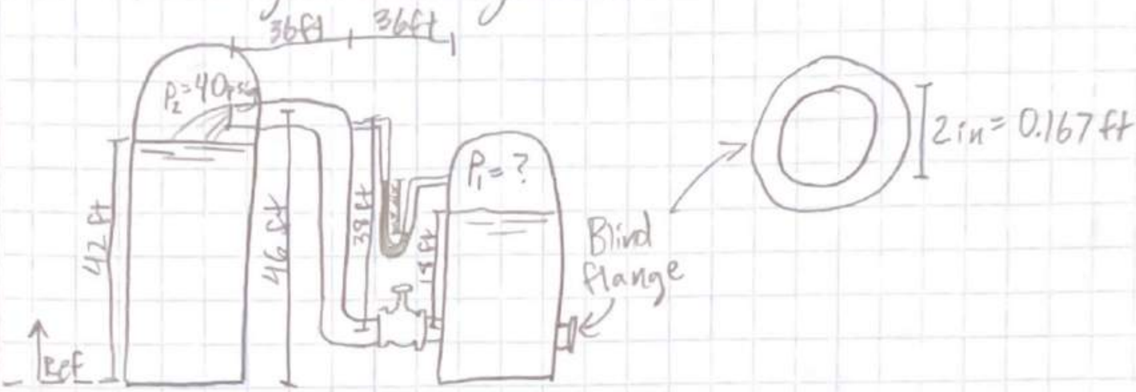
Fluid Mechanics Test #2

Brady Semtner

① Purpose: Find force magnitude + location on a blind flange on the bottom of the right tank if the flow rate is 150 gpm.

Drawings + Diagrams:



Sources: Mott + Untener: Applied Fluid Mechanics 8th edition

Design considerations:

Assume:

- Incompressible fluids
- Steady flow
- Isothermal at 77°F
- Alcohol + mercury don't mix

Data + Variables:

$Q = 150\ gpm = 0.3342\ \frac{ft^3}{s}$ $\gamma_{EA} = 49.01\ \frac{lb}{ft^3}$ $D_{pipe} = 0.1723\ ft$

$P_2 = 40\ psig = 5760\ \frac{lb}{ft^2}$ $\gamma_{Hg} = 844.9\ \frac{lb}{ft^3}$ $f_T = 0.019$ $f = 2.99\times10^{-5}\ \frac{1}{ft}$

Materials:

Ethyl Alcohol, Mercury, Air

Procedure & Calculations:

- First I'll find pressure at P_1
- I'll use these equations

$$\Delta P = \gamma h \quad Q = vA \quad \frac{P_1}{\gamma} + \cancel{\frac{v_1^2}{2g}}^{0} + z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_{L1-2}$$

- $Q = vA$ used to find v_2

$$v_2 = \frac{Q}{A} \quad A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.1723)^2 = 0.02332 \text{ ft}^2$$

$$v_2 = \frac{0.3342 \frac{ft^3}{s}}{0.02332 ft^2} = 14.333 \frac{ft}{s}$$

- Darcy's equation + energy losses at valve, entrance, & elbows

$$h_L = f \cdot \frac{L}{D} \cdot \frac{v_2^2}{2g} + K_{entrance} \cdot \frac{v_2^2}{2g} + K_{gatevalve} \cdot \frac{v_2^2}{2g} + 2K_{elbows} \frac{v_2^2}{2g}$$

$$h_L = f \cdot \frac{L}{D} \cdot \frac{v_2^2}{2g} + 0.5 \frac{v_2^2}{2g} + 8 f_T \frac{v_2^2}{2g} + 2 \cdot 30 f_T \frac{v_2^2}{2g}$$

$$f = \frac{\varepsilon}{D} = \frac{5.0 \times 10^{-6}}{0.1723 ft} = 2.9 \cdot 10^{-5}$$

