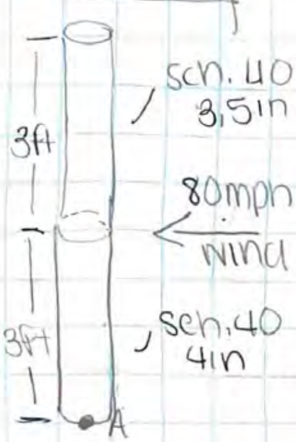
Amber Samm
MET330
Test 3
UIN: 01319568

① Purpose: compute the moment at the base of the vertical pole.

Drawing:



Design considerations
① incompressible
② isothermal
③ steady state

Source: Mott, Robert L.; Utener, Joseph A, Applied Fluid Mechanics, 7th, 2015

Materials: wind.

Data:
L_{top}: Length 3ft, $d_o = 4$ in $= 0.333$ft
L_{bottom}: Length 3ft, $d_o = 4.5$in $= 0.375$ft
Air @ 71°F: $\rho = 0.00232$ slugs/ft^3, $\mu = 0.3801 \times 10^{-6}$ $\frac{lb \cdot s}{ft^2}$
$u = \frac{80 mi}{h} = 117$ ft/s
$d_{top} = 3$ft $\quad d_{bottom} = 1.5$ft

Procedure / calculations:

Determine equation for moment:
$$F_{top} \times d_{top} + F_{bottom} \times d_{bottom} = M_{base}$$

Equation for drag coefficient
$$C_D = \frac{F/A}{\frac{1}{2}\rho u^2}$$

Calculate Reynolds number to get C_D:
$$Re_{top} = \frac{\rho u D}{\mu} = \frac{0.00232 \frac{slug}{ft^3} \cdot 117 \frac{ft}{s} \cdot 0.333ft}{0.3801 \times 10^{-6} \frac{lb \cdot s}{ft^2}} = 237{,}804 = 2.38 \times 10^5$$
$$Re_{bottom} = \frac{\rho u D}{\mu} = \frac{0.00232 \frac{slug}{ft^3} \cdot 117 \frac{ft}{s} \cdot 0.375ft}{0.3801 \times 10^{-6} \frac{lb \cdot s}{ft^2}} = 267{,}798 = 2.68 \times 10^5$$

$C_{D top} = 1.2$
$A_{top} = 3ft \cdot 0.333ft = 0.999 ft^2$

Rearrange to solve F:
$$F_{top} = C_{D_{top}} \tfrac{1}{2}\rho u^2 \cdot A_{top} = 1.2(0.5 \cdot 0.00232 \, slug/ft^3 \cdot (117 \tfrac{ft}{s})^2 \cdot 0.999 ft^2 = 19.04 lb_f$$

$C_{D bottom} = 1.2$
$A_{bottom} = 3ft \cdot 0.375ft = 1.125ft^2$
$$F_{bottom} = C_{D_{bot}} \tfrac{1}{2}\rho u^2 \cdot A_{bott.} = 1.2(0.5 \cdot 0.00232 \, slug/ft^3 \cdot (117 \tfrac{ft}{s})^2 \cdot 1.125 ft^2 = 21.44 lb_f$$

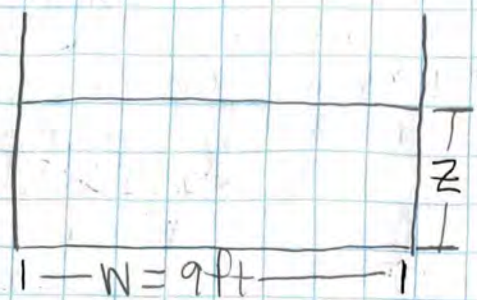
Calculate moment
$$M_{base} = (19.04 lb_f) \cdot (3ft) + (21.44 lb_f) \cdot (1.5ft) = \boxed{89.28 lb_f \cdot ft}$$

Summary
The moment at the base of the pole is 89.28 $lb_f \cdot ft$.

Analysis: If the length of the pole were to double, the moment would more than double. If the length were to decrease the moment experienced at the base would decrease significantly

② Purpose: compute the liquid height in an open channel. Is the flow supercritical or subcritical?

Drawing:



Sources: MOTT, Robert L; UTENER, JOSEPH A, APPLIED FLUID MECHANICS, 7th, 2015.

materials: water.

