$\rho_{water} = 998 kg/m^3$

$\epsilon = 3.0 \times 10^{7} m$ (table 8.2)

$\nu = 1.05 \times 10^{-5} m^2/s$ (table A.2)

$V_{criteria} = 3 m/s$

Calculations/Procedures:

Apply Bernoullis to ensure proper Q's

$$\cancel{h_A}^{0} + \cancel{\frac{P_A}{\gamma}}^{0} + \frac{V_A^2}{2g} + z_A = \cancel{\frac{P_B}{\gamma}}^{0} + \cancel{\frac{V_B^2}{2g}}^{0} + z_B + \cancel{h_L}^{0} + \cancel{h_R}^{0}$$

$$\frac{V_A^2}{2g} = z_B - z_A = 1m$$

$$V_A^2 = 2(9.81 m/s^2) = \sqrt{19.62}$$

$V_A = 4.429 m/s$ (for water to reach 1m)

Flow Rate $\dot{M}$:

$$Q = VA = 4.429 m/s \cdot \left(\frac{\pi}{4}(0.10m^2 - 0.07m^2)\right) = 0.0177 m^3/s$$

PVC pipe diameters:

$Q = \frac{\pi}{4} D^2 V$ rearrange to solve for D: $D = \sqrt{\frac{4Q}{\pi V}} =$

$$D = \sqrt{\frac{4(0.0177 m^3/s)}{\pi (3 m/s)}} = \sqrt{\frac{0.0708}{9.425}} = 0.08667m = 86.67mm \text{ (inside)}$$

125mm pipe is the smallest pipe I can choose based on Req. $D_i \geq 86.67$.

Apply Bernoulli to find pump power required.

$$h_A + \cancel{\frac{P_A}{\gamma}} + \cancel{\frac{V_A^2}{2g}} + \cancel{z_A} = \cancel{\frac{P_B}{\gamma}} + \cancel{\frac{V_B^2}{2g}} + \cancel{z_B} + h_L + \cancel{h_R}^{0}$$

Simplified: $h_A = h_L$

125mm $D = 0.1154 m$

$A = 1.046 \times 10^{-2} m^2$

$$h_L = h_{pipe} + h_{annu} + h_{ent.annu} + (4) h_{elb.} + h_{ent.}$$

$$h_{ent} = 0.5 \frac{(1.692 m/s)^2}{2(9.81)} = 0.0731m$$

$$V = Q/A = \frac{0.0177}{1.046 \cdot 10^{-2}} = 1.692 m/s$$

$$h_{ent_{annu}} = 2.0\left(\frac{(1.692)^2}{2(9.81)}\right) = 0.2918$$

h_{elb}: $\frac{Le}{D} = 30$ (table 10.4) $Le = 30 \cdot 0.1154 = 3.462$

Hydraulic radius for energy loss at annulus: (Fig 9.9)

$A = \frac{\pi}{4}(D^2 - d^2)$ $WP = \pi(D+d)$

$= \frac{\pi}{4}(0.10^2 - 0.07^2)$ $= \pi(.10+0.07)$ $R = \frac{A}{WP} = \frac{0.004}{0.5341} =$

$= 0.004$ $= 0.5341$ $0.0075\,m$

(9-6) $D = 4R : 4(0.0075m) = 0.03m$

$Re = \frac{VD}{\nu} = \frac{4.429 \cdot 0.03}{1.05\times10^{-5}\,m^2/s} = 1.27\times10^4$ $\frac{D}{\varepsilon} = \frac{0.03}{3.0\times10^{-7}} = 100{,}000$

$f = 0.017$ (table 10.5)

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$h_{annun} = f\frac{L}{De}\frac{V^2}{2g} = 0.0175\left(\frac{1.8m}{0.03m}\right)\frac{(4.429m/s)^2}{2(9.81m/s^2)} = \underline{1.05m}$

$h_{pipe} = f\frac{L + \Sigma Le}{D}\frac{V^2}{2g}$ $Re = \frac{VD}{\nu} = \frac{1.692 \cdot 0.1154}{1.05\times10^{-5}} = 1.8\times10^4$

$= 0.017\frac{(18+20+4\times3.462)m}{0.1154m}\frac{(1.692m/s)^2}{2(9.81m/s^2)}$ $\frac{D}{\varepsilon} = \frac{0.1154}{3\times10^{-7}} = 384{,}666$

$= 1.114m$

Calculate $\Sigma h_A = 1.114m + 1.05m + 0.2918m + 0.0731m = 2.5289m$

pump power: $P_{pump} = \rho g \cdot Q \cdot h_A$

$= 998\,kg/m^3 \cdot 9.81\,m/s^2 \cdot 0.0177\,m^3/s \cdot 2.5289m$

$\boxed{= 438.23\ W}$

Convert watts to HP:

$\boxed{0.588\ HP}$

Calculate electric power using efficiency:

$P_{electric} = \frac{0.588}{0.92} = \boxed{0.639\ HP}$

<u>Summary</u>

The pump power required for the flow configuration shown is 0.588 HP. With an efficiency of 92%, the electrical power requirement is 0.639HP.

<u>Analysis</u>:

The benefit of using PVC pipe is that it is generally smooth w/ and next to nothing roughness factor. PVC is practical for this decorative fountain, but a bigger one with higher pressure would benefit from iron or steel fixtures.