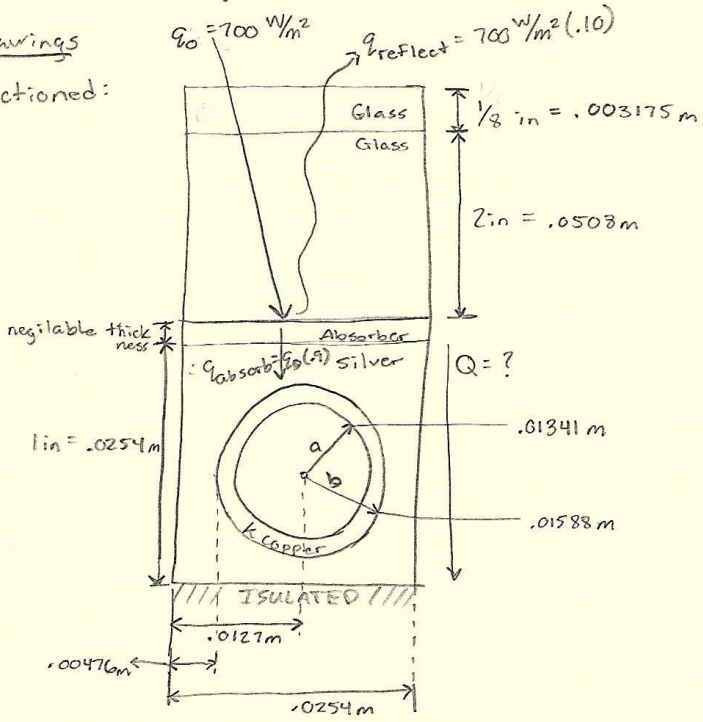
MET440	Test 4	Carter Fishback	1

Purpose: Solve the problem from Test 1, but instead, calculate the Convection Coefficients For the Ambient air, air space, and the Water in the pipes.

Sources: Heat Transfer Fundamentals, 1st Ed., Yildiz Bayazitoglu & Necati Ozisik, Begell House Inc.

Drawings

Sectioned:



Design Considerations

1.) Incompressible fluids (water)
2.) 1D Heat Transfer
3.) Steady state
4.) Conservation of energy
5.) Materials have certain Conductive properties
6.) Thermal Resistances / Circuits
7.) No Radiation
8.) Uniform wall heat flux

Variables

$q_o = 700\ W/m^2$
$T_{wi} = 20°C$
$T_{wo} = 45°C$
$T_{Air} = 25°C$
$L = 1m$ (Assumed Length)

$a = .01341m$
$b = .01588m$
$A = .0254\ m^2$
$L_{Air} = .0508m$
$L_{silver} = .0254\ m$

$k_{silver} = 419\ W/m·°C$
$k_{glass} = 1.177\ W/m·°C$
$k_{copper} = 386\ W/m·°C$

h_o = Air = ?
h_1 = Airspace = ?
h_2 = water = ?

MET 440 | Test 4 | Carter Fishback | 2

Procedure

1. Find all 8 resistances in circuit (R_{eq})
2. Q from sun reflects @ absorber plate (calculate ζ %)
3. Conservation of energy ($Q_{total} = Q_{reflected} + Q_{Absorbed}$)
4. Find Absorber Plate temp w/ Q_{total}
   a. Assume h_0, h_1, h_2
   b. Guess Asorber plate temp, T_{AP}
   C. Compute h_0, h_1, h_2

h_0: Pg 347

i. Compute $T_m = T_w - .25(T_w - T_\infty)$: $T_{w_p} = T_{\infty_0} + \frac{Q_0}{h_0 A}$ $\quad Q_0 = \frac{T_{wf} - T_\infty}{\frac{1}{h_0 A}}$

ii. Read Properties @ T_m: P_r, α, k, ν, and β @ $\left(\frac{T_w + T_\infty}{2}\right)$

iii. Compute $Gr_L = \frac{g\beta(T_{\infty_0} - T_{w_0})L^3}{\nu^2}$: $L = \frac{\text{Area}}{\text{Perimeter}}$