Design considerations: ① incompressible ② isothermal ③ steady state

Data:

W = 9 ft

Q = 350 ft³/s

S = 0.001

unfinished concrete
n = 0.017 (table 14.1)

Procedure / calculations: Use mannings equation (U.S.)

(14-11) $Q = Av = \left(\frac{1.49}{n}\right) A R^{2/3} S^{1/2}$ $A = 9z$ $R = \frac{A}{WP} = \frac{9z}{9+2z}$ $WP = 9+2z$

$350 ft^3/s = \left(\frac{1.49}{0.017}\right) \cdot 9z \left(\frac{9z}{9+2z}\right)^{2/3} \cdot (0.001)^{1/2}$

$350 ft^3/s = 24.945 \cdot z \left(\frac{9z}{9+2z}\right)^{2/3} =$

$14.031 - z\left(\frac{9z}{9+2z}\right)^{2/3} = f(z)$ Solve by trial + error:

z	f(z)
5	5.146
6	2.769
7	0.327
7.5	−0.912
7.25	−0.291
7.125	0.01
7.0625	0.1728

$= \boxed{z = 7.125 ft}$

Use Froude Number equation:

$Fr = \frac{v}{\sqrt{g y_h}} =$

$A = 9ft \cdot 7.125 = 64.125 ft^2$

$v = \frac{Q}{A} = \frac{350}{64.125} = 5.46 ft/s$

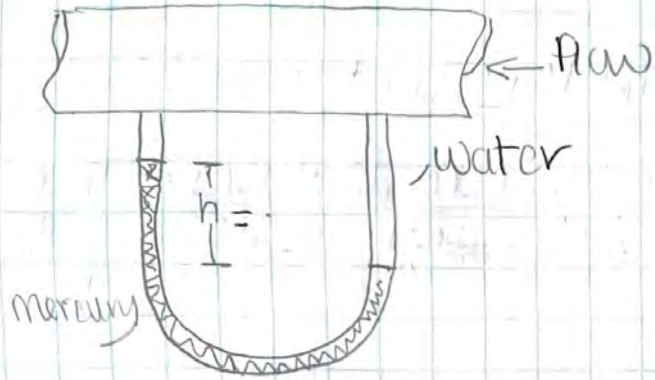
$Fr = \frac{5.46 ft/s}{\sqrt{32.2 \times 7.125}} = \frac{5.46}{15.15} = 0.360$ $Fr < 1$ $\boxed{\text{subcritical}}$

Summary:
The liquid height in the channel is 7.125ft and the Froude number is 0.360 which means the flow is subcritical.

Analysis
If the depth of fluid were to decrease and/or the velocity increase this would increase the Froude number, eventually moving the flow into a super-critical state.

③ Purpose: compute the (height) reading of a mercury manometer

Drawing:



Sources: Mott, Robert L.; Utener, Joseph A., Applied Fluid Mechanics, 7th, 2015.

Materials: mercury, water

Design Considerations: ① Incompressible ② Isothermal ③ Steady state

Data: $Q = 12000\,gpm = 26.73\,ft^3/s$

24" Sch 40: $D_i = 22.626\,in = 1.886\,ft$

$SW_{water} = 62.4\,lb/ft^3$ $SW_{Hg} = 844.9\,lb/ft^3$

$\beta = 0.5$ $\mu_{water} = 1.89\times10^{-5}\,lb\cdot s/ft^2$ $\rho = 1.94\,\frac{slugs}{ft^3}$

$d = 0.5\times1.8853 = 0.9427$

$A_1 = \frac{\pi}{4}D_i^2 = 0.6979\,ft^2$

$\left(\frac{A_1}{A_2}\right)^2 = \left(\frac{1}{\beta^2}\right)^2 = \frac{1}{\beta^4}$ sub into (h) equation

Procedure / calculations:

Use standard orifice flow equation:

$$Q = CA_1\sqrt{\frac{2gh((\gamma_m/\gamma_f)-1)}{(A_1/A_2)^2-1}}$$

Rearrange to solve for h:

$$h = \frac{\left(\frac{Q}{CA_1}\right)^2\left[(A_1/A_2)^2-1\right]}{2g\left[(\gamma_{Hg}/\gamma_w)-1\right]} = \frac{\left(\frac{4Q}{C\pi D_i^2}\right)^2\left((1/\beta)^4-1\right)}{2g\left[(\gamma_{Hg}/\gamma_w)-1\right]} = \frac{\left(\frac{4\cdot26.73\,ft^3/s}{0.603\,\pi\cdot1.886^2}\right)^2\left[(1/0.5)^4-1\right]}{2\cdot32.2\,ft/s^2\left[\frac{844.9\,lb/ft^3}{62.4\,lb/ft^3}-1\right]}$$

