Carter Fishback

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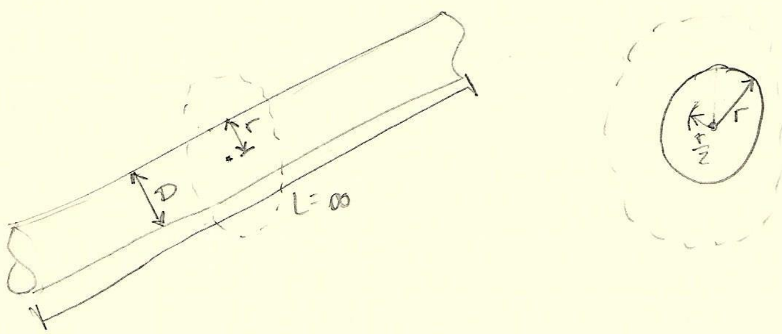
1. Purpose

To Determine a solution using the infinte body model for Transient Flow in a long rod. The solution must be used to solve for the radius that would result a temperature of 30°C at the center of the rod when time, t, equals 1338s. At that time, also Find the temp at the surface and when r/2.

Drawings



Sources

Bayazitoglu, Y. & Ozisik, N., "A Textbook for Heat Transfer fundamentals" Begell House Inc., (2012)

Design Considerations

- Transient Flow through a cylinder body
- Long cylinder assumption means infinite L can be assumed
- Time dependent
- Constant properties

Data and variables

$T_i = 70°C = 343\,K$

$T_{F @ center} = 30°C = 303\,K$

$T_\infty = 25°C = 298\,K$

$t = 1338\,s$

r_i = radius of cylinder

$r_o = r/2$

$\rho = 1190\ kg/m^3$

$C_p = 1465\ J/kg \cdot K$

$k = 0.19\ W/m \cdot k$

$h_\infty = 20.42\ W/m^2 \cdot k$

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Procedure

1. Use radial - Infinite cylinder equation and solve for r
2. Iterate for r value:
   - Guess r
   - Find Bi
   - Interpolate from table 5.1 to get C_1 and ζ_1
   - Plug into equation with guessed r
   - Compute LHS and RHS
   - find % difference
3. Use r to find half the radius
4. Use $\theta = \theta_0 J_0(\zeta_1^2 r_0)$ equation to find the temp at the given time and $\frac{r}{2}$ location
5. Repeat step 4 at the surface of the cylinder

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Calculations

$$B_i = \frac{h\left(\frac{r_o}{3}\right)}{k} \longrightarrow B_i < .1 = \text{Lump Capacitance}$$

$$C(r,t) = \frac{T(r,t) - T_\infty}{T_i - T_\infty} \qquad \alpha = \frac{k}{\rho c_p} \qquad B_i = \frac{h r_o}{k}$$

$$\theta_o^* = C_1 e^{(-\zeta_1^2 \tau)} \qquad \theta^* = \theta_o^* J_o(\zeta_1 r^*) \qquad r^* = \frac{r}{r_o} \qquad \tau = \frac{\alpha t}{r_o^2}$$

Temp @ center point After $T_{center} = 293\,k$ and $t = 1338\,s$

$$\frac{T(r,t) - T_\infty}{T_i - T_\infty} = C_1 e^{(-\zeta_1^2 \tau)} \longrightarrow \ln\left(\frac{T(r,t) - T_\infty}{C_1 (T_i - T_\infty)}\right) = -\zeta_1^2 \left(\frac{\alpha t}{r_o^2}\right)$$

$$\frac{T(r,t) - T_\infty}{T_i - T_\infty} = C_1 \exp\left[-\zeta_1^2 \cdot \left(\frac{\left(\frac{k}{\rho c_p}\right) t}{r_o^2}\right)\right] \qquad r = \sqrt{\frac{-\zeta_1^2 (\alpha t)}{\ln\left[\left(\frac{T(r,t) - T_\infty}{T_i - T_\infty}\right)\frac{1}{C_1}\right]}}$$