IV. compute $Gr_L Pr$: Pick equation on Pg 347 according to value

V. compute $Nu_0 = 0.13(Gr_L Pr)^{1/3}$ for $GrPr < 2\times10^8$

or

$Nu_0 = 0.16(Gr_L Pr)^{1/3}$ FTO $5\times10^8 < GrPr < 10^{11}$

VI. Compute $h_{0_{new}} = Nu_0 \frac{k}{L}$

h_1: Pg 361

i. Compute $T_f = \frac{T_h + T_c}{2}$: $T_h = \frac{Q_r}{h_1 A} + T_{AP}$ (Assumed) $T_c = Q_r\left(\frac{1}{h_1 A} + \frac{L_{glass}}{k_{glass} A}\right) + T_{AP}$

ii. Read Properties @ T_F: β, ν, Pr, k δ = Thickness of space

iii. Compute $Ra = \frac{g\beta(T_h - T_c)\delta^3}{\nu^2}(Pr)$

IV. Compute $Nu_1 = 1 + 1.44\left[1 - \frac{1708}{Ra}\right] + \left[\left(\frac{Ra}{5830}\right)^{1/3} - 1\right]$

V. Compute $h_1 = \frac{k}{\delta} Nu_1$

h_2: Pg 296

i. Compute $T_b = \frac{T_i + T_o}{2}$

ii. Read Properties @ T_b: ρ, k, ν, C_p, Pr

iii. Compute $Q_{water} = Q_A = \frac{(T_{AP} - T_{water})}{\left(R_{silver} + R_{pipe} + \left(\frac{1}{2\pi a(h_2)}\right)\right)}$ (h_2 assumed)

IV. Compute $\dot{m}_{water} = \frac{Q_{water}}{C_p(T_o - T_i)}$

V. Compute Velocity, $u_m = \frac{\dot{m}_{water}}{\rho A}$

VI. Compute $Re = \frac{u_m D}{\nu}$

VII. Compute $Nu_2 = 0.023 Re^{0.8} Pr^n$: Pg 311 $n = 0.3$ (cheating)

VIII. Compute $h_2 = \frac{Nu_2 k}{D}$

D. Compare to old h values

i. If h values = same, solve for T_{AP}

ii. If Not the same, Plug new h values back in and Retry Steps A-d.

MET 440 | Test 4 | Carter Fishback | 3

5. heat absorbed by water

$$Q_A = \dot{m} C_p (T_0 - T_i) = (W)$$

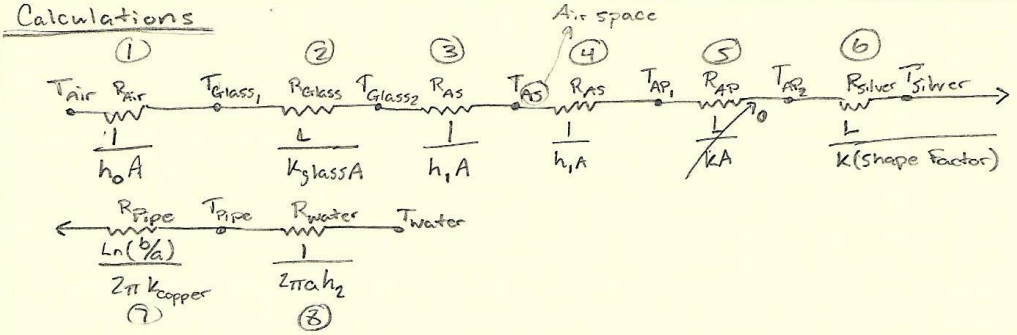
$$E = (J) = (W)\left(\frac{m}{\frac{m}{s}}\right) = \left(\frac{J}{s}\right)(m)\left(\frac{s}{m}\right) = Q_A\left(\frac{D}{V}\right)$$

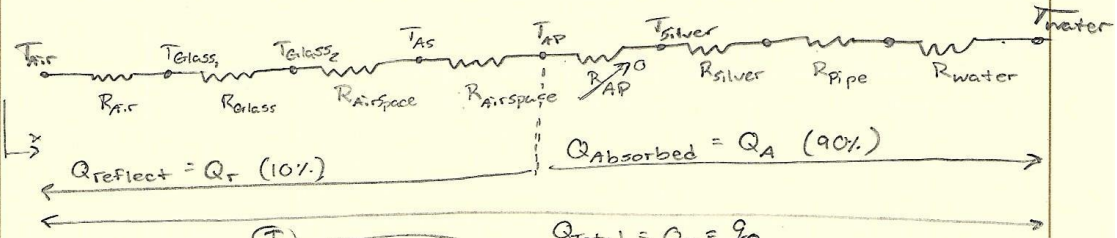
6. Flowrate, $\dot{m}$, from iterations

7. $\eta = \frac{E_{water}}{E_{absorbed}}$

MET 440 | Test 4 | Carter Fishback | 4

Calculations





$Q_o = 700 \frac{W}{m^2}(.0254 m^2) = 17.78 W$

(I) $Q_T = Q_A - Q_r = 14.224$

$Q_r = 700(.1)(.0254) = 1.778 W$

$Q_A = 700(.9)(.0254) = 16.002$

Equations For Resistances:

$R_{Air} = \frac{1}{h_o A}$

$R_{Glass} = \frac{L}{k_{glass} A}$

$R_{AS} = \text{Air space} = \frac{1}{h_1 A}$

$R_{AP} = \text{Absorber Plate} = \frac{L}{kA} \to 0 = \text{negligable}$

$R_{Silver} = \frac{L}{k_{Silver}(SF)}$ → Shape factor : case 5 (Table 4.1)

$SF = \frac{2\pi L}{Ln(\frac{8z}{\pi D})}$

This is because we are looking at a singular section on the entire Absorber.

$R_{Pipe} = \frac{Ln(b/a)}{2\pi k_{copper}}$

$R_{Water} = \frac{1}{2\pi a h_2}$

MET440 | Test 4 | Carter Fishback | 5

Temp @ absorber plate:

$Q_A = 16.002$ W ⟶ directly into the plate

$$Q_A = \frac{T_{AP} - T_{wi}}{R_{AP}^{\to 0} + \left(\frac{L_{silver}}{k_{silver}\left(\frac{2\pi L}{\ln\left(\frac{8z}{\pi b}\right)}\right)}\right) + \left(\frac{\ln(b/a)}{2\pi k_{copper}}\right) + \left(\frac{1}{2\pi r_2 (h_2)(L)}\right)} = 16.002\ W$$

(h_2: Assumed)

$$Q_r = \frac{T_{AP} - T_{Air}}{\left(\frac{1}{h_i A}\right) + \left(\frac{L_{glass}}{k_{glass} A}\right) + \left(\frac{1}{h_o A}\right)} = 1.778\ W$$

(h_i: Assumed; h_o: Assumed)

$Q_A + Q_r = Q_o = 17.78\ W$

* Iterations for T_{AP} on Excel sheet *

$$Q_o = \frac{(T_{AP} - T_{wi})}{R_{silver} + R_{pipe} + \frac{1}{2\pi a h_2}} + \frac{(T_{AP} - T_{Air})}{\frac{1}{h_i A} + R_{glass} + \frac{1}{h_o}}$$

($\frac{1}{2\pi a h_2} = Req_1$; $\frac{1}{h_i A} + R_{glass} + \frac{1}{h_o} = Req_2$)

$$Q_o = \frac{T_{AP}}{Req_1} - \frac{T_{wi}}{Req_1} + \frac{T_{AP}}{Req_2} - \frac{T_{Air}}{Req_2}$$

$$Q_o + \frac{T_{wi}}{Req_1} + \frac{T_{Air}}{Req_2} = \frac{T_{AP}}{Req_1} + \frac{T_{AP}}{Req_2}$$

$$Q_o + \frac{T_{wi}}{Req_1} + \frac{T_{Air}}{Req_2} = T_{AP}\left(\frac{1}{Req_1} + \frac{1}{Req_2}\right)$$

$$T_{AP} = \frac{Q_o + \frac{T_{wi}}{Req_1} + \frac{T_{Air}}{Req_2}}{\left(\frac{1}{Req_1} + \frac{1}{Req_2}\right)}$$

MET 440	Test 4	Carter Fishback	6

From Iterations, the Following was found true:

$h_o = 31.922 \ W/m^2 \cdot K$

$h_i = 3.263 \ W/m^2 \cdot K$

$h_2 = 1126.984 \ W/m^2 \cdot K$

$T_{ap} = 28.569°C$ or $301.719 \ K$

a.) $Q_{water} = \dfrac{T_{AP} - T_{wi}}{R_{eq}} = \dfrac{28.59 - 20}{\left(R_{silver} + R_{Pipe} + \dfrac{1}{(1126.984)(.0134m)(1m)(2\pi)}\right)}$