Calculate reynolds number to find C:

$$Re = \frac{4Q\rho}{\pi D_i\mu} = \frac{4\cdot26.73\,ft^3/s\cdot1.94\,slugs/ft^3}{\pi(1.886\,ft)(1.89\times10^{-5}\,lb\cdot s/ft^2)} = 1.84\times10^6$$

$C = 0.603$ (figure 15.7)

$h = 4.6765 = \boxed{4.707\,ft}$

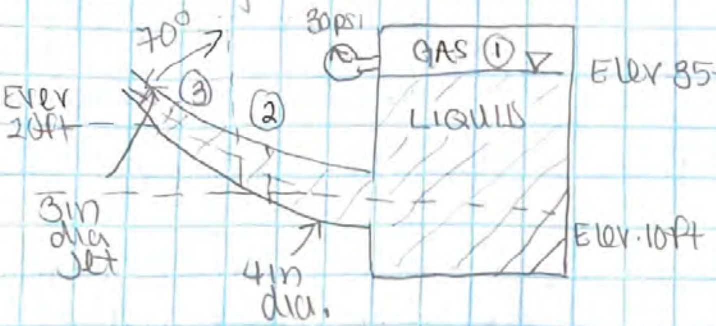
Summary

The height of the mercury on the manometer would read 4.707 ft.

Analysis: If the flow rate were to increase the height reading would also increase because the manometer height is proportional to the flow rate squared. If the fluid density changed by replacing mercury with oil, the difference in SW would decrease and the height deflection would be larger as well.

④ Purpose: compute the force on the curved pipe section.

Drawing:



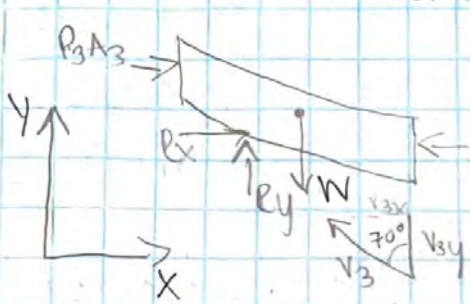
Sources: Mott, Robert L; Untener, Joseph A, Applied Fluid Mechanics, 7th, 2015.

Materials: fluid, gas

Design Considerations: ① Incompressible ② Isothermal ③ Steady state ④ neglect any energy losses

Data: $L_{curved} = 10ft$
$P_1 = 30psi = 4320 lb/ft^2$
$P_3 = 0$ (gage)
$D_2 = 4in = 0.333 ft$ $D_3 = 3in = 0.25ft$
$SW_{liquid} = 55 lb/ft^3$

FBD of Curved Pipe:



$\sin 70 = \frac{V_{3x}}{77.6} =$
$V_{3x} = 72.92 ft/s$

Calculations/Procedures:

Apply Bernoulli's to solve V_3: $\frac{P_1}{\gamma} + \frac{\cancel{V_1^2}^{0}}{\cancel{2g}} + z_1 = \frac{\cancel{P_3}^{0}}{\cancel{\gamma}} + \frac{V_3^2}{2g} + z_3 = \frac{4320 lb/ft^3}{55 lb/ft^3} + 35ft = \frac{V_3^2}{2(32.2)} + 20ft =$

$V_3 = 77.6 ft/s$

Solve for Q:

$Q = V_3 A_3 = 77.6 \frac{ft}{s} \cdot \frac{\pi}{4}(0.25ft)^2 = 3.809 ft^3/s$

$Q = V_2 A_2 = \frac{4Q}{\pi D_i^2} = \frac{4 \cdot 3.809 ft^3/s}{\pi (0.333)^2} = \underline{43.74 ft^3/s}\ V_{2x}$

Apply Bernoulli's (P_2): $\frac{P_1}{\gamma} + \frac{\cancel{V_1}^{0}}{\cancel{2g}} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \Rightarrow \frac{P_2}{\gamma} = \frac{P_1}{\gamma} - \frac{V_2^2}{2g} + (z_1 - z_2)$

$= \frac{4320 lb/ft^2}{55 lb/ft^3} - \frac{(43.74 \frac{ft}{s})^2}{2 \times 32.2 ft/s^2} + 25ft = 78.54 - 29.707 + 25 = 73.84 ft$