$$L = 36 + 38 + 36 = 110 \text{ ft}$$

$h_L = 5.78$ ft Plugged into excel sheet from last test

Plug into Bernoulli's

$$P_1 = \left(\frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_{L1-2} - z_1\right)\gamma$$

$$P_1 = \left(\frac{5760 \frac{lb}{ft^2}}{49.01 \frac{lb}{ft^3}} + 46 ft + \frac{(14.333 \frac{ft}{s})^2}{2(32.2 \frac{ft}{s^2})} + 5.78 ft \; 26 ft\right) \cdot 49.01 \frac{lb}{ft^3} \Rightarrow P_1 = 7179.62 \frac{lb}{ft^2}$$

- Now plug into $\Delta P = \gamma h$ & add surface pressure (P_1) to find pressure at flange depth

$$\Delta P = \gamma h + P_1$$

$$\Delta P = (49.01 \frac{lb}{ft^3})(18 ft) + 7179.62 \frac{lb}{ft^2} = 8061.8 \frac{lb}{ft^2}$$

- Use $F = P \cdot A$ to find force magnitude and $L_p = \frac{I_c}{L_c A} + L_c$ to find location

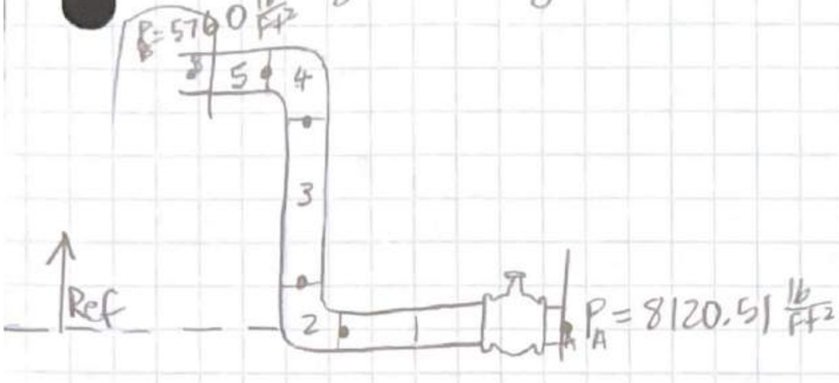
$$F = 8061.8 \frac{lb}{ft^2} \cdot 0.02332 ft^2 = \boxed{188.0 lb = F}$$

Centroid of circle $= \frac{D}{2} \Rightarrow$ Location at centroid. Checked with math: $L_p = \frac{I_c}{L_c A} + L_c = \frac{0.00146 ft^2}{(18 ft)(0.02332 ft^2)} + 18 ft \Rightarrow \boxed{L_p = 18.0035 ft}$

$$I_c = \frac{\pi D^2}{64} = \frac{\pi (0.1723)^2}{64} = 0.00146 ft^2$$

② <u>Purpose:</u> Find Horizontal + vertical forces in the system.

<u>Drawings + Diagrams:</u>



<u>Sources:</u> Mott + Untener: Applied Fluid Mechanics 8th edition

<u>Design Considerations:</u>

Assume:
- Incompressible Fluids
- Steady Flow
- Isothermal at 77°F

<u>Data + Variables:</u>

$\rho_{EA} = 1.53 \frac{slugs}{ft^3}$ $Q = 150gpm = 0.334 \frac{ft^3}{s}$ $v = 14.333 \frac{ft}{s}$

