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MET 330
10/29/2022

Test 2

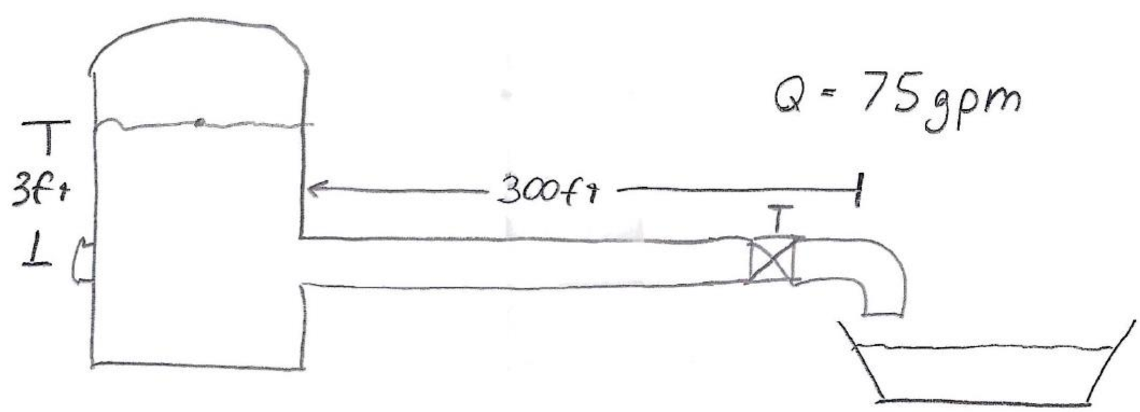
Problem 1

Purpose:

a. Determine the depth of the open channel.
b. Determine all relevant forces in the x and y directions.
c. Determine the largest log of a square cross section that could be carried by the open channel.
d. Determine the pressure drop across a flow nozzle with diameter ratio of 0.5
e. Determine the risk of cavitation along with pressure increment if the valve was closed suddenly
f. Determine the drag force on a log half the size of the one determined in part c if it was stuck in the open channel
g. Determine the force acting on the blind flange in the storage tank.

Drawings and Diagrams:

System Drawing



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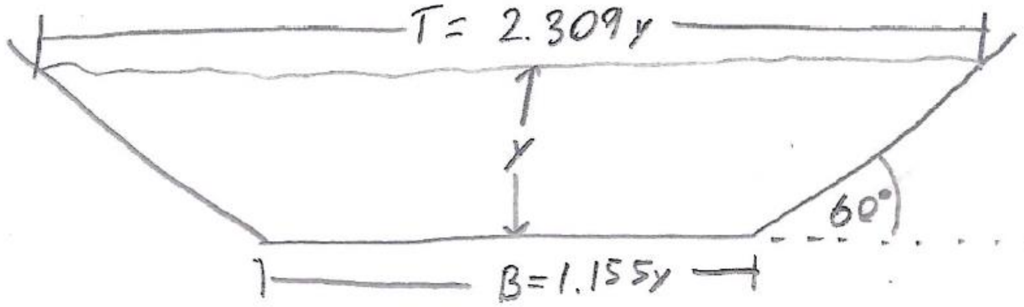
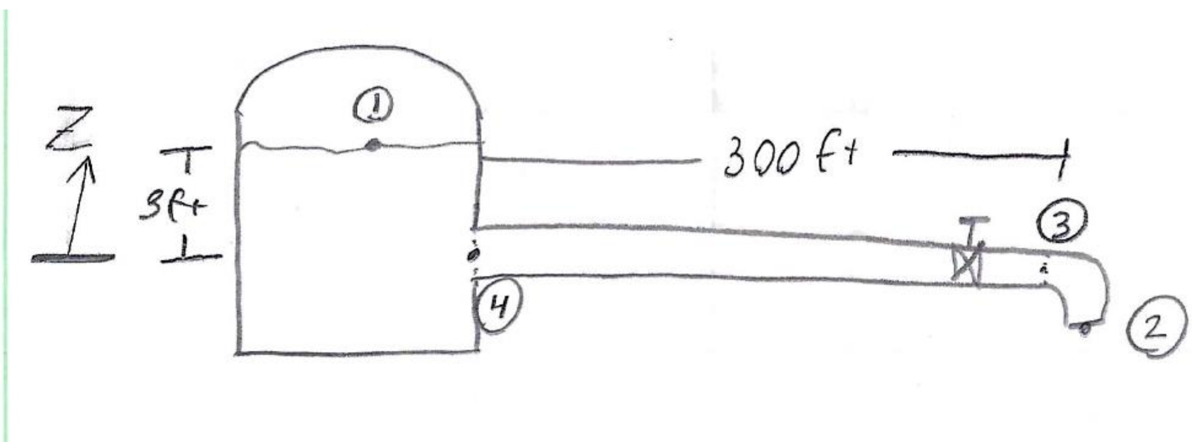
Channel Drawing