$= \dfrac{8.569 \ °C}{(0.00027 + 6.97 \times 10^{-5} + 94.886)\ °C/W} = 787. \ W = Q$

From Tables

b.) $Q_{water} = \dot{m} \ C_p (T_o - T_i)$

$\dot{m} = \dfrac{Q_{water} \left(\frac{kg \ m^2}{s^2}\right)}{(4179.7)(T_o - T_i)(°C)} = .00753 \ \dfrac{kg}{s} = \dot{m}_{water}$

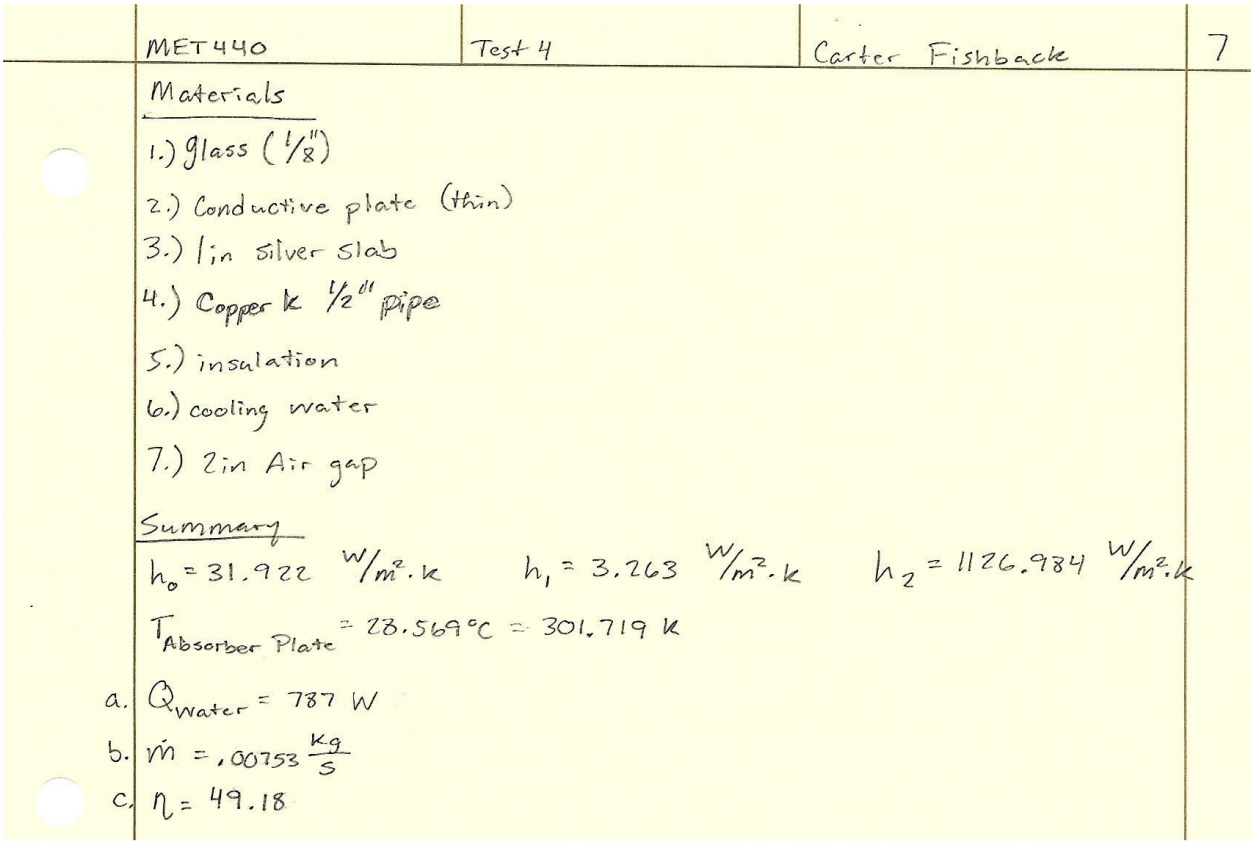
$\left(\frac{J}{kg \cdot °C}\right)$

c.) $Q_{reflected} = \dfrac{T_{...}}{\left(R_{glass} + \dfrac{1}{h_i A} + \dfrac{1}{h_o A}\right)} = \dfrac{28.569 - 25.\ C}{(0.1062 + 12.0656 + 1.233)}$