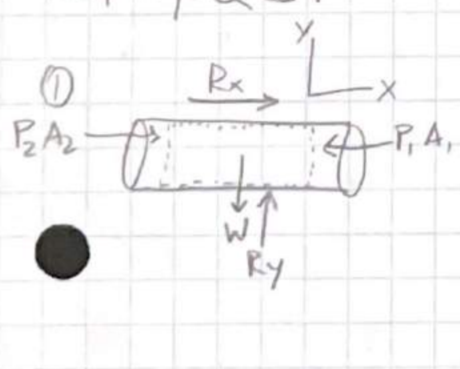
$A = 0.0233 ft^2$

<u>Materials:</u>

Ethyl Alcohol

<u>Procedure + Calculations:</u>

$\Sigma F = \rho Q \Delta v$



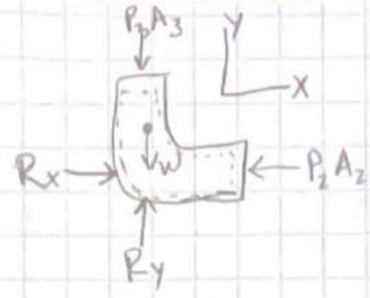
(Y) $R_{y_1} - W = \rho Q(\cancel{V_{2y} - V_{1y}})^0$

$R_{y_1} = W$

(X) $P_2A_2 - P_1A_1 + R_{x_1} = \cancel{\rho Q(v_{2x} - v_{1y})}^0$

$R_{x_1} = P_2A_2 - P_1A_1$

(2)



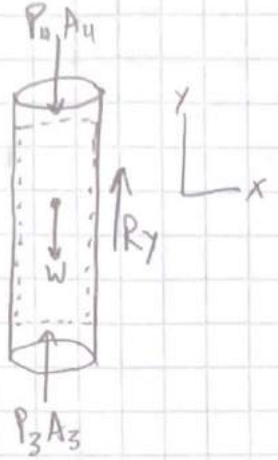
(x) $R_{x_2} - P_2A_2 = \rho Q(v_{2x} - v_{1x})$

$\underline{R_{x_2} = P_2A_2 + \rho Q(-v_{1x})}$

(y) $R_{y_2} - W - P_3A_3 = \rho Q(v_{2y} - v_{1y})$

$\underline{R_{y_2} = W + P_3A_3 + \rho Q(v_{2y})}$

(3)

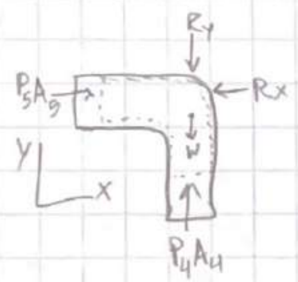


(y) $R_{y_3} - W + P_3A_3 - P_4A_4 = \rho Q\cancel{(v_{2y} - v_{1y})}^0$

$\underline{R_{y_3} = W + P_4A_4 - P_3A_3}$

(x) Nothing in x direction

(4)



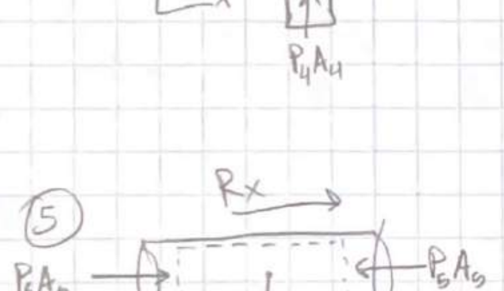
(x) $P_5A_5 - R_{x_4} = \rho Q(v_{2x} - v_{1x})$

$\underline{R_{x_4} = P_5A_5 - \rho Q(v_{2x})}$

(y) $P_4A_4 - W - R_{y_4} = \rho Q(v_{2y} - v_{1y})$

$\underline{R_{y_4} = P_4A_4 - W - \rho Q(0 - v_{1y})}$

(5)



(x) $R_{x_5} + P_6A_6 - P_5A_5 = \rho Q\cancel{(v_{2x} - v_{1x})}^0$

$\underline{R_{x_5} = P_5A_5 - P_6A_6}$

(y) $R_{y_5} - W = \rho Q\cancel{(v_{2y} - v_{1y})}^0$

$\underline{R_{y_5} = W}$

$\Sigma R_x = R_{x_1} + R_{x_2} + \cancel{R_{x_3}}^0 + R_{x_4} + R_{x_5}$

$R_x = P_2A_2 - P_1A_1 + P_2A_2 + \rho Q(-v) + P_5A_5 - \rho Q(v) + P_5A_5 - P_6A$