Bernoulli's Equation Drawing



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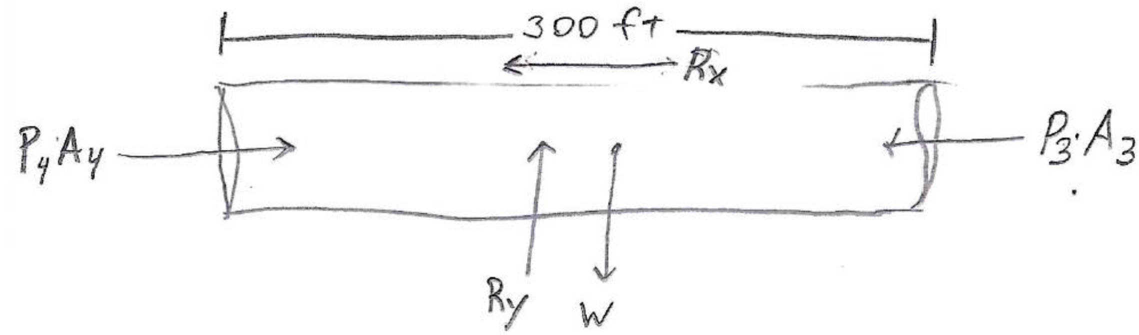
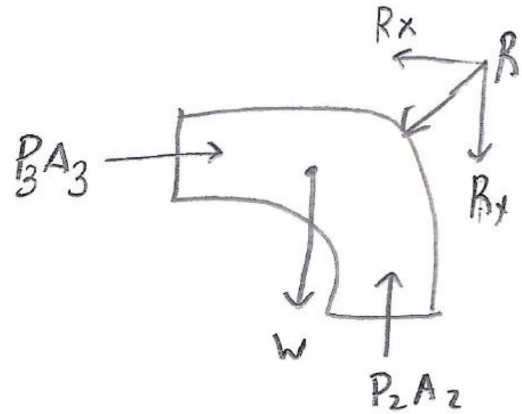
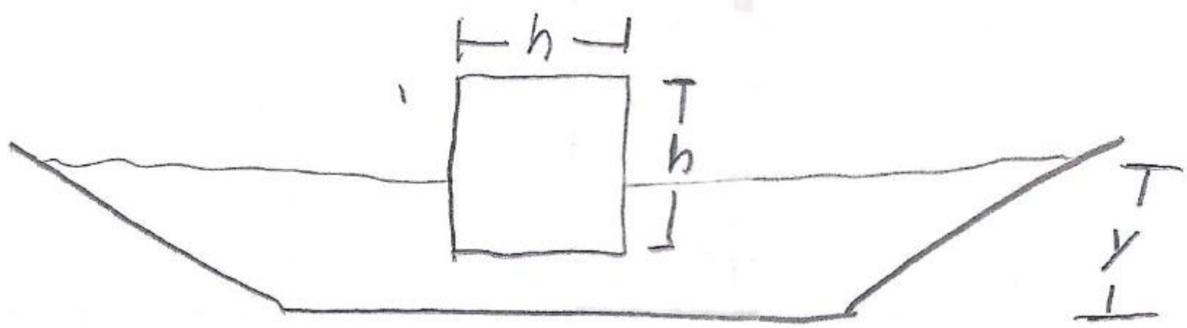
Forces Drawing