$P_2 = 73.84 \cdot 55 = 4061.07 lb/ft^2$

Resolve Rx + Ry: $\Sigma F_x = \rho Q (V_{3x} - V_{2x}) \rightarrow R_x - P_2A_2 = \rho Q (V_{3x} - V_{2x}) \Rightarrow$

$R_x = \frac{\gamma}{g} Q (V_{3x} - V_{2x}) + P_2 A_2 \Rightarrow \frac{55 lb/ft^3}{32.2 ft/s^2} \cdot 3.809 \frac{ft^3}{s} \times (-72.92 + 43.74)\frac{ft}{s} + 4061 \frac{lb}{ft^2} \times \frac{\pi}{4}(0.333)^2 =$

$R_x = 163.84$

4-cont Solve for R_y: $\Sigma F_y = \rho Q(V_{3y} - V_{2y}) \Rightarrow R_y - W = \rho Q(V_{3y} - V_{2y})$

$V_{3y} = V_3 \cos 70 = 26.54\,ft/s$

$W = \gamma \times V$ $\qquad 55 \cdot 0.669$

$V = \frac{\pi}{4} D_2^2 (L_{curved}) = \frac{\pi}{4}(0.2922^2 \cdot 10ft = 0.669$ $\quad W = 36.83\,lbf$

$R_y = \frac{55\,lb/ft^3}{32.2\,ft/s^2} \cdot 3.809 \frac{ft^3}{s} \times 26.54 \frac{ft}{s} + 36.83\,lbf =$

$\boxed{R_y = 209.5\,lbf}$

Calculate the resultant force: $R = \sqrt{R_x^2 + R_y^2}$

$R = \sqrt{(163.84)^2 + (209.5)^2} = 265.56\,lbf$

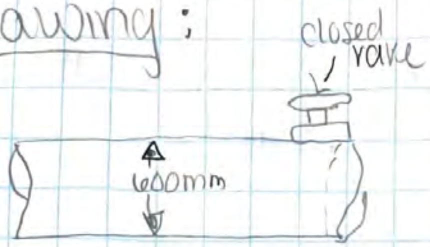
$\tan^{-1}(\theta) = \frac{209.5}{163.48} = \boxed{52.03^\circ}$

Summary: The resultant force is 265.56 lbf at 52.03°. The force in the x is 163.84 lbf and the force in the y is 209.5 lbf.

Analysis: If the length of the curved pipe section increased the force in the y direction would increase and also the resultant force.

⑤ Purpose: Calculate the pressure increment when valve suddenly closes.

Drawing:



Sources: Mott, Robert L.; Untener, Joseph A., Applied Fluid Mechanics, 7th, 2015.

Materials: Water

Design Considerations: ~~① incompressible~~ ② solid state ③ isothermal

Data:

$D = 600mm = 0.6m$

$v = 2.5m/s$

$E = 2.0 \times 10^7 N/cm^2 = 2 \times 10^{11} N/m^2$

$\delta = 10mm = 0.010m$

$\rho = 1000 kg/m^3$

$E_0 = 2.03 \times 10^5 N/cm^2 = 2.03 \times 10^9 N/m^2$

Procedure / Calculations:

Determine equations:

$\Delta p = \rho c v$ (9.1.4)

water hammer equation

Velocity of a wave (9.1.4)

$$c = \sqrt{\frac{E_0/\rho}{1 + \frac{E_0 \cdot D}{E \cdot \delta}}} = \sqrt{\frac{\frac{2.03 \times 10^9 N/m^2}{1{,}000 kg/m^3}}{1 + \frac{2.03 \times 10^9 N/m^2 \cdot 0.6m}{2.0 \times 10^{11} N/m^2 \cdot 0.1m}}} = \sqrt{\frac{2.03 \times 10^6}{1.609}} = \boxed{1{,}123.23 m/s}$$

$$\Delta p = 1000 kg/m^3 \cdot 1123.23 m/s \cdot 2.5 m/s$$

$$= 2808075 \text{ Pa or } \boxed{2.81 \times 10^6 N/m^2}$$

Summary: The pressure increment is $2.81 \times 10^6 N/m^2$ when the valve suddenly closes

Analysis: If the pipe diameter was larger, the wave speed would decrease and so would the pressure increment. An increase in velocity would also increase the water hammer; doubling velocity would double the hammer as they are directly proportional to one another.