$R_x = A(P_2 - P_1 + P_2 + P_5 + P_5 - P_6) - 2\rho Q(v)$

$\underline{R_x = A(2P_2 + 2P_5 - P_1 - P_6) - 2\rho Q(v)}$

$$\Sigma R_y = R_{y1} + R_{y2} + R_{y3} + R_{y4} + R_{y5}$$

$$R_y = W + W + \cancel{P_3 A} + \rho Q(v) + W + P_4 A - \cancel{P_3 A} + P_4 A - \cancel{W} + \rho Q(v) + \cancel{W}$$

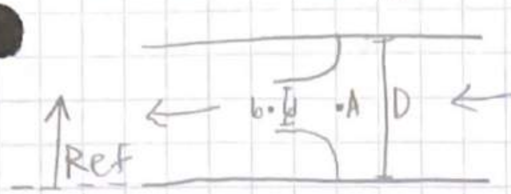
$$\underline{R_y = 3W + 2P_4 A + 2\rho Q(v)}$$

$$2\rho Q v = 2\left(1.53 \frac{\text{lb-s}^2}{\text{ft}^3}\right)\left(0.334 \frac{\text{ft}^3}{\text{s}}\right)\left(14.333 \frac{\text{ft}}{\text{s}}\right) = 14.65 \text{ lb}\cdot\text{ft}$$

$$\boxed{\begin{aligned} R_x &= 0.0233\,\text{ft}^2(2P_2 + 2P_5 - P_1 - P_6) - 14.65\ \text{lb-ft} \\ R_y &= 3W + 2P_4(0.0233\,\text{ft}^2) + 14.65\ \text{lb-ft} \end{aligned}}$$

③ Purpose: Find pressure drop across flow nozzle

Drawings + Diagrams:



Sources: Mott + Untener: Applied Fluid Mechanics 8th edition

Design Considerations:

Assume:
- Incompressible fluids
- Steady flow
- Isothermal at 77°F

Data + Variables:

From problem ①

$Q = 0.3342\ \frac{ft^3}{s}$ $\beta = 0.5$ $v_1 = 14.333\ \frac{ft}{s}$ $\rho_{EA} = 1.53\ \frac{slugs}{ft^3}$

$D = 0.1723\ ft$ $\gamma_{EA} = 49.01\ \frac{lb}{ft^3}$ $\eta_{EA} = 2.1\cdot10^{-5}\ \frac{lb\cdot s}{ft^2}$

Materials:

Ethyl Alcohol

Procedure + Calculations:

- Find d:

$\beta = \frac{d}{D} \Rightarrow 0.5 = \frac{d}{0.1723ft} \Rightarrow d = 0.08615\ ft$

- Find Area in tube + in throat of nozzle:

$A_1 = \frac{\pi D^2}{4} = \frac{\pi(0.1723)^2}{4} = \underline{0.0233\ ft^2}$

$A_2 = \frac{\pi d^2}{4} = \frac{\pi(0.08615)^2}{4} = \underline{0.005829\ ft^2}$

- Find N_R:

$N_R = \frac{\rho v D}{\eta} = \frac{(1.53\ \frac{slugs}{ft^3})(14.333\ \frac{ft}{s})(0.1723\ ft)}{2.1\cdot10^{-5}\ \frac{lb\cdot s}{ft^2}} = 180051.78$

- Find C:

$C = 0.9975 - 6.53\sqrt{\frac{\beta}{N_R}} = 0.9975 - 6.53\sqrt{\frac{0.5}{180051.78}} \Rightarrow \underline{C = 0.9866}$