Log Cross Section



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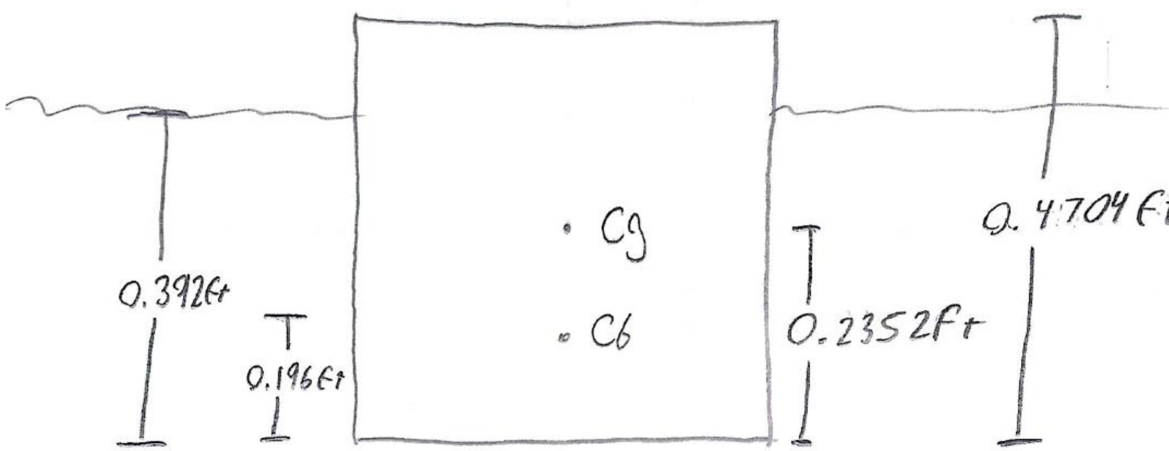
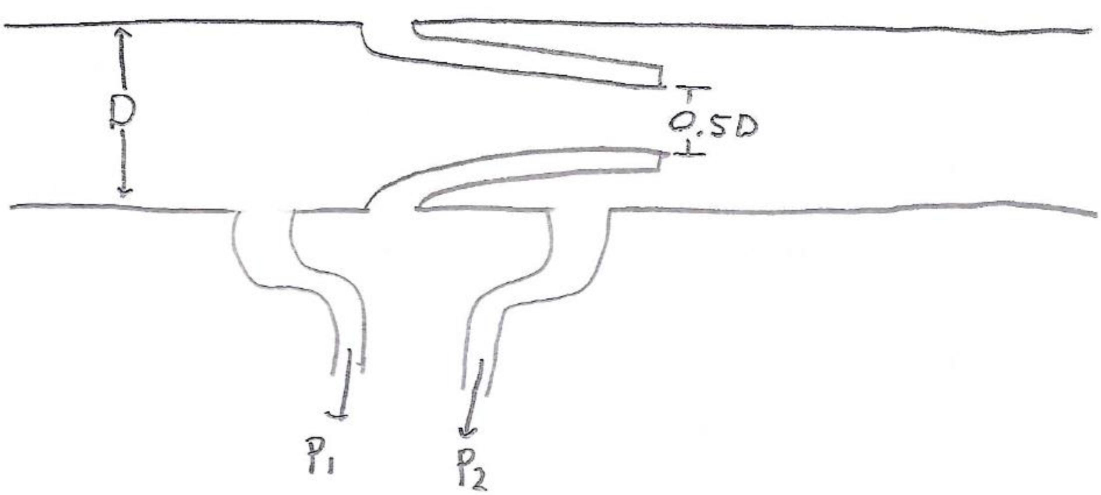
Log Stability