$Q_{reflected} = .2266 \ W$

$Q_s = 700 \ W/m^2 \ (.0254 m^2) = 17.78 W (.9) = 16.002$ $\quad \eta \ \dfrac{Q_{water}}{Q_{s@90\%}}$

$\eta = \dfrac{787 \ W}{W} = 49.18$



MET440 | Test 4 | Carter Fishback | 7

Materials

1.) glass (1/8")
2.) Conductive plate (thin)
3.) 1in silver slab
4.) Copper k 1/2" pipe
5.) insulation
6.) cooling water
7.) 2in Air gap

Summary

$h_o = 31.922 \ W/m^2.k$ $h_1 = 3.263 \ W/m^2.k$ $h_2 = 1126.984 \ W/m^2.k$

$T_{Absorber\ Plate} = 28.569°C = 301.719 \ K$

a. $Q_{water} = 787 \ W$
b. $\dot{m} = .00753 \frac{kg}{s}$
c. $\eta = 49.18$

Analysis

Solving for the equation found on page 5, the temperature at the absorber plate was assumed with three convection coefficients. The iteration process used the assumed values to calculate new h values and temperature to then be reiterated. The process yielded values with a percent error of < 1%. The coefficients found were very similar to those from the first test and made logical sense to use. With the final convective coefficients, I was able to solve for the Q to the water, mass flowrate, and efficiency ratio.

Iterations

Constants		
Variable	Value	Units
Solar Flux, qs	700.0000	W/m^2
qs reflected	70.0000	W/m^2
qs absorbed	630.0000	W/m^2
Q absorbed	16.0020	W
Q Reflected	1.7780	W
Q Total	17.7800	W
Temp Water inlet	293.1500	K
Temp Water outlet	318.1500	K
Temp Abient Air	298.1500	K
Length of panel	1.0000	m
pipe inner Dia, a	0.0134	m
pipe outter Dia, b	0.0159	m
Surface Area Panel	0.0254	m^2
Length Air	0.0508	m
Length Silver	0.0254	m
Length Glass	0.0032	m
k silver	419.0000	W/m*K
k glass	1.1770	W/m*K
k copper	386.0000	W/m*K
Shape Factor (silver)	0.2244	
Resistance Calculations		
R glass	0.106202209	C/W
R silver	0.00027	C/W
R pipe	6.97065E-05	C/W

