Fluid Mechanics Test #1

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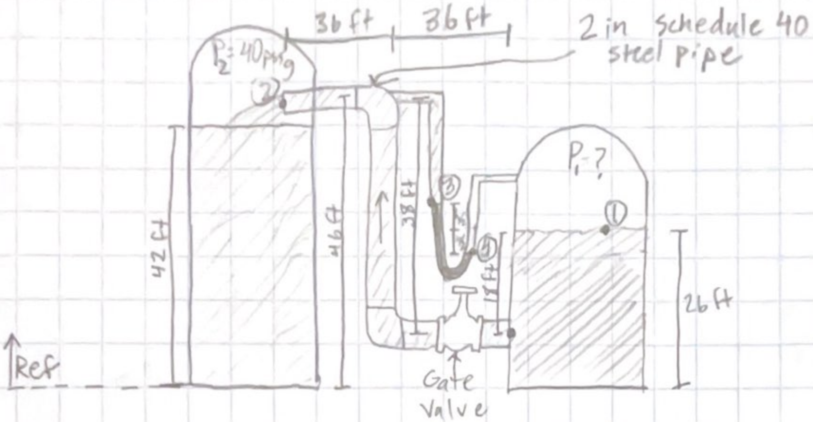
① **Purpose:** Determine the required air pressure in the tank needed to move $250 \frac{gal}{min}$ of ethyl alcohol. This must be solved by hand and in excel. Then determine manometer reading in picture.

Drawings & Diagrams



Sources: Mott & Untener. Applied Fluid Mechanics 8th edition

Design considerations:

Based on the problem description, I assume:

- Incompressible fluids
- Steady flow
- Isothermal at T = 77°F
- Alcohol & mercury do not mix

Data & Variables

$Q = 250 \frac{gal}{min} = 0.557 \frac{ft^3}{s}$

$\gamma_{Ethyl\ Alc} = 49.01 \frac{lb}{ft^3}$

$D_{pipe} = 2\text{ in} = 0.167\text{ ft}$

$P_2 = 40\text{ psig} = 5760 \frac{lb}{ft^2}$

$\gamma_{Hg} = 844.9 \frac{lb}{ft^3}$

$f_T = 0.019$

$f = 2.99 \times 10^{-5} \frac{1}{ft}$

Materials

Ethyl Alcohol, Mercury, Air

Procedure + Calculations:

- I need to find the pressure at P_1
- I will use the following equations

$$\Delta P = \gamma h \qquad Q = vA \qquad \frac{P_1}{\gamma} + \overset{0}{\cancel{\frac{v_1^2}{2g}}} + \underbrace{z_1}_{26\text{ft}} = \underbrace{\frac{P_2}{\gamma}}_{5760\frac{lb}{ft^2}} + \underbrace{\frac{v_2^2}{2g}}_{25.6\frac{ft}{s}} + \underbrace{z_2}_{46\text{ft}} + \underbrace{h_{L1-2}}_{18.44}$$

$$h_L = K\left(\frac{v^2}{2g}\right) \qquad h_L = f \times \frac{L}{D} \times \frac{v^2}{2g}$$

- Use $Q = vA$ to find v_2

$$v_2 = \frac{Q}{A}$$

$$A = \pi r^2 = \pi\left(\tfrac{1}{12}\text{ft}\right)^2 = 0.0218\text{ft}^2$$

$$v_2 = \frac{0.557\frac{ft^3}{s}}{0.0218\text{ft}^2} = 25.6\frac{ft}{s}$$

Darcy's equation + the energy losses at the valve, entrance, + elbows

$$h_L = f \cdot \frac{L}{D} \cdot \frac{v_2^2}{2g} + K_{entrance} \cdot \frac{v_2^2}{2g} + K_{gate\ valve} \cdot \frac{v_2^2}{2g} + 2K_{elbows}\frac{v_2^2}{2g}$$

$$h_L = f \cdot \frac{L}{D} \cdot \frac{v_2^2}{2g} + 0.5\frac{v_2^2}{2g} + 8f_T\frac{v_2^2}{2g} + 2 \cdot 30f_T\frac{v_2^2}{2g}$$

$$f = \frac{\varepsilon}{D} = \frac{5.0\times10^{-6}}{0.167\text{ft}} = 2.99\times10^{-5}\frac{1}{ft}$$

$$L = 36\text{ft} + 38\text{ft} + 36\text{ft} = 110\text{ft}$$

$$h_L = 2.99\times10^{-5}\frac{1}{ft} \cdot \frac{110\text{ft}}{0.167\text{ft}} \cdot \frac{(25.6\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + 0.5 \cdot \frac{(25.6\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + 8 \cdot 0.019 \cdot \frac{(25\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + 2 \cdot 30 \cdot 0.019 \cdot \frac{(25.6\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} = 18.44\text{ ft}$$