Flow Nozzle



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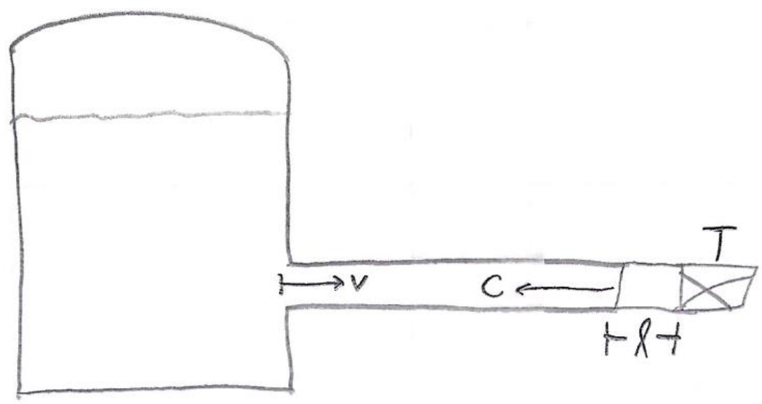
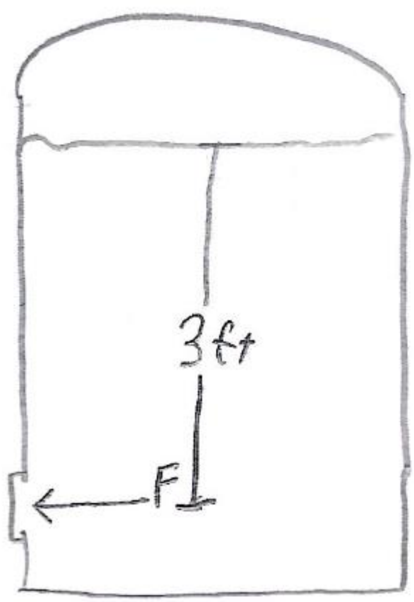
Cavitation



Blind Flange



Sources:

- Mott, R., Untener, J.A., "Applied Fluid Mechanics", 7th edition. Pearson Education, Inc, (2015)

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Design Considerations:

- Constant Properties
- Incompressible Fluids
- Isothermal

Data and Variables:

$Temp = 60°F$

$Q = 75GPM$

$L_{pipe} = 300\,ft$

1.5in Schedule 40 Steel Pipe

Modulus of Elasticity of Steel $= 200GPa$

$\rho_{hickory} = 830kg/m^3$

$S = 0.001$

$T = 2.309y$

Procedure:

h. Draw a sketch of the channel, the given information is used to determine that this is efficient trapezoid and table 14.3 can be used. Since the material is provided, unfinished concrete, the Manning number can also be determined. Using Manning's equation, and the values A and R from table 14.3 the value for the depth of the water, y can be determined.
i. Draw a free body diagram for the straight section of the pipe as well as the 90 deg elbow. The force equation can be used with statics to determine the reaction force values. Add the horizontal and vertical sections from each section to determine the overall vertical and horizontal forces. Bernoulli's equation will be used to determine the pressure at all locations, with the tank being the other reference point.
j. Draw a cross sectional sketch of the log. The value for the depth of the water was determined in part a. For the log to barely be floating the log cannot have a width higher than the depth of the water. The density is provided and can be used to determine the specific weight. Set the buoyant force equal to the weight and this creates a ratio of the volume displaced to the total volume, this can be used to determine the width of the cross sections. The center of gravity is easily determined because of the square cross section. I'm not sure how to determine the length since it doesn't seem to be needed for determining the maximum size? I selected an arbitrary length. The center of buoyancy is determined using the depth that is submerged and the metacenter is found

