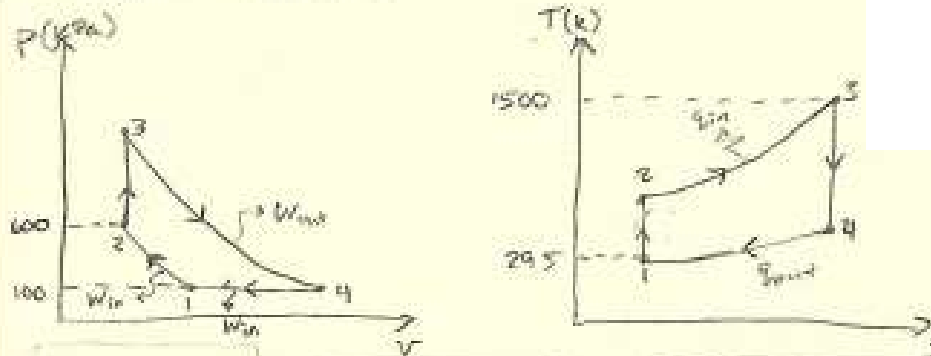


## 13) Isentropic Process (Air)

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① (1→2)

$$P_1 = 100 \text{ kPa}$$

$$T_1 = 295 \text{ K}$$

$$V_1 = 0.857 \text{ m}^3$$

② (2→3)

$$P_2 = 600 \text{ kPa}$$

$$T_2 = 490 \text{ K}$$

$$V_2 = 0.234 \text{ m}^3$$

③ (3→4)

$$P_3 = 1839.74 \text{ kPa}$$

$$T_3 = 1500 \text{ K}$$

$$V_3 = V_2 = 0.234 \text{ m}^3$$

④ (4→1)

$$P_4 = 100 \text{ kPa}$$

$$T_4 = 720 \text{ K}$$

$$V_4 = 2.07 \text{ m}^3$$

$$R_{\text{air}} = 0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$V_1 = \frac{RT_1}{P_1} = \frac{(0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(295 \text{ K})}{100 \text{ kPa}} = 0.847 \text{ m}^3$$

$$\frac{P_2}{P_1} = \frac{P_{12}}{P_{11}} \Rightarrow \frac{600 \text{ kPa}}{100 \text{ kPa}} = \frac{P_2}{1.3068} \Rightarrow P_{r2} = 7.3408 \Rightarrow T_2 = 490 \text{ K}$$

$$V_2 = \frac{RT_2}{P_2} = \frac{(0.287)(490 \text{ K})}{600 \text{ kPa}} = 0.234 \text{ m}^3$$

$$P_3 = \frac{RT_3}{V_3} = \frac{(0.287)(1500 \text{ K})}{0.234} = 1839.74 \text{ kPa}$$

$$\frac{P_4}{P_3} = \frac{P_{r4}}{P_{r3}} \Rightarrow \frac{100 \text{ kPa}}{1839.74 \text{ kPa}} = \frac{P_{r4}}{601.9} \Rightarrow P_{r4} = 32.72 \rightarrow T_4 = 720 \text{ K}$$

$$V_4 = \frac{RT_4}{P_4} \rightarrow V_4 = 2.0664 \text{ m}^3$$

① → ②  
(Isentropic)② → ③  
( )

③ → ④

④ → ①  
(Isobaric)

Processes:

1→2

$$Q_{in} - Q_{out} + W_{in} - W_{out} = \Delta U \rightarrow W_{in} = \Delta U = c_v(T_2 - T_1)$$

$$W_{in} = 0.718(490 - 295) = 140.01 \frac{\text{kJ}}{\text{kg}}$$

2 → 3

$$\cancel{q_{in}} - \cancel{q_{out}} + \cancel{w_{in}} - \cancel{w_{out}} = \Delta U$$

$$q_{in} = C_v (\Delta T) \rightarrow q_{in} = .718 (1500 - 2190) = \underline{725.18 \frac{kJ}{kg}}$$

3 → 4

$$\cancel{q_{in}} - \cancel{q_{out}} + \cancel{w_{in}} - \check{w_{out}} = \Delta U$$

$$-w_{out} = .718 (720 - 1500) \rightarrow \underline{w_{out} = 560.04 \frac{kJ}{kg}}$$