Iterations																						
Convection Assumed			Guessed	Compute New Convection Coefficient													New Calculated	% difference				
ho	h1	h2	Temp ap (C)	ho		h1		h2									New Temp AP (C)	Temp AP	ho	h1	h2	
50	10	1000	50	Tw (K)	312.150000000000	Tc (K)	316.150000000000	Tb (C)	32.500000000								28.51212491	42.98%	44.86%	74.48%	-12.70%	
				Tm (K)	308.650000000000	Th (K)	330.338827527613	Q water	16.002000000													
				T for B (K)	305.150000000000	Tf (K)	280.244413763806	rho	996.813750000													
				Pr	0.707809700000	Pr	0.713531680000	k	0.616375000													
				alpha	0.000022292691	alpha	0.000018604315	viscosity	0.000004490													
				k	0.026305567000	k	0.024671373600	Cp	4.179675000													
				viscosity	1.69E-05	viscosity	1.41E-05	Pr	5.345000000													
				Beta	3.28E-03	Beta	3.57E-03	mass flow rate	0.153141094													
				Gr	1.38E+08	Ra	3.26E+05	velocity	1.087752993													
				Gr*Pr	9.78E+07	Nu	5.255598462771	Re	3248.904199775													
				Nu	59.901249993538	h1 (New)	2.552417975721	Nu	24.518939007													
				ho (New)	27.571939546610			h2 (New)	1126.984416894													
27.57194	2.552418	1126.98	28.51212491	Tw (K)	323.538130523668	Tc (K)	274.237149589707	Tb (C)	32.500000000								28.58431797	-0.25%	-20.67%	-61.90%	0.00%	
				Tm (K)	317.191097892751	Th (K)	329.275927764787	Q water	16.002000000													
				T for B (K)	310.844065261834	Tf (K)	300.669389087540	rho	996.813750000													
				Pr	0.704218200000	Pr	0.699155400000	k	0.616375000													
				alpha		alpha		viscosity	0.000004490													
				k	0.027543002000	k	0.027887084000	Cp	4.179675000													
				viscosity	1.82E-05	viscosity	1.34E-05	Pr	5.345000000													
				Beta	3.22E-03	Beta	3.33E-03	mass flow rate	0.153141094													
				Gr	2.13E+08	Ra	1.32E+06	velocity	1.087752993													
				Gr*Pr	1.50E+08	Nu	7.527503523553	Re	3248.904199775													
				Nu	69.034127668155	h1 (New)	4.132285887236	Nu	24.518939007													
				ho (New)	33.270465729348			h2 (New)	1126.984416894													
33.27047	7.146	1126.98	28.58431797	Tw (K)	319.189681430805	Tc (K)	291.938628072309	Tb (C)	32.500000000								28.56507568	0.07%	5.78%	61.63%	0.00%	
				Tm (K)	313.929761073104	Th (K)	311.718835392813	Q water	16.002000000													
				T for B (K)	308.669840715402	Tf (K)	283.040103660252	rho	996.813750000													
				Pr	0.704935400000	Pr	0.712035640000	k	0.616375000													
				alpha		alpha		viscosity	0.000004490													
				k	0.027295894000	k	0.025095607800	Cp	4.179675000													
				viscosity	1.79E-05	viscosity	1.49E-05	Pr	5.345000000													
				Beta	3.24E-03	Beta	3.53E-03	mass flow rate	0.153141094													
				Gr	1.83E+08	Ra	4.07E+05	velocity	1.087752993													
				Gr*Pr	1.29E+08	Nu	5.550031327382	Re	3248.904199775													
				Nu	65.633813031680	h1 (New)	2.741760028931	Nu	24.518939007													
				ho (New)	31.347919568304			h2 (New)	1126.984416894													
31.34792	2.74176	1126.98	28.56507568	Tw (K)	320.480030497711	Tc (K)	276.184030201483	Tb (C)	32.500000000								28.57596533	-0.04%	-1.91%	-35.54%	0.00%	
				Tm (K)	314.897522873283	Th (K)	327.434948693563	Q water	16.002000000													
				T for B (K)	309.315015248855	Tf (K)	298.775459246040	rho	996.813750000													
				Pr	0.704722440000	Pr	0.730238540000	k	0.616375000													
				alpha		alpha		viscosity	0.000004490													
				k	0.027369268400	k	0.020244288400	Cp	4.179675000													
				viscosity	1.80E-05	viscosity	8.78E-06	Pr	5.345000000													
				Beta	3.23E-03	Beta	3.35E-03	mass flow rate	0.153141094													
				Gr	1.92E+08	Ra	2.86E+06	velocity	1.087752993													
				Gr*Pr	1.35E+08	Nu	9.325336712750	Re	3248.904199775													
				Nu	66.707643930972	h1 (New)	3.716236335433	Nu	24.518939007													
				ho (New)	31.946446388073			h2 (New)	1126.984416894													
31.94645	3.716236	1126.98	28.57596533	Tw (K)	320.061670283970	Tc (K)	282.889703666341	Tb (C)	32.500000000								28.56873167	0.03%	0.60%	13.97%	0.00%	
				Tm (K)	314.583752712978	Th (K)	320.751054511467	Q water	16.002000000													
				T for B (K)	309.105835141985	Tf (K)	292.080675422563	rho	996.813750000													