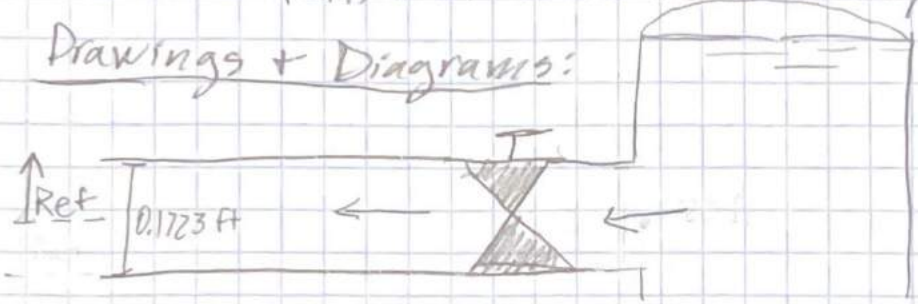
· Plug into this equation to find pressure drop:

$$V_1 = C\sqrt{\frac{\frac{2g(P_1-P_2)}{\gamma}}{\left(\frac{A_1}{A_2}\right)^2-1}} \qquad 14.333\ \tfrac{ft}{s} = 0.9866\sqrt{\frac{\frac{(2)(32.2\frac{ft}{s^2})(\Delta P)}{49.01\frac{lb}{ft^3}}}{\left(\frac{0.0233\,ft^2}{0.005829\,ft^2}\right)-1}}$$

· This gives: $\boxed{\Delta P = 482.08\ \tfrac{lb}{ft^2}}$

(4) Purpose: Find pressure increment in pipe if valve closes, if it would fail, + if there would be any cavitation.

Drawings + Diagrams:



Sources: Mott + Untener: Applied fluid mechanics 8th edition

Design Considerations:

Assume:

· Isothermal at 77°F

· Incompressible fluid (Ethyl Alcohol)

Data + Varidbles:

$E = 200\ GPa$ $\quad E_{steel\ (seamless)} = 1.00$ $\quad D_{outer\ diameter} = 2.375\ in = 0.1979\ ft$ $\quad D_{inner} = 0.1723\ ft$

$Y_{steel} = 0.40$ $\quad \delta = t_{basic\ wall\ thickness} = 0.154\ in = 0.0128\ ft$ $\quad \rho_{EA} = 1.53\ \tfrac{slugs}{ft^3}$

$E_{0\ Ethyl\ Alcohol} = 130{,}000\ psi = 18720000\ \tfrac{lb}{ft^2}$

Materials:

Ethyl Alcohol, steel

Procedure + Calculations:

$$P = P_{oper} + \Delta P \qquad \Delta P = \rho V C \qquad C = \frac{\sqrt{\frac{E_0}{\rho}}}{\sqrt{1+\frac{E_0 D_{inner}}{E \delta}}}$$

$$t = \frac{P D_{outer}}{2(SE + PY)}$$ for the pipe not to break: $t < t_{actual}$

$t < 0.0128\ ft$

$E = 200\ GPa = 2.9 \cdot 10^7\ Psi = 4176000000\ \frac{lb}{ft^2}$

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + \cancel{Z_A} = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + \cancel{Z_B} + h_{L_{A-B}}$$

$P_{oper} = 8061.8\ \frac{lb}{ft^2}$ ← From problem ① → $V = 14.333\ ft/s$

$$C = \frac{\sqrt{\frac{18720000\ \frac{lb}{ft^2}}{1.53\ \frac{slugs}{ft^3}}}}{\sqrt{1+\frac{(18720000\frac{lb}{ft^2})(0.1723ft)}{(4176000000\frac{lb}{ft^2})(0.0128ft)}}} = 3396.9\ \frac{ft}{s}$$

$$\Delta P = \rho V C = (1.53\ \tfrac{slugs}{ft^3})(14.333\ \tfrac{ft}{s})(3396.9\ \tfrac{ft}{s}) = 74476.7\ \tfrac{lb}{ft^2}$$

$$P = P_{oper} + \Delta P = 8061.8\ \tfrac{lb}{ft^2} + 74476.7\ \tfrac{lb}{ft^2} = \boxed{82538.5\ \tfrac{lb}{ft^2}}$$

$$t = \frac{P D_{outer}}{2(SE+PY)} = \frac{(82538.5\frac{lb}{ft^2})(0.1979\ ft)}{2((2882189.92\frac{lb}{ft^2})(1.00)+(82538.5\frac{lb}{ft^2})(0.4))}$$ $S = 20\ ksi = 2882189.92\ \frac{lb}{ft^2}$