4 → 1

$$+q_{out} + w_{in} = \Delta U$$

$$-q_{out} = .718 (720 - 295) = -121.3$$

$$\underline{q_{out} = -121.3 \frac{kJ}{kg}}$$

$$\begin{aligned} w_{in} &= P_{114} (\Delta V) = 100 \text{ kPa} (2.07 \text{ m}^3 - .857 \text{ m}^3) \\ &= 100 (1.213) \\ &= \underline{121.3 \frac{kJ}{kg}} \end{aligned}$$

$$\boxed{W_{net} = -140.01 \frac{kJ}{kg} - 121.3 \frac{kJ}{kg} + 560.04 \frac{kJ}{kg} = 298.73 \frac{kJ}{kg}}$$

$$q_{in} = 725.18 \frac{kJ}{kg}$$

$$\eta = \frac{298.73}{725.18} = .411 \rightarrow \boxed{41.1 \%}$$

9-18  $T_1 = 350\text{ K}$   $T_2 = 1200\text{ K}$

$P_1 = 150\text{ kPa}$

from Table A-17

$P_2 = 300\text{ kPa}$

@ 350 K -  $P_{r1} = 2.379$ ,  $U = 250.02$ ,  $V_r = 422.2$

$W_{\text{net}} = .5\text{ kJ}$

$S^* = 1.35$

$P_{\text{max}} = ?$

@ 1200 K -  $P_{r2} = 238$ ,  $U = 933.33$ ,  $V_r = 14.49$

$S^* = 3.18$

$Q_{\text{in}} = ?$

$W_{\text{net}} = ?$   $R = .287\text{ kJ/kg}\cdot\text{K}$

a)  $\frac{P_2}{P_1} = \frac{P_{r2}}{P_{r1}}$   $P_1$  is highest pressure

$P_1 = \frac{P_{r1}}{P_{r2}} (P_2) \Rightarrow P_1 = \frac{238}{2.379} (300\text{ kPa}) = \boxed{P_1 = 30.012\text{ MPa}}$

b)  $\eta = \frac{W_{\text{net}}}{Q_{\text{in}}} = 1 - \frac{T_2}{T_1} \Rightarrow \frac{500\text{ J}}{Q_{\text{in}}} = 1 - \frac{350\text{ K}}{1200\text{ K}}$

$Q_{\text{in}} = \frac{500\text{ J}}{.29166} = \boxed{Q_{\text{in}} = 705.8\text{ J}}$

c)  $\Delta S = (S_2 - S_1) - R \ln\left(\frac{P_2}{P_1}\right)$

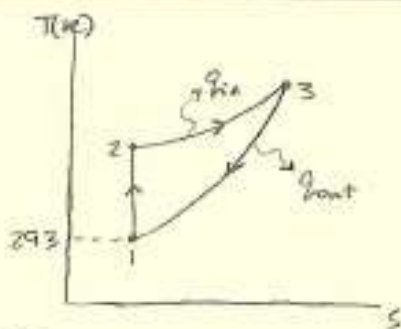
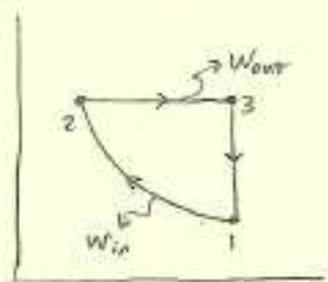
$\Delta S = R \ln\left(\frac{P_2}{P_1}\right) \rightarrow .287 \ln\left(\frac{300\text{ kPa}}{150\text{ kPa}}\right) = .2\text{ kJ/kg}\cdot\text{K}$

$W = \Delta S(\Delta T)$   
 $= .2\text{ kJ/kg}\cdot\text{K} (1200\text{ K} - 350\text{ K})$

$W = 170\text{ kJ/kg}$

$W_{\text{net}} = .5\text{ kJ}$ , therefore mass =  $\frac{.5\text{ kJ}}{170\text{ kJ/kg}} = \boxed{m = .003\text{ kg}}$

22) R



$$C_v = 0.7 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad R = 0.2 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad \gamma = 5$$