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using the equation of the moment of inertia/volume. The stability will then be determined based on whether the metacenter is above or below the center of gravity

k. A sketch of the flow nozzle was drawn. Since we know both diameters, beta can be determined. The Reynolds number can be calculated and then the value for the discharge coefficient can be found. $P_1 - P_2 = h(\gamma_m - \gamma_f)$ will be used but h needs to be determined. This equation will be used $h = \frac{\left(\frac{Q}{A_1 \times C}\right)^2 \times \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}{2g\left(\frac{\gamma_m}{\gamma_w} - 1\right)}$. Once that is complete the pressure drop can be found.

l. For this section since the valve closed suddenly this equation can be used $\Delta P = \rho A C$ to find the pressure increment. To determine the value of C this equation can be used: $C = \frac{\sqrt{\frac{E_o}{\rho}}}{\sqrt{1 + \frac{E_o D}{E\delta}}}$ The values for δ,ρ,D,E_o can be found in tables in the textbook, while E was given in the problem statement. Once C is determined the pressure change can be determined. To decide if cavitation is an issue the pressure will have to be compared to the saturation pressure at the temperature given in the problem. If the pressure is greater than there are no issues but if it is lower cavitation will be a problem.

m. Assuming that the log has the same square cross section, the projected area can be determined. We know the velocity of the fluid and the density. Since this is a square projected area, the C_D=1.16. $F_D = C_D \frac{\rho V^2}{2} A$ can be used to determine the drag force.

n. The force can be determined by the pressure times the area. The pressure will be determined by the gamma h equation, and the area is the same as the pipe.

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Calculations:

a) Water Channel Depth

$A = 1.73y^2$

$WP = 3.46y$

$R = y/2$

$S = 0.001$

The material is unfinished concrete $\therefore$ n=0.017

$$\text{AR}^{2/3} = \frac{nQ}{S^{1/2}} \longrightarrow (1.73y^2)\left(\frac{y}{2}\right)^{2/3} = \frac{(0.017)(0.167\,ft^3/s)}{0.001^{1/2}}$$

$$(1.73y^2)\left(\frac{y}{2}\right)^{2/3} = 0.0898$$

Using Excel:

y	A	R	RHS	LHS	% diff
1	1.73	0.5	0.089777	1.089832	-1114%
2	6.92	1	0.089777	6.92	-7608%
0.5	0.4325	0.25	0.089777	0.171638	-91%
0.4	0.2768	0.2	0.089777	0.094664	-5%
0.39	0.263133	0.195	0.089777	0.088484	1%
0.392	0.265839	0.196	0.089777	0.089699	0%

$$\boxed{y = 0.392\,ft}$$

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b) Applying Bernoulli's equation to determine unknown pressures

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L$$

$$\frac{p_1}{\gamma}+\frac{\cancel{V_1}^2}{2g}+z_1=\frac{\cancel{p_2}}{\gamma}+\frac{V_2^2}{2g}+\cancel{z_2}+h_L$$

$$\frac{p_1}{\gamma}+z_1=\frac{V_2^2}{2g}+h_L$$

$$Q=VA\longrightarrow V=Q/A$$

$$\frac{p_1}{\gamma}+z_1=\frac{(Q/A)^2}{2g}+h_L$$

$$p_1=\gamma\left(\frac{(Q/A)^2}{2g}+h_L-z_1\right)$$

Energy Losses

$$h_L=h_{L,inlet}+h_{L,valve}+h_{L,elbow}$$

$$h_L=K_{inlet}\left(\frac{V^2}{2g}\right)+K_{valve}\left(\frac{V^2}{2g}\right)+K_{elbow}\left(\frac{V^2}{2g}\right)+f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$$

$$K_{inlet}=0.5$$

$$K_{valve}=340f$$

$$K_{elbow}=30f$$

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Reynolds Number

$$V = Q / A = \left(0.167 \frac{ft^3}{s}\right) / \left(0.01414\, ft^2\right) = 11.8104\, ft/s$$

$$\text{Re} = \frac{\rho V D}{\eta} = \frac{\left(1.94 \frac{slugs}{ft^3}\right)\left(11.8104\, ft/s\right)\left(0.1342\, ft\right)}{2.34 \times 10^{-5} \frac{lb-s}{ft^2}}$$

