Devin Clarke
MET 330
11/19/2022

Test 3

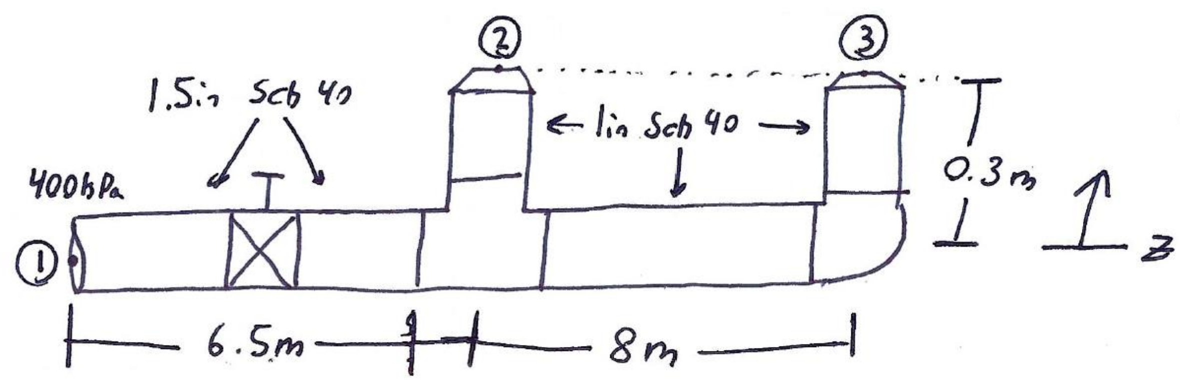
Please grade problem 1 and use problem 2 for the opportunity for extra credit on Test 2

Problem 1

Purpose:

To determine the flow rate delivered to each sprinkler head, and then determine the best method to ensure the same flow rate is delivered to each head as well as to regulate the velocity.

Drawings and Diagrams:



Sources:

Mott, R., Untener, J.A., "Applied Fluid Mechanics", 7th edition. Pearson Education, Inc, (2015)

Design Considerations:

- Constant Properties
- Incompressible Fluids
- Could not find a k value for a wide open ball valve, I used the k value for a wide open gate valve.
- Reductions were assumed to be 60 degree gradual reductions, and the diameter ratio was used.

Data and Variables:

Section 1: 1.5in Schedule 40 Steel Piping

Section 2 & 3: 1in Schedule 40 Steel Piping

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$P_1 = 400kPa$
$K_{sprinkler} = 50$
$L_1 = 6.5m$
$L_2 = 0.3m$
$L_3 = 8.3m$

Procedure:

- Create a drawing with all relevant information and choose a reference point.
- 3 points were chosen to be used in Bernoulli's equation
  - The beginning of the pipe
  - The first sprinkler head
  - The second sprinkler head
- Bernoulli's equation was written from point 1 to point 2 and from point 1 to point 3 considering all minor losses.
- Both equations were rewritten to solve for Q_2 and Q_3. For the third equation I used $Q_1 = Q_2 + Q_3$ which is valid due to conservation of mass.
- At this point iterations were completed in Excel. Since Q_1 was in the equation for Q_2 and Q_3 a value had to be guessed. In addition, f_1, f_2, and f_3 were also guessed.
- Using the guessed values Q_2 and Q_3 were solved for.
- Using the solved for Q_2 and Q_3 a new f_2 and f_3. Then a new Q_1 and f_1 were calculated.
- Iterations were run until the % difference for f_1, f_2, f_3 and Q_1 were less than 1%.