Used in Iterations

* Iteration is used to compare the LHS to the RHS *

- r was guessed to Find $B_i = \frac{h r_o}{k}$
- B_i used to fix C_1 and ζ_1
- Iterations on excel

$$\frac{303\,K - 298\,K}{343\,K - 298\,K} = C_1 e^{\left[-\zeta_1^2 \left(\frac{\left(\frac{0.19\,W/m.k}{(1190\,\frac{kg}{m^3})(1465)}\right)(1338\,s)}{r^2}\right)\right]}$$

$$.1111 = C_1 e^{\left[-\zeta_1^2 \left(\frac{(1.46\times 10^{-4})}{r^2}\right)\right]}$$

$$\ln\left(\frac{.1111}{C_1}\right) = -\zeta_1^2 \left(\frac{1.46\times 10^{-4}}{r^2}\right)$$

$$\frac{\ln\left(\frac{.1111}{C_1}\right)}{-\zeta_1^2} = (1.46\times 10^{-4}) \frac{1}{r^2} \longrightarrow r^2 = \frac{(1.46\times 10^{-4})(-\zeta_1^2)}{\ln\left(\frac{.1111}{C_1}\right)}$$

$$\boxed{r = \sqrt{\frac{(1.46\times 10^{-4})(-\zeta_1^2)}{\ln\left(\frac{.1111}{C_1}\right)}}}$$

- Interpolation was done to calculat the exact C_1 and z_1 values From B_i charts.

By Iteration, $\boxed{r = 0.01\,m = 10\,mm \quad @ \quad 0.076\%}$

$\boxed{D = .02\,m = 20\,mm}$

Calculations Cont.

Temp @ the surface

- Bi is same as in Excel Radius solution
- ζ_1 is same as Excel Radius solution

Equation: $\frac{T(r,t) - T_\infty}{T_i - T_\infty} = \left(C_1 e^{(-\zeta_1^2 \tau)}\right) J_0\left(\zeta_1\left(\frac{r_0}{r}\right)\right)$

$Bi = \frac{hr}{k} = \frac{(20.42)(.01)}{.19} = 1.07$

from Excel iterations

$C_1 = 1.216$ $\quad \tau = \frac{\alpha t}{r^2} = \frac{(1.09\times10^{-7})(1338\,s)}{(.01\,m)^2} = 1.458$

$\zeta_1 = 1.28$

$\frac{T(r,t) - 298}{343 - 298} = \left[(1.216)e^{(-1.28^2(1.458))}\right] J_0\left(1.28\left(\cancel{\frac{.01}{.01}}\right)^1\right)$

Table B.4

$\frac{T(r,t) - 298}{45} = .1116\ \cancel{J_0(1.28)}$ Interpolation

$\frac{T(r,t) - 298}{45} = .1116(.6303)$

$\frac{T(r,t) - 298}{45} = .0703$

$T(r,t) = (.0703)(45) + 298 = 301.16$

$\boxed{T(\text{surface}) = 301.16\ K = 28.014\ °C}$

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Calculations Cont.

Temp @ $\frac{r}{2}$

Equation : $\frac{T(r,t) - T_\infty}{T_i - T_\infty} = \left(C_1 e^{(-\zeta_1^2 \tau)}\right) J_0\left(\zeta_1 \left(\frac{r_0}{r}\right)\right)$

C_1 = const

ζ_1 = Const

$\tau = 1.458$

$$\frac{T(r,t) - 298}{343 - 298} = \left[\left(1.216 e^{(-1.28^2(1.458))}\right] J_0\left(\cancel{1.28(.5)}\right)\right) \rightarrow \text{Interpolation}$$

$$\frac{T(r,t) - 298}{45} = .1116\ \cancel{J_0(.64)} \rightarrow .8997$$

$$T(r,t) = .1004(45) + 298$$

$$\boxed{T\left(\frac{r}{2}\right) = 302.52\ K = 29.37\ ^\circ C}$$

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Summary

The radius, r, found to cause the given conditions is $r = 10\,mm$ or $D = 20\,mm$. At the time, $t = 838$ seconds, and radius, $r = 10\,mm$, The temp at $\frac{r}{2}$ is $T_{\frac{r}{2}} = 29.37\,°C$ and $T_{surface} = 28.014\,°C$.

Materials

- Long Plastic rod
- Low Convective Fluid

Analysis

The Fluid that the rod was in had a low convection coefficient and the rod had a low thermal conductivity, these two factors contributed to the slow cooling of the rod. To calculate the radius, a long infinite cylinder was considered, this is because the head Transfer through the end faces can be Assumed negligable.

