

Homework #7

Case 1	Case 2	Case 3
11	37	59
6	11	15
	p and a are published	
4	13	17
5	9	20
9	11	35
10	36	7
	A and B exchange keys	
1	36	46
1	36	46

Case 1

$$A = \alpha^a \text{ mod } p \rightarrow 6^4 \text{ mod } 11 = 9$$

$$B = \alpha^b \text{ mod } p \rightarrow 6^5 \text{ mod } 11 = 10$$

Exchange public keys

$$A = 10^4 \text{ mod } 11 = 1$$

$$B = 9^5 \text{ mod } 11 = 1$$

Case 2

$$A = 11^{13} \text{ mod } 37 = 11$$

$$B = 11^9 \text{ mod } 37 = 36$$

Exchange public keys

$$A = 36^{13} \text{ mod } 37$$

$$36^{13} = -1^{13} \text{ mod } 37 \rightarrow -1^{13} = - \text{ mod } 37$$

$$-1 + 37 = 36 \rightarrow 36^{13} \text{ mod } 37 = 36$$

$$B = 11^9 \text{ mod } 37 = 36$$

Case 3

$$A = 15^{17} \text{ mod } 59 = 35$$

$$B = 15^{20} \text{ mod } 59 = 7$$

Exchange keys

$$A = 7^{17} \bmod 59 = 46$$

$$B = 35^{20} \bmod 59 = 46$$

For the given three cases where Alice is trying to send encrypted data to Bob, and Bob is trying to decrypt it, using Elgamal encryption scheme, fill the values in the table.

Case 1	Case 2	Case 3
11	31	59
7	3	2
6	9	3
4	29	8
	p, a and B are sent to Alice	
4	5	7
3	26	10
3	6	56
7	7	9
10	11	56
	Alice sends kE and y to Bob	
3	30	56
	Verify Alice and Bob computed same key	
3	30	56
8	20	9

Case 1

$$P = 11, a = 7, d = 6, i = 4, x = 7$$

$$B = a^d \bmod p$$

$$7^6 \bmod 11 = 4$$

$$kE = a^i \bmod p \rightarrow 7^4 \bmod 11 = 3$$

$$kM = B^i \bmod p \rightarrow 4^4 \bmod 11 = 3$$

Alice encrypts

$$Y = x * kM \bmod p \rightarrow 7 * 3 \bmod 11 = 10$$

Bob computed

$$kM = kE^d \text{ mod } p \rightarrow 3^6 \text{ mod } 11 = 3$$

Verify that Bob indeed computed the same KM

$$Km \text{ mod } p \rightarrow 3 \text{ mod } 11 = 3$$

$$Y * kM \text{ mod } p \rightarrow 10 * 3 \text{ mod } 11 = 8$$

Case 2

$$P = 31, \alpha = 3, d = 9, i = 5, x = 7$$

$$3^9 \text{ mod } 31 = 29$$

$$3^5 \text{ mod } 31 = 26$$

$$29^5 \text{ mod } 31 = 6$$

$$7 * 6 \text{ mod } 31 = 11$$

Alice sends kE to bob

$$26^9 \text{ mod } 31 = 30$$

Verify that Bob indeed computed the same KM

$$30 \text{ mod } 31 = 30$$

$$11 * 30 \text{ mod } 31 = 20$$

Case 3

$$P = 59, \alpha = 2, d = 3, l = 7, x = 9$$

$$B = 2^3 \text{ mod } 59 = 8$$

$$2^7 \text{ mod } 9 = 10$$

$$kM = B^i \text{ mod } p \rightarrow 8^7 \text{ mod } 59 = 56$$

$$9 * 56 \text{ mod } 59 = 56$$

$$10^3 \text{ mod } 59 = 56 \rightarrow 56 \text{ mod } 59 = 56$$

$$56 * 56 \text{ mod } 59 = 9$$