Calculations:

Bernoulli's Equation from point 1 to 2:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^{\ 2}}{2g} + z_2 + h_{L,1\to2}$$
$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = + \frac{V_2^{\ 2}}{2g} + z_2 + h_{L,1\to2}$$

Energy losses from 1 to 2:

$$h_{L,1\to2} = \left(f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g}\right) + \left(K_{tee,turn}\frac{V_1^2}{2g}\right) + \left(K_{valve}\frac{V_1^2}{2g}\right) + \left(K_{reduction}\frac{V_1^2}{2g}\right) + \left(f_2 \frac{L_2}{D_2}\frac{V_2^{\ 2}}{2g}\right) + \left(K_{head}\frac{V_2^{\ 2}}{2g}\right)$$
$$h_{L,1\to2} = \frac{V_1^2}{2g}\left[\left(f_1\frac{L_1}{D_1}\right) + \left(K_{tee,turn}\right) + \left(K_{valve}\right) + \left(K_{reduction}\right)\right] + \frac{V_2^{\ 2}}{2g}\left[\left(f_2\frac{L_2}{D_2}\right) + \left(K_{head}\right)\right]$$

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Combining:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=+\frac{V_2^{\ 2}}{2g}+z_2+\frac{V_1^2}{2g}\left[\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,turn}\right)+\left(K_{valve}\right)+\left(K_{reduction}\right)\right]+\frac{V_2^{\ 2}}{2g}\left[\left(f_2\frac{L_2}{D_2}\right)+\left(K_{head}\right)\right]$$

$$\frac{p_1}{\gamma}-z_2=\frac{V_2^{\ 2}}{2g}+\frac{V_1^2}{2g}\left[\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,turn}\right)+\left(K_{valve}\right)+\left(K_{reduction}\right)\right]+\frac{V_2^{\ 2}}{2g}\left[\left(f_2\frac{L_2}{D_2}\right)+\left(K_{head}\right)\right]-\frac{V_1^2}{2g}$$

$$\frac{p_1}{\gamma}-z_2=\frac{V_1^2}{2g}\left[\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,turn}\right)+\left(K_{valve}\right)+\left(K_{reduction}\right)-1\right]+\frac{V_2^{\ 2}}{2g}\left[\left(f_2\frac{L_2}{D_2}\right)+\left(K_{head}\right)+1\right]$$

$$\frac{p_1}{\gamma}-z_2=Q_1^{\ 2}\left(\frac{8}{g\pi^2D_1^{\ 4}}\right)\left[\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,turn}\right)+\left(K_{valve}\right)+\left(K_{reduction}\right)-1\right]+Q_2^{\ 2}\left(\frac{8}{g\pi^2D_2^{\ 4}}\right)\left[\left(f_2\frac{L_2}{D_2}\right)+\left(K_{head}\right)+1\right]$$

Solving for Q_2

$$\frac{p_1}{\gamma}-z_2=Q_1^{\ 2}\left(\frac{8}{g\pi^2D_1^{\ 4}}\right)\left[\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,turn}\right)+\left(K_{valve}\right)+\left(K_{reduction}\right)-1\right]+Q_2^{\ 2}\left(\frac{8}{g\pi^2D_2^{\ 4}}\right)\left[\left(f_2\frac{L_2}{D_2}\right)+\left(K_{head}\right)+1\right]$$

$$\boxed{\sqrt{\frac{\frac{p_1}{\gamma}-z_2-Q_1^{\ 2}\left(\frac{8}{g\pi^2D_1^{\ 4}}\right)\left[\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,turn}\right)+\left(K_{valve}\right)+\left(K_{reduction}\right)-1\right]}{\left(\frac{8}{g\pi^2D_2^{\ 4}}\right)\left[\left(f_2\frac{L_2}{D_2}\right)+\left(K_{head}\right)+1\right]}}=Q_2}$$

Bernoulli's Equation from point 1 to 3

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_3}{\gamma}+\frac{V_3^{\ 2}}{2g}+z_3+h_{L,1\to 3}$$

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=\frac{V_3^{\ 2}}{2g}+z_3+h_{L,1\to 3}$$