Iteration Process

Constants			
Variable		Value	Unit
Initial Temp	Ti	343.15	K
Center Temp	Tc	303.15	K
Fluid Temp	Tf	298.15	K
Time	t	1338	s
Density	Rho	1190	kg/m^3
Specific Heat	Cp	1465	j/kg*K
Convection Coefficient	hf	20.42	W/m^2*K
Thermal Conductivity	k	0.19	W/m*K
C Constant	C1	Based on Tables	
z Constant	z1	Based on Tables	
Radius	r	0.010000	m
		10	mm

Problem 1 Iterations								
r (m) guessed	Biot Number	alpha	C1	z1	Theta	LHS	RHS	% Diff
0.025	2.69	1.09E-07	1.39	1.730	0.11	0.02500	0.01314	47.43%
0.020	2.15	1.09E-07	1.35	1.628	0.11	0.02000	0.01244	37.81%
0.010	1.07	1.09E-07	1.216	1.280	0.11	0.01000	0.00999	0.0759%
0.009	0.97	1.09E-07	1.202	1.241	0.11	0.00900	0.00971	-7.86%
0.0099	1.06	1.09E-07	1.2150	1.2764	0.11	0.00990	0.00997	-0.67%
0.00995	1.07	1.09E-07	1.2163	1.2799	0.11	0.00995	0.00999	-0.41%

Test 3: Problem 2

Author

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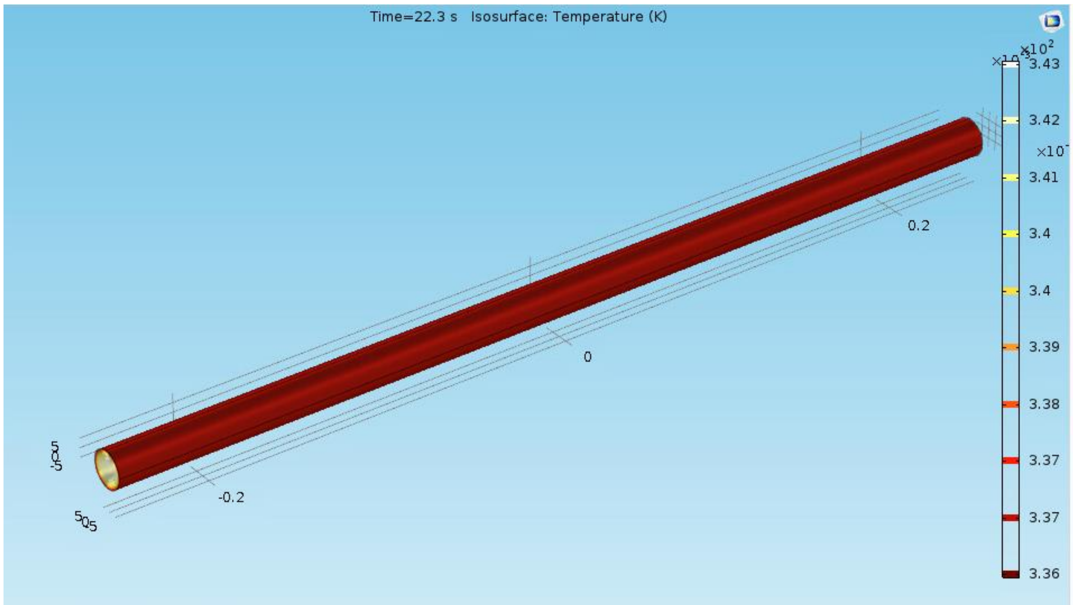