$\text{Re} = 131,402.3085 \therefore$ flow is turbulent

Friction Factor

$\varepsilon = 1.5 \times 10^{-4}\, ft$

$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7\left(\frac{D}{\varepsilon}\right)} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log\left(\frac{1}{3.7\left(\frac{0.1342\, ft}{1.5 \times 10^{-4}\, ft}\right)} + \frac{5.74}{131,402.3085^{0.9}}\right)\right]^2}$$

$f = 0.0203$

Energy Losses Continued:

$$h_L = K_{inlet}\left(\frac{V^2}{2g}\right) + K_{valve}\left(\frac{V^2}{2g}\right) + K_{elbow}\left(\frac{V^2}{2g}\right) + f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$$

$$h_L = \left(\frac{V^2}{2g}\right)\left(K_{inlet} + K_{valve} + K_{elbow} + f\left(\frac{L}{D}\right)\right)$$

$$h_L = \left(\frac{V^2}{2g}\right)\left(0.5 + 340 f + 30 f + f\left(\frac{L}{D}\right)\right)$$

$$h_L = \left(\frac{11.8104\, ft/s^2}{2(32.2\, ft/s^2)}\right)\left(0.5 + 340(0.0203) + 30(0.0203) + (0.0203)\left(\frac{300\, ft}{0.1342\, ft}\right)\right)$$

$h_L = 115.64\, ft$

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$$p_1 = 62.4 lb/ft^3 \left(\frac{\left(0.167 \frac{ft^3}{s} / 0.01414 ft^2\right)^2}{2(32.2 ft/s^2)} + 115.64 ft - 3 ft \right)$$

$$\boxed{p_1 = 247.79 lb/ft^2}$$

$$\frac{p_3}{\gamma} + \frac{\cancel{V_3}^2}{2g} + \cancel{z_1} = \frac{\cancel{p_2}}{\gamma} + \frac{\cancel{V_3}^2}{2g} + \cancel{z_2} + h_L$$

$$p_3 = \frac{h_L}{\gamma} = \frac{30 f \left(\frac{V^2}{2g}\right)}{\gamma} = \frac{30(0.0203)\left(\frac{(11.8104 ft/s)^2}{2(32.2 ft/s^2)}\right)}{62.4 lb/ft^3}$$

$$\boxed{p_3 = 0.02113 lb/ft^2}$$

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Forces in the Straight Section of the Pipe

$$F_x = \rho Q(V_{2x} - V_{1x}) = P_4 A_4 - P_3 A_3 - R_x$$

$$R_x = P_4 A_4 - P_3 A_3 - \rho Q(V_{2x} - \cancel{V_{1x}})$$

$$R_x = \left(434.99\frac{lb}{ft^2}\right)\left(0.01414\,ft^2\right) - \left(0.02113 lb/ft^2\right)\left(0.01414\,ft^2\right)$$

$$\boxed{R_x = 6.15 lb}$$

$$F_y = \rho Q(V_{2y} - V_{1y}) = R_y - W$$

$$R_y = \rho Q(\cancel{V_{2y}} - \cancel{V_{1y}}) + W$$

$$\boxed{R_y = W}$$

Forces in the Elbow of the Pipe

$$F_x = \rho Q(V_{2x} - V_{1x}) = P_3 A_3 - R_x$$

$$R_x = P_3 A_3 - \rho Q(\cancel{V_{2x}} - V_{1x})$$

$$R_x = \left(0.02113 lb/ft^2\right)\left(0.01414\,ft^2\right) - 1.94\frac{slugs}{ft^3}\left(0.167\frac{ft^3}{s}\right)(-11.8104\,ft/s)$$