①

$$P_1 =$$

$$T_1 = 293 \text{ K}$$

$$V_1 =$$

②

$$P_2 = P_3$$

$$T_2 = 585.36 \text{ K}$$

$$V_2 =$$

③

$$P_3 = P_2$$

$$T_3 = 2926.8 \text{ K}$$

$$V_3 = V_1$$

$$C_v + R = C_p = 1 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$\frac{C_p}{C_v} = \gamma = 1.43$$

$$\gamma = \frac{V_1}{V_2}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \rightarrow T_2 = T_1 (\gamma)^{\gamma-1} \rightarrow T_2 = 293 (5)^{1.43-1} = 585.36$$

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \rightarrow V_3 T_2 = V_2 T_3 \rightarrow \left(\frac{V_3}{V_2}\right) T_2 = T_3 \rightarrow \gamma T_2 = T_3 \rightarrow 5 (585.36) = 2926.8$$

Processes

$$1 \rightarrow 2: q_{in} - q_{out} + W_{in} - W_{out} = C_v (T_2 - T_1)$$

$$W_{in} = 0.7 (585.36 - 293) = 204.652 \frac{\text{kJ}}{\text{kg}}$$

$$2 \rightarrow 3: q_{in} - q_{out} + W_{in} - W_{out} = C_v (T_3 - T_2)$$

$$q_{in} = C_p (T_3 - T_2)$$

$$q_{in} - W_{out} = .7 (2926.8 - 585.36)$$

$$q_{in} = 2341.44 \frac{\text{kJ}}{\text{kg}}$$

$$W_{out} = 2341.44 - .7 (2926.8 - 585.36) = 702.43 \frac{\text{kJ}}{\text{kg}}$$

$$3 \rightarrow 1: q_{in} - q_{out} + W_{in} - W_{out} = C_v (T_1 - T_3)$$

$$-q_{out} = .7 (293 - 2926.8) = -1843.66$$

$$q_{out} = 1843.66 \frac{\text{kJ}}{\text{kg}}$$

$$W_{net} = -204.652 + 702.43 = 497.78 \frac{\text{kJ}}{\text{kg}}$$

$$\eta = \frac{497.78}{2341.44} = .213 = \boxed{21.3\%}$$

$$1 \rightarrow 2: W_{in} = C_v (T_2(r)^{k-1} - T_1) \rightarrow W_{in} = C_v T_1 ((r)^{k-1} - 1)$$

$$2 \rightarrow 3: W_{out} = C_p (r T_1(r)^{k-1} - T_1(r)^{k-1}) - C_v (r T_1(r)^{k-1} - T_1(r)^{k-1})$$

$$W_{out} = C_p T_1(r)^{k-1} (r - 1) - C_v T_1(r)^{k-1} (r - 1)$$

$$\eta = \frac{[C_p T_1(r)^{k-1} (r - 1) - C_v T_1(r)^{k-1} (r - 1)] - [C_v T_1 ((r)^{k-1} - 1)]}{[C_p T_1(r)^{k-1} (r - 1)]}$$

9-31

$$C.R. = 10.5$$

$$P_{in} = 8 \text{ kPa}$$

$$T_{in} = 40^\circ\text{C} (273)$$

$$n = 2500 \text{ rpm}$$

$$W_{out} = 90 \text{ kW}$$

$$\eta = ?$$

$$Q_{in} = ?$$

Table A-2

$$\text{Air @ } 300 \text{ K } C_p = 1.005, C_v = 0.718$$

Table A-17

$$T_1 @ 310 \text{ K } U_1 = 221.25 \quad U_\eta = 572.3$$

$$\eta_{\text{eff}} = 1 - \frac{1}{r^{k-1}}$$

$$k = \frac{C_p}{C_v} = \frac{1.005}{0.718} = 1.4$$

$$\eta = 1 - \frac{1}{10.5^{(1.4-1)}}$$

$$= 1 - 0.3904$$

$$\boxed{\eta = 60.9\%}$$

$$\eta = \frac{W_{net}}{Q_{in}} \rightarrow 0.609 = \frac{90 \text{ kW}}{Q_{in}} \rightarrow \boxed{Q_{in} = 147.8 \text{ kW}}$$