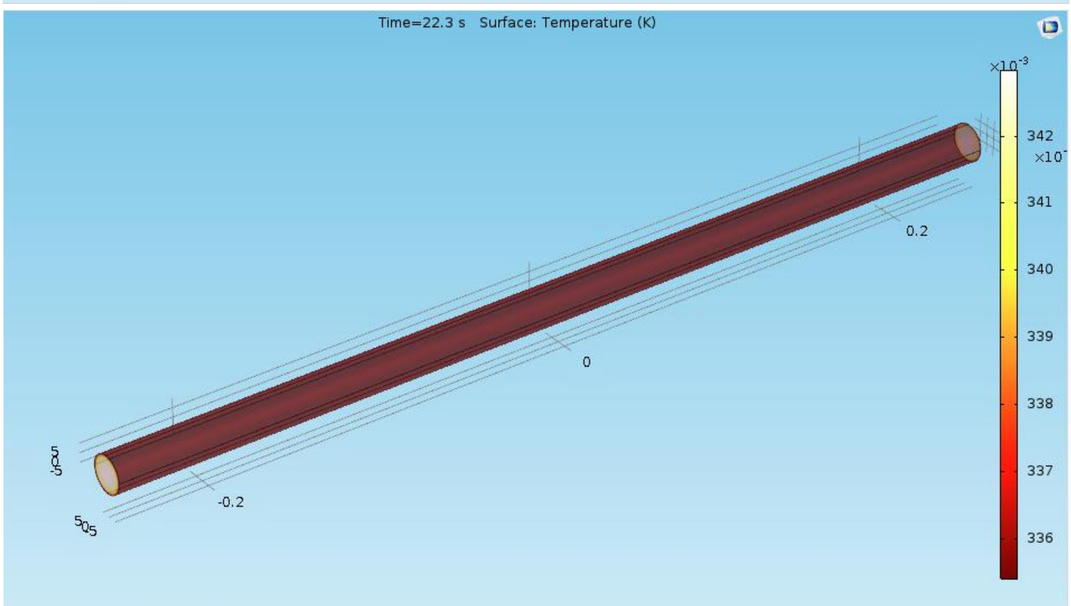

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Contents

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1 Purpose

Verify the time that it takes for 3 points along the circles radius to reach a calculated temperature through computational methods. Compute the percent error between the two methods.

2 Sources

Bayazitoglu, Y. & Ozisik, N., "A Textbook for Heat Transferfundamentals", Begell House Inc., (2012)

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3 Data and Variables

Date	Nov 3, 2019 6:01:43 PM

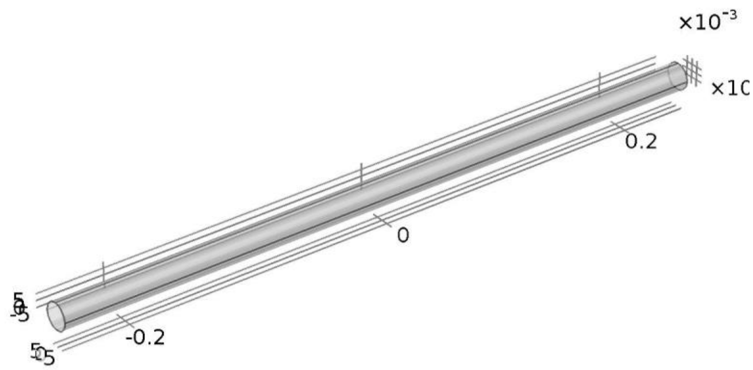
3.1 Variables

Parameters

Name	Expression	Value	Description
ro	10[mm]	0.01 m	Radius of the Rod
L	1[m]	1 m	Length of the Rod
Ti	343[K]	343 K	Initial Rod Temp
Tf	298[K]	298 K	Ambient Temp
Tc	303[K]	303 K	Temperature at center at t
to	1338[s]	1338 s	Time of interest
tp	25[min]	1500 s	Time range
rho1	1190[kg/m^3]	1190 kg/m^3	Density
Cp1	1465[J/kg/K]	1465 J/(kg·K)	Specific Heat
ko	0.19[W/m/K]	0.19 W/(m·K)	Thermal Conductivity
ho	20.42[W/m^2/K]	20.42 W/(m^2·K)	Convection Coefficient