$t = 0.0028\ ft$ $\boxed{0.0028\ ft < 0.0128\ ft \therefore \text{Pipe won't break}}$

there will be cavitation if $P_{oper} > P_{sat_{@T=77°F}}$

The point with the lowest pressure in the system is in the left tank, which is not vaporizing. Therefore the system will not experience cavitation because the place with the lowest pressure is not vaporizing.

Part A

Summary:

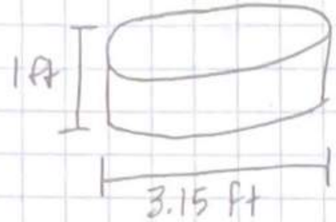
The system pumps ethyl Alcohol. If a blind flange is placed at the Bottom of the right tank The force on it will be 188.0 lb at its centroid. As the fluid flows through the pipes, it exerts forces on the pipes at the straight portions and the elbows. If a flow nozzle were to be placed in the pipe, it would experience a pressure drop of 482.08 $\frac{lb}{ft^2}$ across it. If the valve were suddenly closed, the pipe would be able to withstand the pressure and no cavitation is present in the system.

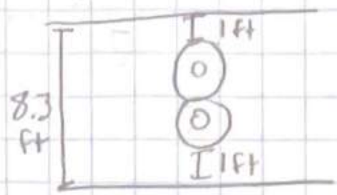
Analysis:

If a flow nozzle were to be placed in the system, there is a good chance it would experience cavitation. It would create a low pressure environment + it might damage the flow nozzle. My best solution for this would be to use a cheaper instrument to measure the flow rate like an orifice plate. When it is damaged, it will be cheaper to replace.

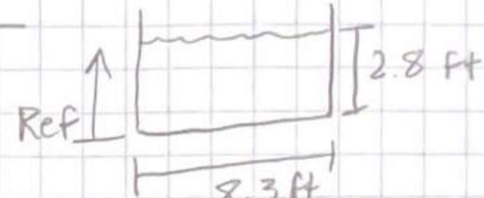
(5) Purpose: Design a lazy river that won't drown a 4-year-old + determine it's flow rate

Drawings + Diagrams:





Giving 1ft on either side to give room for legs



Sources: Mott + Ontener: Applied Fluid mechanics 8th edition

Design Considerations:

Assume:

- Average height of 4-year-old in America = 3.33 ft
- Height up to neck ≈ 2.8 ft
- Open channel
- Trowel finished concrete

Data + Variables:

Slope = 0.1% ⇒ $S = 0.001$ Depth = 2.8 ft

$n = 0.013$ Width = 8.3 ft

Materials:

Water

Procedure + Calculations:

- Find A, WP, R

$A = h \cdot b = 2.8\,ft \cdot 8.3\,ft = 23.24\,ft^2$ $R = \frac{A}{WP} = \frac{23.24\,ft^2}{13.9\,ft} = 1.67\,ft$

$WP = 2h + b = 2 \cdot 2.8\,ft + 8.3\,ft = 13.9\,ft$

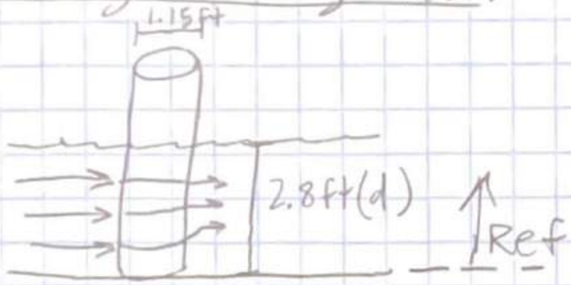
- Find Q:

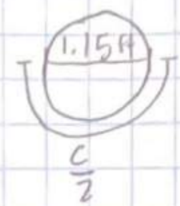
$Q = \left(\frac{1.49}{n}\right) A R^{2/3} S^{\frac{1}{2}}$

$Q = \left(\frac{1.49}{0.013}\right)(23.24\,ft^2)(1.67\,ft)^{\frac{2}{3}}(0.001)^{\frac{1}{2}} \Rightarrow \boxed{Q = 118.57\,\frac{ft^3}{s}}$

(6) Purpose: Find drag force a 4-year-old would experience in the lazy river.

Drawings + Diagrams:





$C = \pi(D)$

$C = \pi(1.15ft)$

$C = 3.613$

$\frac{C}{2} = 1.8 ft$

Sources:

Mott + Untener: Applied Fluid Mechanics 8th edition

Design considerations:

Assume:

- Child is a cylinder
- $D = 1.15$ ft
- $T = 80°F$

Data + Variables:

$Q = 118.57 \frac{ft^3}{s}$ $\rho = 1.93 \frac{slugs}{ft^3}$ $\nu = 9.15 \cdot 10^{-6} \frac{ft^2}{s}$

$A_{LR} = 23.24 ft^2$ $T = 80°F$ $C_D = 2 \cdot 10^{-1}$ (from graph 17-4)

$D = 1.15$ ft $d = 2.8$ft

Materials:

water, 4-year-old child

Procedure + Calculations:

- Find v:

$v = \frac{Q}{A_{LR}} = \frac{118.57 \frac{ft^3}{s}}{23.24 ft^2} \Rightarrow v = 5.1 \frac{ft}{s}$

- Find N_R

$N_R = \frac{vD}{\nu} = \frac{(5.1 \frac{ft}{s})(1.15 ft)}{9.15 \cdot 10^{-6} \frac{ft^2}{s}} = 6.4 \times 10^5 \Rightarrow$ Use graph 17-4 to find C_D

$C_D = 2 \cdot 10^{-1}$

Find drag force:

$A_{child} = d \cdot \frac{c}{2} = 2.8ft \cdot 1.8 ft = 5.07 ft^2$

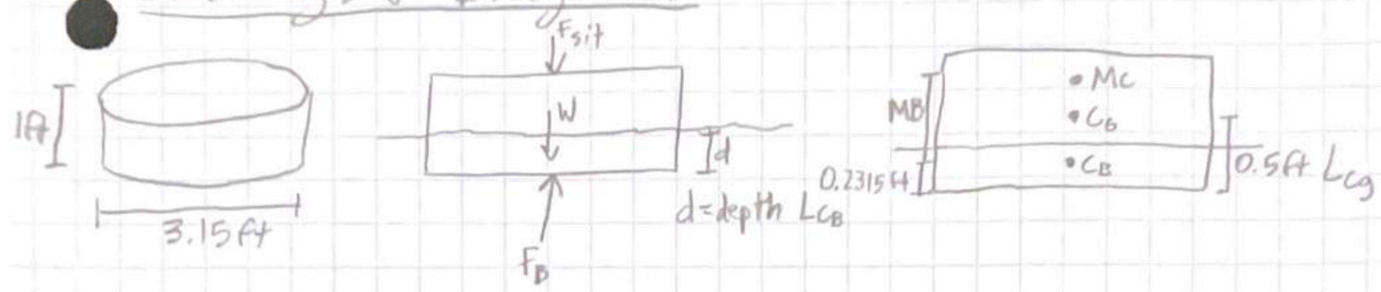
$F_D = C_D \left(\frac{\rho v^2}{2}\right) A_{child}$