				Pr	0.704791652000	Pr		k	0.616375000													
				alpha		alpha		viscosity	0.000004490													
				k	0.027345421720	k	0.025271478800	Cp	4.179675000													
				viscosity	1.80E-05	viscosity	1.52E-05	Pr	5.345000000													
				Beta	3.24E-03	Beta	3.42E-03	mass flow rate	0.153141094													
				Gr	1.89E+08	Ra	7.24E+05	velocity	1.087752993													
				Gr*Pr	1.33E+08	Nu	6.426362380889	Re	3248.904199775													
				Nu	66.366151432286	h1 (New)	3.196922847830	Nu	24.518939007													
				ho (New)	31.755212578290			h2 (New)	1126.984416894													
31.75521	3.196923	1126.98	28.56873167	Tw (K)	320.193625067041	Tc (K)	279.822676199217	Tb (C)	32.500000000								28.57378268	-0.02%	-0.19%	-12.04%	0.00%	
				Tm (K)	314.682718800281	Th (K)	323.803614660955	Q water	16.002000000													
				T for B (K)	309.171812533520	Tf (K)	295.140469230869	rho	996.813750000													
				Pr	0.704769806000	Pr		k	0.616375000													
				alpha		alpha		viscosity	0.000004490													
				k	0.027352948660	k	0.020490072400	Cp	4.179675000													
				viscosity	1.80E-05	viscosity	8.93E-06	Pr	5.345000000													
				Beta	3.23E-03	Beta	3.39E-03	mass flow rate	0.153141094													
				Gr	1.90E+08	Ra	2.40E+06	velocity	1.087752993													
				Gr*Pr	1.34E+08	Nu	8.880630670620	Re	3248.904199775													
				Nu	66.474505446212	h1 (New)	3.581983570840	Nu	24.518939007													
				ho (New)	31.815813380037			h2 (New)	1126.984416894													
31.81581	3.581984	1126.98	28.57378268	Tw (K)	320.151637727710	Tc (K)	282.181537824262	Tb (C)	32.500000000								28.56879294	0.02%	0.06%	10.60%	0.00%	
				Tm (K)	314.651228295782	Th (K)	321.454855057443	Q water	16.002000000													
				T for B (K)	309.150818863855	Tf (K)	292.786658616590	rho	996.813750000													
				Pr	0.704776736000	Pr		k	0.616375000													
				alpha		alpha		viscosity	0.000004490													
				k	0.027350560960	k	0.025667261598	Cp	4.179675000													
				viscosity	1.80E-05	viscosity	1.59E-05	Pr	5.345000000													
				Beta	3.23E-03	Beta	3.42E-03	mass flow rate	0.153141094													
				Gr	1.89E+08	Ra	6.86E+05	velocity	1.087752993													
				Gr*Pr	1.33E+08	Nu	6.337601081763	Re	3248.904199775													
				Nu	66.440074759539	h1 (New)	3.202143009240	Nu	24.518939007													
				ho (New)	31.796558440905			h2 (New)	1126.984416894													
31.79656	3.202143	1126.98	28.56879294	Tw (K)	320.164961188362	Tc (K)	279.858432614411	Tb (C)	32.500000000								28.56969112	0.00%	-0.61%	-2.40%	0.00%	
				Tm (K)	314.661220891271	Th (K)	323.767980795672	Q water	16.002000000													
				T for B (K)	309.157480594181	Tf (K)	295.104774090630	rho	996.813750000													
				Pr	0.709176749200	Pr		k	0.616375000													
				alpha		alpha		viscosity	0.000004490													
				k	0.025834556412	k	0.025851318738	Cp	4.179675000													
				viscosity	1.64E-05	viscosity	1.62E-05	Pr	5.345000000													
				Beta	3.23E-03	Beta	3.39E-03	mass flow rate	0.153141094													
				Gr	2.27E+08	Ra	7.32E+05	velocity	1.087752993													
				Gr*Pr	1.61E+08	Nu	6.443653190849	Re	3248.904199775													
				Nu	70.765433127174	h1 (New)	3.279073473893	Nu	24.518939007													
				ho (New)	31.989388873904			h2 (New)	1126.984416894													
31.98939	3.279073	1126.98	28.56969112	Tw (K)	320.032256105588	Tc (K)	280.372197516222	Tb (C)	32.500000000								28.56949056	0.00%	0.21%	0.50%	0.00%	
				Tm (K)	314.561692079191	Th (K)	323.256012245139	Q water	16.002													
				T for B (K)	309.091128052794	Tf (K)	294.591907364459	rho	996.813750000													
				Pr	0.709076950600	Pr		k	0.616375000													
				alpha		alpha		viscosity	0.000004490													
				k	0.025868941566	k	0.025810597654	Cp	4.179675000													
				viscosity	1.65E-05	viscosity	1.61E-05	Pr	5.345000000													
				Beta	3.24E-03	Beta	3.39E-03	mass flow rate	0.153141094													
				Gr	2.25E+08	Ra	7.22E+05	velocity	1.087752993													
				Gr*Pr	1.60E+08	Nu	6.421623593531	Re	3248.904199775													
				Nu	70.522905193763	h1 (New)	3.262715410592	Nu	24.518939007													
				ho (New)	31.922185713421			h2 (New)	1126.984416894													