Fluid Mechanics Test 3

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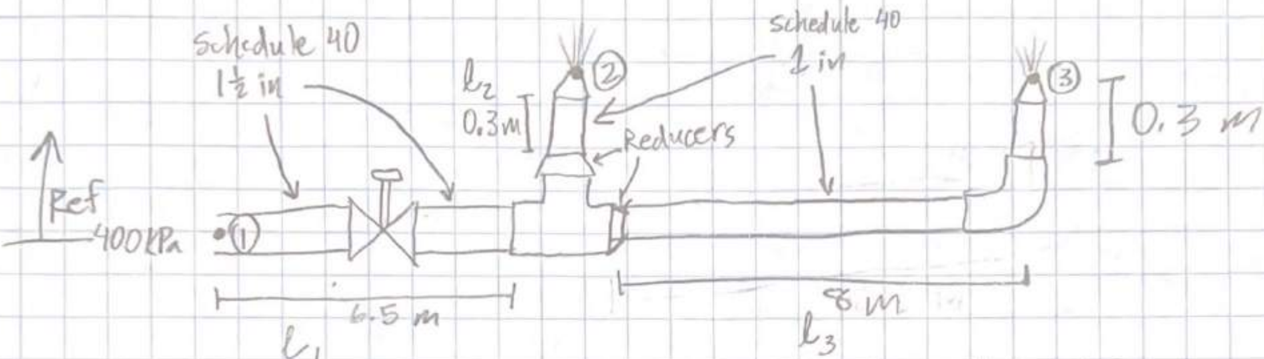
① <u>Purpose:</u> Find the flows through each sprinkler. Propose ways to make the flows equal. Compare velocity to the critical velocity + propose ways to keep it below the critical velocity

Drawings + Diagrams:



Sources: Mott + Untener: Applied Fluid Mechanics 8th edition

Design Considerations:

Assume:

- Incompressible fluids
- Steady flow
- Isothermal
- Parallel system
- Reducers are 30° angles
- Valve is similar to gate valve
- Temp is 20°C

Data + Variables

$K_{\text{sprinkler head}} = 50$

$K_{elbow} = 30 f_T$

$K_{Tee_{thru}} = 20 f_T$

$K_{Tee_{Turn}} = 60 f_T$

$P_1 = 400\ kPa$

$P_2 = 101.325\ kPa$

$P_3 = 101.325\ kPa$

$K_{valve} = 8 f_T$

$\gamma_{water} = 9.79\ \frac{kN}{m^3}$

$\rho = 998\ \frac{kg}{m^3}$

$\nu = 1.02 \times 10^{-6}\ \frac{m^2}{s}$

$\eta = 1.02 \times 10^{-3}\ Pa \cdot s$

$K_{Reducer} = 0.05 \leftarrow$ from figure 10.12

$\varepsilon = 4.6 \times 10^{-5}\ m$

$D_1 = 0.0409\ m$

$D_2 = D_3 = 0.0266\ m$

$g = 9.81\ m/s$ $\quad \varepsilon = 4.6\times10^{-5}\ m$

· Materials:
Water, steel

Procedure + calculations:

$$\frac{D_1}{D_2} = \frac{40.9\,mm}{26.6\,mm} = 1.54 \Rightarrow K_{Reducer} = 0.05$$

$$Q = vA \qquad v = \frac{Q}{A} \qquad v = \frac{4Q}{\pi D^2} \qquad v^2 = \frac{16Q^2}{\pi^2 D^4}$$

$$A = \frac{\pi (D)^2}{4}$$

Area plugged in — Will be used in Bernoulli's later

$$A_1 = \frac{\pi (D_1)^2}{4} = \frac{\pi(0.0409m)^2}{4} = 0.00131\ m^2$$

$$A_2 = \frac{\pi (D_2)^2}{4} = \frac{\pi(0.0266m)^2}{4} = 0.00056\ m^2 = A_3$$

Bernoulli's used from point 1 to point 2:

$1\to2$

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + \cancel{z_1}^{\,0} = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_{L_{1\to2}}$$

Rearranged + equation for v^2 plugged in:

$$\frac{P_1 - P_2}{\gamma} + \frac{8Q_1^2}{g\pi^2 D_1^4} - \frac{8Q_2^2}{g\pi^2 D_2^4} - z_2 = h_{L_{1\to2}}$$

For pipe energy losses, Darcy-Weisbach equation used:

$h_L = f\frac{l}{D}\cdot\frac{v^2}{2g}$ equation for v^2 plugged in: $h_L = f\frac{l}{D}\cdot\frac{8Q_1^2}{g\pi^2 D_1^4}$

$$h_{L_{1\to2}} = f_1\cdot\frac{l_1}{D_1}\cdot\frac{8Q_1^2}{g\pi^2 D_1^4} + K_{valve}\cdot\frac{8Q_1^2}{g\pi^2 D_1^4} + K_{\substack{Tee\\turn}}\cdot\frac{8Q_1^2}{g\pi^2 D_1^4} + K_{Red}\cdot\frac{8Q_2^2}{g\pi^2 D_2^4} + f_2\frac{l_2}{D_2}\cdot\frac{8Q_2^2}{g\pi^2 D_2^4} + K_{sprinkler}\cdot\frac{8Q_2^2}{g\pi^2 D_2^4}$$

$$h_{L_{1\to2}} = \left(f_1\frac{l_1}{D_1} + K_{valve} + K_{\substack{Tee\\turn}}\right)\frac{8Q_1^2}{g\pi^2 D_1^4} + \left(f_2\frac{l_2}{D_2} + K_{Red} + K_{sprinkler}\right)\frac{8Q_2^2}{g\pi^2 D_2^4}$$

Plug into Bernoulli's:

$$\frac{P_1 - P_2}{\gamma} + \frac{8Q_1^2}{g\pi^2 D_1^4} - \frac{8Q_2^2}{g\pi^2 D_2^4} - z_2 = \left(f_1\frac{l_1}{D_1} + K_{valve} + K_{\substack{Tee\\turn}}\right)\frac{8Q_1^2}{g\pi^2 D_1^4} + \left(f_2\frac{l_2}{D_2} + K_{Red} + K_{sprinkler}\right)\frac{8Q_2^2}{g\pi^2 D_2^4}$$

Solve for Q_2:

$$Q_2 = \sqrt{\frac{\frac{P_1 - P_2}{\gamma} - z_2 - \left(f_1 \frac{l_1}{D_1} + K_{valve} + K_{\substack{Tee\\turn}} - 1\right)\frac{8Q_1^2}{g\pi^2 D_1^4}}{\left(f_2 \cdot \frac{l_2}{D_2} + K_{Red} + K_{sprinkler} + 1\right)\frac{8Q_2^2}{g\pi^2 D_2^4}}}$$

Bernoulli's used from point 1 to point 3:

$\underline{1 \to 3}$

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + \cancelto{0}{z_1} = \frac{P_3}{\gamma} + \frac{v_3^2}{2g} + z_3 + h_{L_{1\to3}}$$

Rearranged + equation for v^2 plugged in

$$\frac{P_1 - P_3}{\gamma} + \frac{8Q_1^2}{g\pi^2 D_1^4} - \frac{8Q_3^2}{g\pi^2 D_3^4} - z_3 = h_{L_{1\to3}}$$

For energy losses, Darcy-Weisbach equation used

$$h_{L_{1\to3}} = f_1 \frac{l_1}{D_1} \cdot \frac{8Q_1^2}{g\pi^2 D_1^4} + K_{valve} \frac{8Q_1^2}{g\pi^2 D_1^4} + K_{\substack{Tee\\thru}} \cdot \frac{8Q_1^2}{g\pi^2 D_1^4} + K_{Red} \frac{8Q_3^2}{g\pi^2 D_3^4} + K_{sprinkler} \frac{8Q_3^2}{g\pi^2 D_3^4} + f_3 \frac{l_3}{D_3} \cdot \frac{8Q_3^2}{g\pi^2 D_3^4} + K_{elb} \cdot \frac{8Q_3^2}{g\pi^2 D_3^4}$$

$$h_{L_{1\to3}} = \left(f_1 \frac{l_1}{D_1} + K_{valve} + K_{\substack{Tee\\thru}}\right)\frac{8Q_1^2}{g\pi^2 D_1^4} + \left(f_3 \frac{l_3}{D_3} + K_{Red} + K_{sprinkler} + K_{elb}\right)\frac{8Q_3^2}{g\pi^2 D_3^4}$$

Plug into Bernoulli's

$$\frac{P_1 - P_3}{\gamma} + \frac{8Q_1^2}{g\pi^2 D_1^4} - \frac{8Q_3^2}{g\pi^2 D_3^4} - z_3 = \left(f_1 \frac{l_1}{D_1} + K_{valve} + K_{\substack{Tee\\thru}}\right)\frac{8Q_1^2}{g\pi^2 D_1^4} + \left(f_3 \frac{l_3}{D_3} + K_{Red} + K_{sprinkler} + K_{elb}\right)\frac{8Q_3^2}{g\pi^2 D_3^4}$$