$F_D = (2 \cdot 10^{-1}) \left(\frac{1.93 \frac{slugs}{ft^3} \cdot (5.1 \frac{ft}{s})^2}{2}\right)(5.07 ft^2)$

$\boxed{F_D = 25.45 \text{ lb}}$

7 Purpose: If a 220 lb person sits on the tube, how deep does it go? Is it stable?

Drawings + Diagrams:



Sources: Mott + Untener: Applied Fluid Mechanics 8th edition

Design Considerations:

Assume:

- Innertube is hollow short cylinder
- 220 lb is just a force, not considering shape of person

Data + Variables:

$F_{sit} = 220 lbs$ $\quad \gamma_{air\,80°F} = 0.0736 \frac{lb}{ft^3}$ $\quad D = 3.15 ft$

$W = 2kg = 4.41 lbs$ $\quad \rho_{air} = 2.28 \cdot 10^{-3} \frac{slugs}{ft^3}$ $\quad h = 1 ft$

$\gamma_{H_2O\,80°F} = 62.2 \frac{lb}{ft^3}$

Materials:

Water, air

Procedure + calculations:

- I will add F_{sit} to W to simplify it

$F = F_{sit} + W = 220 lb + 4.41 lb = 224.41 lb$

- F must $= F_B$ because it is floating $\therefore F = F_B = 224.41 lb$

- I will find displaced Volume:

$\frac{F_B}{\gamma_{H_2O}} = V_D = \frac{224.41 lb}{62.2 \frac{lb}{ft^3}} \Rightarrow V_D = 3.61 ft^3 = V_{H_2O}$

- I will use V_{H_2O} to find depth:

$V_{H_2O} = \frac{\pi}{4}(D^2) \cdot d \Rightarrow 3.61 ft^3 = \frac{\pi}{4}(3.15 ft^2) \cdot d \Rightarrow \boxed{d = 0.463 ft}$

- I will find the full mass of the tube with the force on it in slugs

$$m = \frac{W}{g} = \frac{224.14\,lb}{32.2\,\frac{ft}{s^2}} = 6.96\ slugs$$

- I place C_B at centroid of displaced volume + C_g at centroid of the volume of the tube

- $MB > L_{cg} - L_{CB}$ MB must be over C_g to be stable

$MB > 0.5ft - 0.2315\,ft$

$MB > 0.2685\,ft$

MB must be over 0.2685 ft to be stable

- I will find MB

$$MB = \frac{I}{\forall_d} = \frac{\frac{\pi D^4}{64}}{\forall_d} = \frac{\frac{\pi (3.15ft)^4}{64}}{3.61\,ft^3} \Rightarrow \underline{MB = 1.34\,ft}$$

$$I_{circle} = \frac{\pi D^4}{64}$$

$MB = 1.34\,ft$ $1.34ft > 0.2685\,ft$ $\therefore$ $\boxed{\text{stable}}$

Part B

Summary:

A lazy river was designed to fit 2 innertubes of a given size and be shallow enough to be safe for a 4-year-old. I made the river 8.3 ft wide to give space for riders' legs and I made it 2.8 ft deep to be up to the neck of a 4 year-old. I found I would need a flow rate of 118.57 $\frac{ft^3}{s}$. When the 4-year-old stands in the water, it experiences a drag force of 25.45 lb. If a 220 lb individual sat on the tube, it would sink 0.463 ft into the water, but it would be stable.

Analysis:

The innertube can most likely support a much larger person, or perhaps a person holding their child. However calculations were performed assuming the tube was a cylinder so the real tube may be more or less stable. The gaps at the side are also important because if someone becomes trapped under a bunch of innertubes with people in them, they need to have a space to get to the surface. I have experienced this before in lazy rivers. If the slope was increased the required, the flow rate could be increased without having to use jets as much. The flow rate could also be increased if a smoother surface were used for the pool walls and floor.