4 Drawings

4.1 Geometry 1

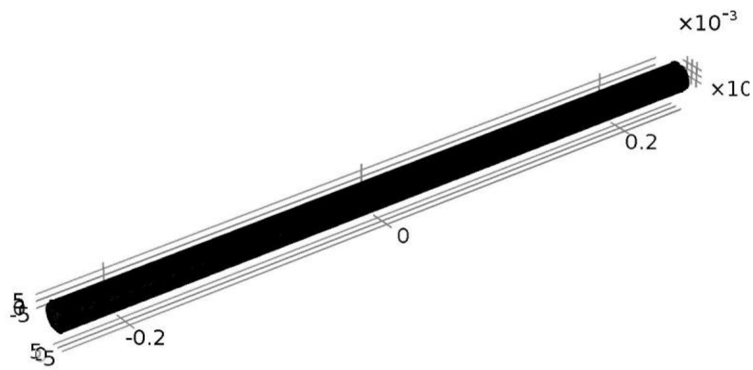


Geometry 1

Units

Length unit	m
Angular unit	deg

4.2 Mesh 1



Mesh 1

4.3 3D Data Points

4.3.1 Center Point

Data

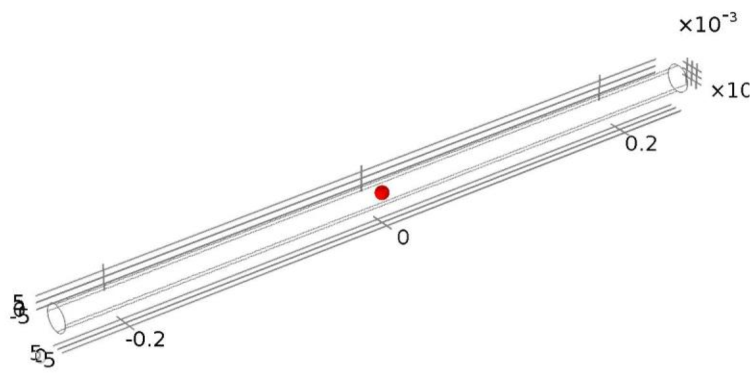
Description	Value
Data set	Study 1/Solution 1

Point data

Description	Value
Entry method	Coordinates

Settings

Description	Value
x	0
y	0
z	0



Data set: Center Point

4.3.2 Half radius

Data

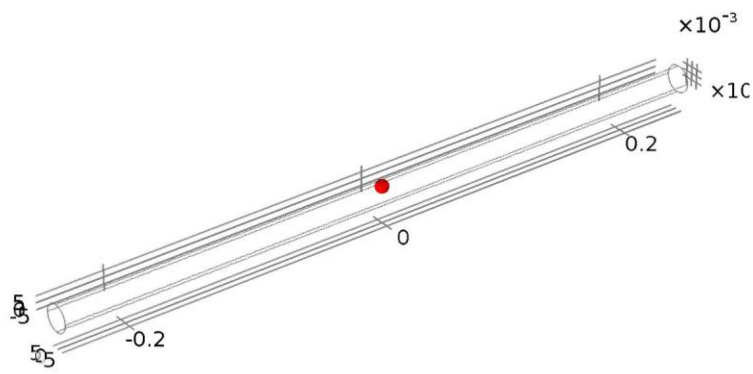
Description	Value
Data set	Study 1/Solution 1

Point data

Description	Value
Entry method	Coordinates

Settings

Description	Value
x	0
y	0
z	0.005



Data set: Half radius

4.3.3 Surface

Data

Description	Value
Data set	Study 1/Solution 1

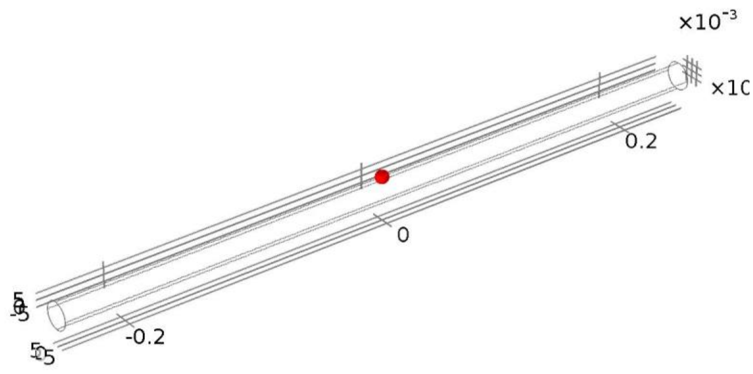
Point data

Description	Value
Entry method	Coordinates

Settings

Description	Value
x	0

Description	Value
y	0
z	0.01



Data set: Surface

4.3.4 Study 1/Solution 1

Solution

Description	Value
Solution	Solution 1
Component	Save Point Geometry 1

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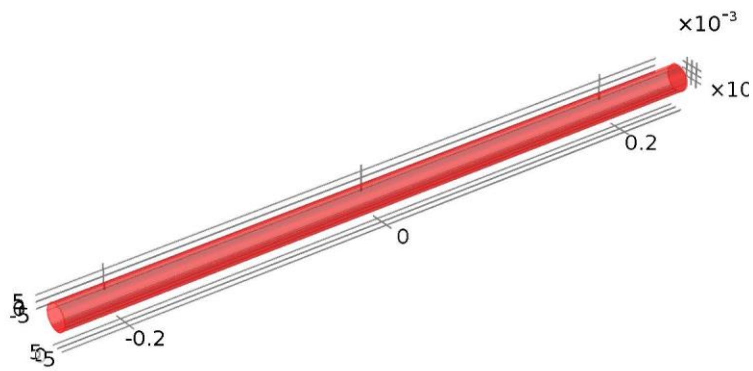