$$\boxed{R_x = 3.827 lb}$$

$$F_y = \rho Q(V_{2y} - V_{1y}) = -P_2 A_2 - W + R_y$$

$$R_y = \rho Q(V_{2y} - \cancel{V_{1y}}) + \cancel{P_2} A_2 + W$$

$$R_y = \rho Q(V_{2y}) + W$$

$$R_y = 1.94\frac{slugs}{ft^3}\left(0.167\frac{ft^3}{s}\right)(11.8104\,ft/s) + W$$

$$\boxed{R_y = 3.83 lb + W}$$

Total Forces:

$$\boxed{\text{Horizontal: 9.97lb}}$$

$$\boxed{Vertical: 3.83 lb + 2W}$$

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c) Determining the largest log the channel can support

$$\gamma_{\log} = \rho g = 830kg/m^3 \cdot 9.81m/s^2 = 8142.3\frac{kg}{m^2 \cdot s^2}$$

$$F_b = \gamma_{water} \cancel{V}_d$$

$$F_b = W$$

$$W = \gamma_{\log} \cancel{V}_{\log}$$

$$\gamma_{\log} \cancel{V}_{\log} = \gamma_{water} \cancel{V}_d$$

$$\cancel{V}_{\log} = \frac{\gamma_{water}}{\gamma_{\log}} \cancel{V}_d = \frac{9810\frac{kg}{m^2 \cdot s^2}}{8142.3\frac{kg}{m^2 \cdot s^2}} \cancel{V}_d = 1.2\cancel{V}_d$$

The log will be 80% submerged in the fluid based on the determined ratio.

Since y=0.392ft, $h = 0.392 \cdot 1.2 = \boxed{0.4704\,ft \times 0.4704\,ft}$

Stability Determination:

Assuming the same square cross section:

$$L_{cg} = \frac{h}{2} = \frac{0.4704\,ft}{2} = 0.2352\,ft$$

$$L_{cb} = \frac{y}{2} = \frac{0.392\,ft}{2} = 0.196\,ft$$

$$MB = \frac{I}{V_d}$$

$$I = \frac{h^4}{12} = \frac{(0.4704\,ft)^4}{12} = 0.00408\,ft^4$$

$$MB = \frac{I}{V_d} = \frac{0.00408\,ft^4}{2h \cdot L} = \frac{0.00408\,ft^4}{2(0.4704\,ft) \cdot L}$$

Assuming L=1ft

$$MB = 0.004336\,ft$$

$$L_{mc} = L_{cb} + MB = 0.196\,ft + 0.004336\,ft = 0.20\,ft$$

$$\boxed{L_{mc} < L_{cg} \therefore \text{Log is not stable}}$$

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d) Determining the pressure drop due to flow nozzle

$$\text{Re} = 131402.31$$

$$\beta = d / D = 0.5$$

$$C = 0.9975 - 6.53\sqrt{\frac{\beta}{\text{Re}}} = 0.9975 - 6.53\sqrt{\frac{0.5}{131402.31}} = 0.9848$$

$$h = \frac{\left(\frac{Q}{A_1 C}\right)^2 \bullet \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}{2g\left(\frac{\gamma_m}{\gamma_w} - 1\right)}$$

$$A_1 = 0.01414\, ft^2$$

$$A_2 = \frac{\pi}{4} D^2 = \frac{\pi}{4}\left(\frac{0.1342\, ft}{2}\right)^2 = 0.00354\, ft^2$$