Energy losses from 1 to 2:

$$h_{L,1\to 2}=\left(f_1\frac{L_1}{D_1}\frac{V_1^2}{2g}\right)+\left(K_{tee,straight}\frac{V_1^2}{2g}\right)+\left(K_{reduction}\frac{V_1^2}{2g}\right)+\left(K_{valve}\frac{V_1^2}{2g}\right)+\left(f_3\frac{L_3}{D_3}\frac{V_3^{\ 2}}{2g}\right)+\left(K_{head}\frac{V_3^{\ 2}}{2g}\right)+\left(K_{elbow}\frac{V_3^{\ 2}}{2g}\right)$$

$$h_{L,1\to 2}=\frac{V_1^2}{2g}\left[\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,straight}\right)+\left(K_{valve}\right)+\left(K_{reduction}\right)\right]+\frac{V_3^{\ 2}}{2g}\left[\left(f_3\frac{L_3}{D_3}\right)+\left(K_{head}\right)+\left(K_{elbow}\right)\right]$$

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Combining:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=\frac{V_3^2}{2g}+z_3+h_{L,1\to3}$$

$$\frac{p_1}{\gamma}-z_3=\frac{V_3^2}{2g}-\frac{V_1^2}{2g}+\frac{V_1^2}{2g}\left[\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,straight}\right)+\left(K_{valve}\right)+\left(K_{reduction}\right)\right]+\frac{V_3^2}{2g}\left[\left(f_3\frac{L_3}{D_3}\right)+\left(K_{head}\right)+\left(K_{elbow}\right)\right]$$

$$\frac{p_1}{\gamma}-z_3=\frac{V_1^2}{2g}\left[\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,straight}\right)+\left(K_{valve}\right)+\left(K_{reduction}\right)-1\right]+\frac{V_3^2}{2g}\left[\left(f_3\frac{L_3}{D_3}\right)+\left(K_{head}\right)+\left(K_{elbow}\right)+1\right]$$

$$\frac{p_1}{\gamma}-z_3=Q_1^2\left(\frac{8}{g\pi^2D_1^4}\right)\left[\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,straight}\right)+\left(K_{valve}\right)+\left(K_{reduction}\right)-1\right]+$$
$$Q_3^2\left(\frac{8}{g\pi^2D_3^4}\right)\left[\left(f_3\frac{L_3}{D_3}\right)+\left(K_{head}\right)+\left(K_{elbow}\right)+1\right]$$

Solving for Q_3

$$\frac{p_1}{\gamma}-z_3=Q_1^2\left(\frac{8}{g\pi^2D_1^4}\right)\left[\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,straight}\right)+\left(K_{valve}\right)+\left(K_{reduction}\right)-1\right]+$$
$$Q_3^2\left(\frac{8}{g\pi^2D_3^4}\right)\left[\left(f_3\frac{L_3}{D_3}\right)+\left(K_{head}\right)+\left(K_{elbow}\right)+1\right]$$

$$\boxed{\sqrt{\frac{\frac{p_1}{\gamma}-z_3-Q_1^2\left(\frac{8}{g\pi^2D_1^4}\right)\left[\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,straight}\right)+\left(K_{valve}\right)+\left(K_{reduction}\right)-1\right]}{\left(\frac{8}{g\pi^2D_3^4}\right)\left[\left(f_3\frac{L_3}{D_3}\right)+\left(K_{head}\right)+\left(K_{elbow}\right)+1\right]}}=Q_3}$$

Due to conservation of mass:

$$\boxed{Q_1=Q_2+Q_3}$$

Values for Q_1, f_1, f_2, and f_3 were assumed and iterations were completed in Excel.

$$\boxed{\begin{aligned}Q_1&=0.004103m^3/s\\Q_2&=0.00211m^3/s\\Q_3&=0.001996175m^3/s\end{aligned}}$$

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Fluid Velocity:

$$V = Q / A$$

$$V_1 = \frac{0.004103 m^3 / s}{\left(\frac{\pi (0.0409)^2}{4}\right)} = 3.12 m / s$$

$$V_2 = \frac{0.00211 m^3 / s}{\left(\frac{\pi (0.0266)^2}{4}\right)} = 3.80 m / s$$

$$V_3 = \frac{0.001996175 m^3 / s}{\left(\frac{\pi (0.0266)^2}{4}\right)} = 3.59 m / s$$

Summary:

This system was a series pipeline designed to deliver flow to two sprinkler heads. With the current design the flow rate at head 1 and head 2 will be slightly different. In addition, the velocity in each section is over the critical velocity. Possible design changes will be discussed in the analysis section.