Data set: Study 1/Solution 1

5 Design Considerations

Computation information

Computation time	1 min 31 s
CPU	Intel(R) Xeon(R) CPU E5-2680 v4 @ 2.40GHz, 2 cores
Operating system	Windows 8

5.1 Time Dependent

Study settings

Description	Value
Include geometric nonlinearity	Off

Times	Unit
range(0,to/60,to)	s

Physics and variables selection

Physics interface	Discretization
Heat Transfer in Solids (ht)	physics

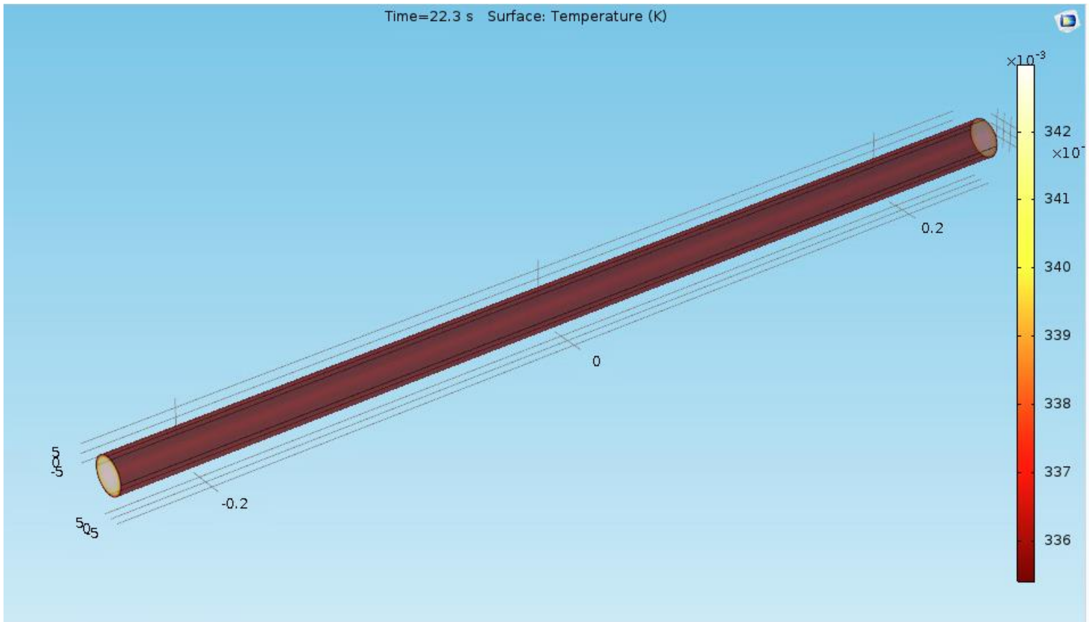
Mesh selection

Geometry	Mesh
Geometry 1 (geom1)	mesh1

6 Results

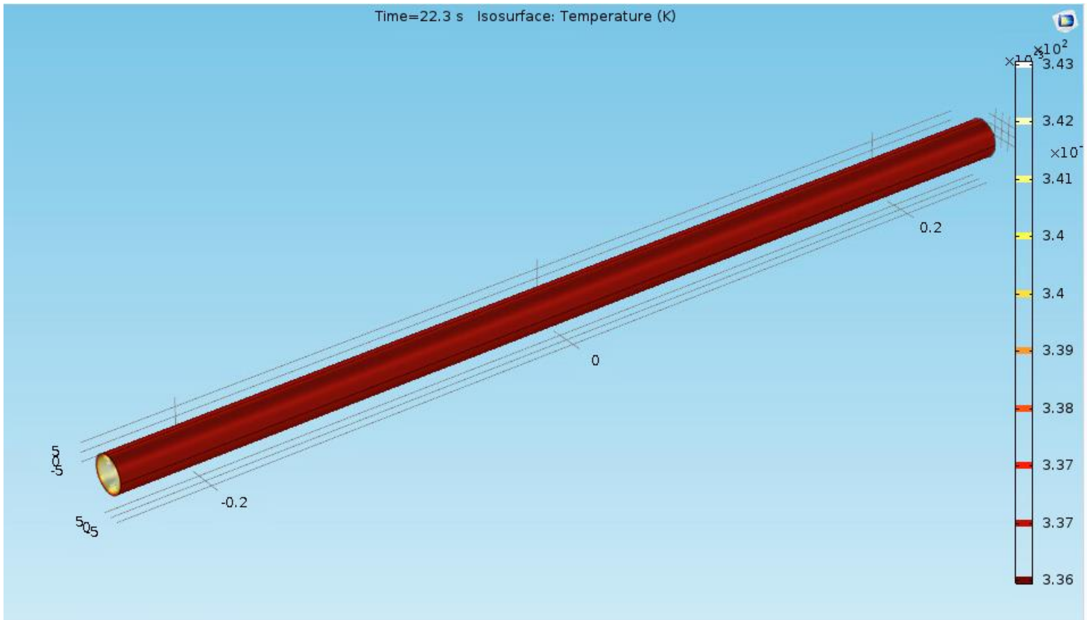
6.1 Summary

6.1.1 Temperature (ht)



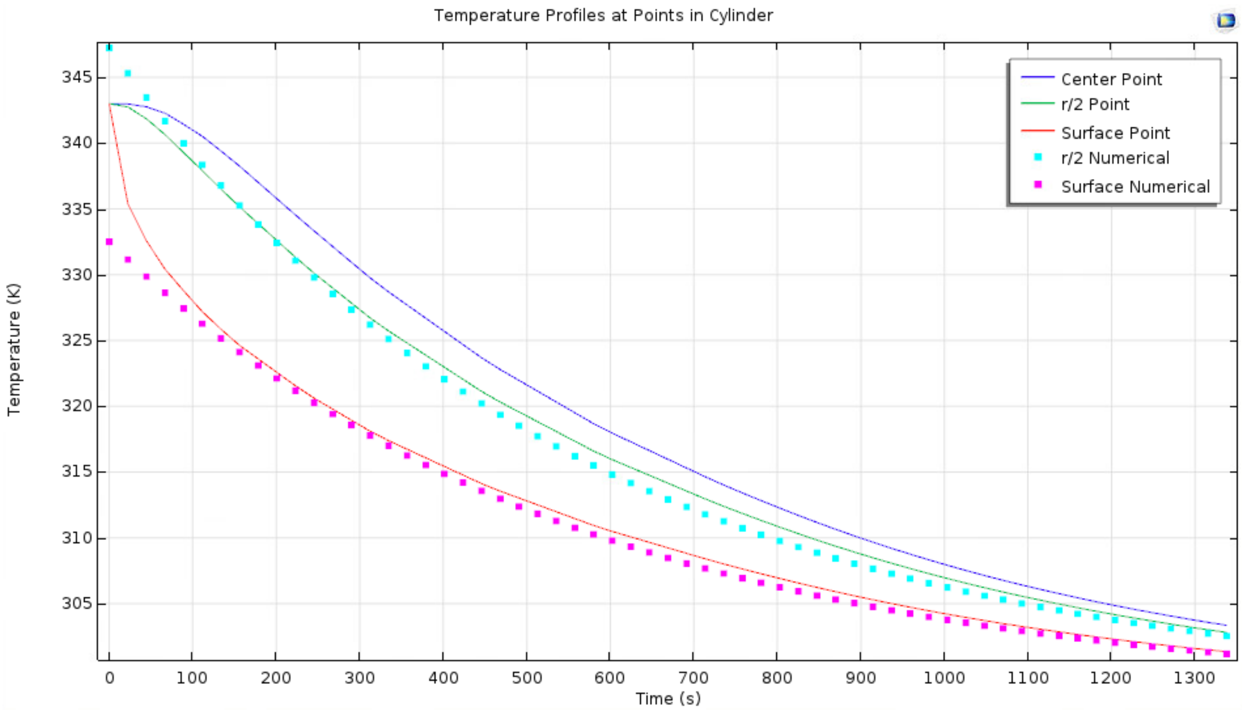
Time=1338 s Surface: Temperature (K)

6.1.2 Isothermal Contours (ht)



Time=1338 s Isosurface: Temperature (K)

6.1.3 1D Plot Group 3



Temperature Profiles at Points in Cylinder

6.2 Analysis

By using COMSOL, we were able to positively verify the calculations done in problem 1. The graph generated from the three points along the radius depicts the change in temperature over time. As calculated, the outside boundary cools off rapidly while the heat transfer slows down as r moves closer to the center. The numerical and analytical comparison proved that there was a small margin of error, however, both show the same trend at each point.