Solve for Q_3:

$$Q_3 = \sqrt{\frac{\frac{P_1 - P_3}{\gamma} - z_3 - \left(f_1 \frac{l_1}{D_1} + K_{valve} + K_{\substack{Tee\\thru}} - 1\right)\frac{8Q_1^2}{g\pi^2 D_1^4}}{\left(f_3 \frac{l_3}{D_3} + K_{Red} + K_{sprinkler} + K_{elb} + 1\right)\frac{8Q_3^2}{g\pi^2 D_3^4}}}$$

For Parallel systems: $Q_{total} = \sum Q_n$

$\therefore Q_1 = Q_2 + Q_3$

$$Q_1 = \sqrt{\frac{\frac{P_1 - P_2}{\gamma} - z_2 - \left(f_1 \frac{l_1}{D_1} + K_{valve} + K_{Tee,\,branch} - 1\right)\frac{8Q_1^2}{g\pi^2 D_1^4}}{\left(f_2 \frac{l_2}{D_2} + K_{Red} + K_{sprinkler}\right)\frac{8}{g\pi^2 D_2^4}}} + \sqrt{\frac{\frac{P_1 - P_3}{\gamma} - z_3 - \left(f_1 \frac{l_1}{D_1} + K_{valve} + K_{Tee,\,thru} - 1\right)\frac{8Q_1^2}{g\pi^2 D_1^4}}{\left(f_3 \frac{l_3}{D_3} + K_{Red} + K_{sprinkler} + K_{elb} + 1\right)\frac{8}{g\pi^2 D_3^4}}}$$

When plugged into excel, this gives:

$Q_1 = 0.0086 \frac{m^3}{s}$

$\boxed{Q_2 = 0.00738 \frac{m^3}{s}}$

$\boxed{Q_3 = 0.00148 \frac{m^3}{s}}$

The flows through each sprinkler are not the same. The flow through the first nozzle is significantly higher than through the second nozzle. I would make them the same by placing a valve after the tee on the pipe before the first sprinkler head. If it is allowed, the length of pipe to the second sprinkler could also be shortened which would reduce losses + increase the flow rate through the second sprinkler.

$Q = VA \qquad V_{critical} = 3 \frac{m}{s}$

$$V_1 = \frac{Q_1}{A_1} = \frac{0.0086 \frac{m^3}{s}}{0.00131 m^2} = 6.565 \frac{m}{s} > V_{critical}$$

$$V_2 = \frac{Q_2}{A_2} = \frac{0.00738 \frac{m^3}{s}}{0.00056 m^2} = 13.179 \frac{m}{s} > V_{critical}$$

$$V_3 = \frac{Q_3}{A_3} = \frac{0.00148 \frac{m^3}{s}}{0.00056 m^2} = 2.64 \frac{m}{s} < V_{critical}$$

The velocity at point 1 is over twice as fast as the critical velocity + the velocity at point 2 is over four times as fast as the critical velocity. The velocity at point 3 is slightly below critical velocity + may be left alone. To bring the velocities at points 1 + 2 down, I would close the valve partially + perhaps add a valve between the tee + the first sprinkler.

Summary:

The system pumps water through a valve and to two sprinkler heads. The flow through the first sprinkler is $0.00738 \frac{m^3}{s}$. This gives a fluid velocity of $13.179 \frac{m}{s}$ which is much greater than the critical velocity of $3 \frac{m}{s}$. The flow through the second sprinkler is $0.00148 \frac{m^3}{s}$. This gives a fluid velocity of $2.64 \frac{m}{s}$ which is below the critical velocity. The speed of the fluid early on the system is high enough to make the flow turbulent, which increases the risk of cavitation.

Analysis:

The flows through the sprinklers are not the same which would lead to uneven coverage of the area being watered. This could be solved by placing a valve in front of the first sprinkler head. This would add losses to the first flow + cause Mother nature to balance the flows more evenly. The length of pipe to the second sprinkler could also be shortened. This would reduce losses for the second flow + therefore increase it, balancing the two out. I think the valve would be the best think to do because if the flow to the first sprinkler were decreased, the velocity would also be decreased + it would be brought down closer to the critical velocity.