Materials:

- 6.5m of 1 ½ in schedule 40 steel pipe
- 8.6m of 1 in schedule 40 steel pipe
- 1 elbow
- 1 tee
- 1 ball valve
- 2 sprinkler heads

Analysis:

The current design of the sprinkler system delivers a different flow rate to each sprinkler head. The difference in flow rate is about 2 GPM, if this is unacceptable a valve could be added to restrict flow to head number 1, this would need to be located after the turn of the tee. The velocity in each section is higher than the critical velocity, the easiest solution to remedy this would be to slightly increase the area of the pipes.

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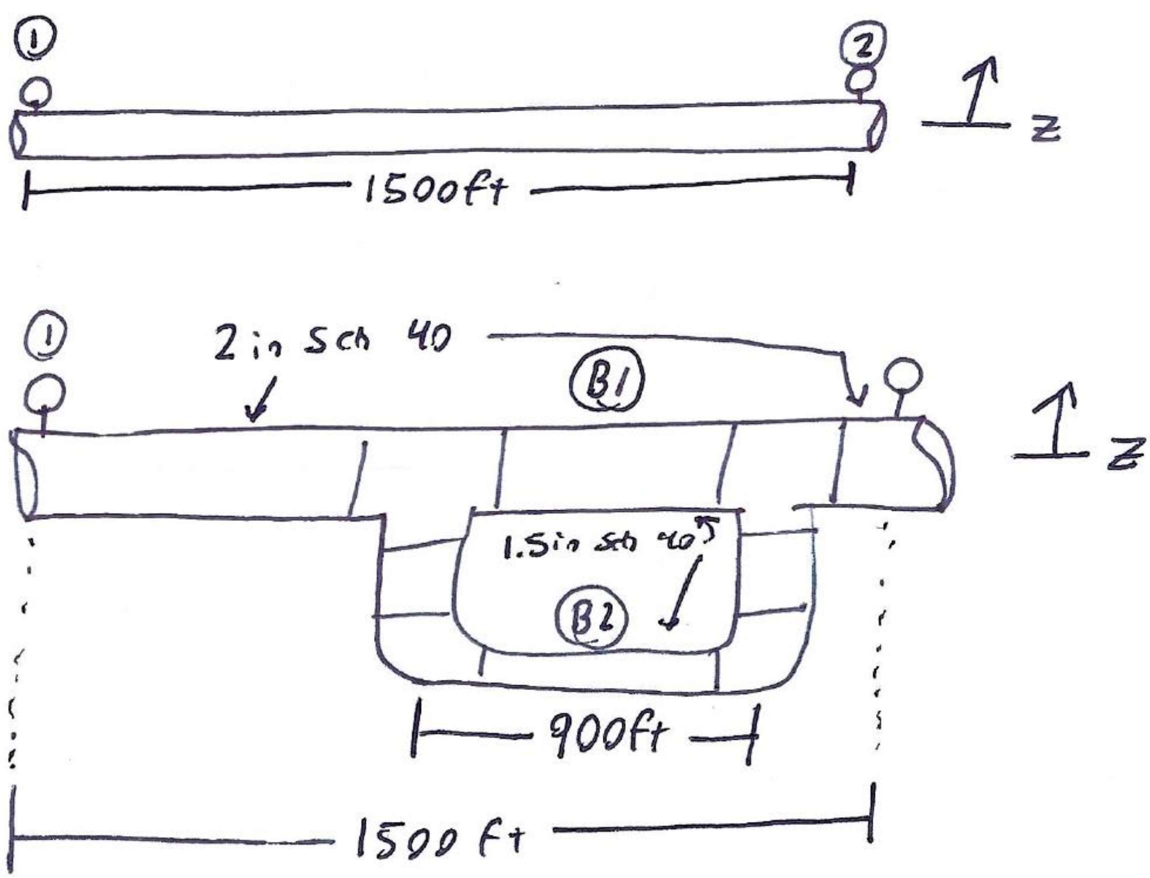
Problem 2

Purpose:

To determine the pressure drop in the first system, and then the flow rate of each branch in the second system using the calculated pressure drop from the first system.