Using Mercury as the manometric fluid

$$\gamma_m = 844.9 \frac{lb}{ft^3}$$

$$\gamma_w = 62.4 \frac{lb}{ft^3}$$

$$h = \frac{\left(\frac{0.167 \frac{ft^3}{s}}{\left(0.01414\, ft^2\right) 0.9848}\right)^2 \bullet \left[\left(\frac{\left(0.01414\, ft^2\right)}{0.00354\, ft^2}\right)^2 - 1\right]}{2g\left(\frac{844.9 \frac{lb}{ft^3}}{62.4 \frac{lb}{ft^3}} - 1\right)} = 0.0266\, ft$$

$$P_1 - P_2 = h(\gamma_m - \gamma_w) = 0.0266\, ft(844.9 \frac{lb}{ft^3} - 62.4 \frac{lb}{ft^3}) = \boxed{23.5 \frac{lb}{ft^2}}$$

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e) Cavitation Calculations

$$E_o = 4.49\times10^7 \frac{lb}{ft^2}$$

$$E = 200GPa = 4.177\times10^9 \frac{lb}{ft^2}$$

$$\delta = 0.0118\,ft$$

$$C = \frac{\sqrt{\frac{E_o}{\rho}}}{\sqrt{1+\frac{E_o D}{E\delta}}} = \frac{\sqrt{\frac{4.49\times10^7 \frac{lb}{ft^2}}{1.94\frac{slugs}{ft^3}}}}{\sqrt{1+\frac{\left(4.49\times10^7 \frac{lb}{ft^2}\right)(0.1342\,ft)}{\left(4.177\times10^9 \frac{lb}{ft^2}\right)(0.0118\,ft)}}} = \boxed{4541.27\,ft/s}$$

$$\Delta P = \rho AC = \left(1.94\frac{slugs}{ft^3}\right)\left(0.01414\,ft^2\right)\left(4541.27\,ft/s\right) = \boxed{124.57\frac{lb}{ft^2}}$$

f) Drag Calculations

$$F_D = C_D\left(\frac{\rho V^2}{2}\right)A$$

$$V = \frac{1.0}{\eta}S^{1/2}R^{2/3} = \frac{1.0}{1.15\times10^{-3}}\left(0.001\right)^{1/2}\frac{0.119m}{2}^{2/3} = 4.19m/s = 13.746\,ft/s$$

$$A = \text{Projected Area of the log} = \frac{h}{2}\cdot\frac{h}{2} = 0.05532\,ft^2$$

Since the projected area is a square $C_D = 1.16$

$$F_D = 1.16\left(\frac{\left(62.4\frac{lb}{ft^3}\right)\left(13.746\,ft/s\frac{ft}{s}\right)^2}{2}\right)0.05532\,ft^2$$

$$\boxed{F_D = 378.31lb}$$

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g) Forces acting upon the blind flange

$$F = pA$$

$$A_{Flange} = 0.01414\,ft^2$$

$$p = \gamma h = \left(62.4\frac{lb}{ft^3}\right)(3\,ft) = 187.2\frac{lb}{ft^2}$$

$$\boxed{F = 187.2\frac{lb}{ft^2}\left(0.01414\,ft^2\right) = 26.47 lb \text{ at the center of the flange}}$$

Summary:

The system designed above would be used to transport logs from the location of the system to a destination using gravity in an open channel. The maximum size of these logs was determined, a flow nozzle was also implemented. The drag forces acting upon a wedged log, and the force acting on the flange on the tank were also calculated.

Materials:

- Pressurized Storage Tank
- 300ft of 1.5 in Schedule 40 steel piping.
- 1 gate valve
- 1 90-degree elbow
- 1 blind flange
- An unfinished concrete channel
- Flow nozzle with a diameter ratio of 0.5

Analysis:

There were a few issues determined during the analysis of the system. There is a risk of cavitation if the valve were to be closed suddenly. This easiest way to mitigate this would be to slowly close the valve, but the pipe diameter could also be increased. Another issue was the stability of the logs, if the largest possible log were to be added to the channel it would not be stable, this could cause the log to wedge in the channel causing a large amount of drag force. This could be mitigated by limiting the size of the logs allowed to travel down the channel.