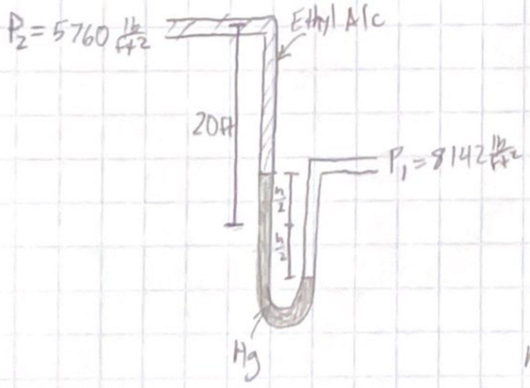
Plug into Bernoulli's

$$P_1 = \left(\frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_{L1-2} - z_1\right)\gamma$$

$$P_1 = \left(\frac{5760\frac{lb}{ft^2}}{49.01\frac{lb}{ft^3}} + 46\text{ft} + \frac{(25.6\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + 18.44\text{ft} - 26\text{ft}\right) \cdot 49.01\frac{lb}{ft^3} \Rightarrow \boxed{P_1 = 8142.69\frac{lb}{ft^2}}$$

Now to find the manometer reading

We will use $\Delta P = \gamma h$



$$\gamma_{Hg} = 844.9\frac{lb}{ft^3}$$

$$\gamma_{Ethyl\ Alc} = 49.01\frac{lb}{ft^3}$$

$$P_2 + \gamma_{EA} \cdot \left(20\text{ft} - \tfrac{h}{2}\text{ft}\right) + \gamma_{Hg}(h\text{ ft}) = P_1$$

$$5760\frac{lb}{ft^2} + \left(49.01\frac{lb}{ft^3}\right)\left(20\text{ft} - \tfrac{h}{2}\text{ft}\right) + 844.9\frac{lb}{ft^3}(h\text{ ft}) = 8142.69\frac{lb}{ft^2}$$

$$\hookrightarrow \boxed{h = 1.71\text{ ft}}$$

The distance between the two tops of the mercury is 1.71 ft. This means that each side has moved 0.85 ft from its original location.

② **Purpose:** Find the pressure in the tank on the right when the flow rate of the system is 0. Then determine what the manometer would read

Drawings & diagrams: Same as problem ①

Sources: Same as problem ①

Design considerations:

Based on the problem description, I assume:

- Incompressible fluids
- Isothermal at $T=77°F$
- Alcohol + Mercury don't mix
- Steady state

Data + variables:

$Q = 0 \frac{ft^3}{s}$ $\quad \gamma_{Ethyl\ alc} = 49.01 \frac{lb}{ft^3}$ $\quad D_{pipe} = 2 in$

$P_2 = 40 psig = 5760 \frac{lb}{ft^2}$ $\quad \gamma_{Hg} = 844.9 \frac{lb}{ft^3}$

Materials

Ethyl Alcohol, Mercury, Air

Procedure + Calculations:

$\Delta P = \gamma h$

I know the pressure in the left tank and since the fluid is stagnant I can use $\Delta P = \gamma h$ to work my way through the pipe + up into the tank on the right

$$P_2 + \gamma_{ethyl\ alc} \cdot 38 ft - \gamma_{ethyl\ alc} \cdot 18 ft = P_1$$

$$5760 \frac{lb}{ft^2} + 49.01 \frac{lb}{ft^3} \cdot 38 ft - 49.01 \frac{lb}{ft^3} \cdot 18 ft = P_1 = \boxed{P_1 = 6740.2 \frac{lb}{ft^2}}$$

Now to find the manometer

We will use $\Delta P = \gamma h$

$$P_2 + \gamma_{EA} \cdot (20 ft - \tfrac{h}{2}) + \gamma_{Hg} (h ft) = P_1$$

$\gamma_{Hg} = 844.9 \frac{lb}{ft^3}$

$\gamma_{EA} = 49.01 \frac{lb}{ft^3}$

$$5760 \frac{lb}{ft^2} + (49.01 \frac{lb}{ft^3})(20 ft - \tfrac{h}{2} ft) + (844.9 \frac{lb}{ft^3})(h ft) = 6740.2 \frac{lb}{ft^2}$$

$\hookrightarrow \boxed{h = 0 ft}$ The height returns to it's original height.

③ Purpose: Use excel to create a graph of the pressure at different flow rates. then use it to determine the flow rate at a pressure of 75 psi.

Procedure:
Used excel spreadsheet created in problem 1.

Using the plot, I determine that when the air pressure in the tank on the right is 75 psig, the flow rate is roughly 426 gpm. Using the spreadsheet, I plugged in different values to check my answer + found that the flow rate is 428.23 gpm at 75 psig. This means that my graph is correct.

SUMMARY:

The system is only able to keep pumping ethyl alcohol until the right tank falls to a pressure of 6740.2 $\frac{lb}{ft^2}$. At first it was operating with a pressure of 8142.69 $\frac{lb}{ft^2}$ in the right tank. If this pressure falls below 6740.2 $\frac{lb}{ft^2}$, then the alcohol will be forced back through the pipes into the right tank. At a pressure of 75 psig in the right tank the flow rate is roughly 426 gpm. The pressure increased linearly with flow rate.

Analysis:

The manometer can be used by technicians to monitor the pressure in the tanks. When the manometer starts to level out, the technicians can close the gate valve in the pipe to shut off the flow + repressurize the tank. However, when this happens all the pressure would only be able to escape through the manometer + it has the potential to push the mercury out of the manometer + into the pipe. Mercury + Ethyl alcohol don't mix but it would still be very difficult to clean the mercury out of the system. Perhaps the manometer could also have a valve built into it to safeguard this from happening.