Drawings and Diagrams:



Sources:

Mott, R., Untener, J.A., “Applied Fluid Mechanics”, 7th edition. Pearson Education, Inc, (2015)

Design Considerations:

- Constant Properties
- Incompressible Fluids
- The vertical section of piping in system 2 will be neglected

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- Reductions were assumed to be 60 degree gradual reductions, and the diameter ratio was used.

Data and Variables:

- Branch 1: 2 in Schedule 40 Steel Pipe
- Branch 1: L=900ft
- Branch 2: 1.5 in Schedule 40 Steel Pipe
- Branch 2: L=900ft

Procedure:

- The first part of the problem involves the straight piping section, a drawing was made and a reference decided upon.
- Bernoulli's equation was written from the left side of the pipe to the right side.
- The only losses to take into account was from friction in the pipe.
- The pressure drop can be solved using the written Bernoulli's equation
- The second part of the problem involves adding a branch making this a parallel system.
- A Bernoulli's equation must be written going from point A to B for each branch of the system.
- Three equations will be made, 1 for each flow rate and 1 equating the total flow rate to each individual flow rate.
- At this point the values for f1 and f2 can be guessed and the flow rate for branch 1 and 2 can be solved for.
- Iterations will be completed in Excel until the % difference between the guessed values for f1 and f2 are within less than 1% of the calculated values.

Calculations:

Bernoulli's equation system 1:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_{L,1\to 2}$$

$$\frac{p_1}{\gamma}+=\frac{p_2}{\gamma}+h_{L,1\to 2}$$

$$\frac{\Delta p}{\gamma}=h_{L,1\to 2}$$

Energy Loss Calculation:

$$h_{L,1\to 2}=\left(f\frac{L}{D}\frac{V^2}{2g}\right)=\left(f\frac{L}{D}Q^2\frac{8}{g\pi^2D^4}\right)$$

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Friction Factor:

$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7\left(\frac{D}{\varepsilon}\right)}\right)+\left(\frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2}$$

$$D = 0.0525m$$

$$\varepsilon = 0.000046m$$

$$\mathrm{Re} = \frac{VD}{\upsilon} = \frac{\frac{4Q}{\pi D}}{\upsilon}$$

$$Q = 65gpm = 0.0041m^3/s$$

$$\upsilon = 1.02x10^{-6}m^2/s$$

$$\mathrm{Re} = \frac{\frac{4Q}{\pi D}}{\upsilon} = \frac{\frac{4\left(0.0041m^3/s\right)}{\pi\left(0.0525m\right)}}{\left(1.02x10^{-6}m^2/s\right)} = 97,484$$

$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7\left(\frac{D}{\varepsilon}\right)}\right)+\left(\frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log\left(\frac{1}{3.7\left(\frac{0.0525m}{0.000046m}\right)}\right)+\left(\frac{5.74}{97,484^{0.9}}\right)\right]^2} = 0.01169$$

Combining:

$$\frac{\Delta p}{\gamma} = h_{L,1\to 2}$$

$$\Delta p = \gamma h_{L,1\to 2} = \gamma\left(f\frac{L}{D}Q^2\frac{8}{g\pi^2D^4}\right)$$

$$\Delta p = \left(9.79kN/m^3\right)\left(\left(0.01169\right)\frac{457.2m}{0.0525m}\left(\left(0.0041m^3/s\right)^2\right)\left(\frac{8}{\left(9.81m/s^2\right)\pi^2\left(0.0525m\right)^4}\right)\right)$$

$$\boxed{\Delta p = 182.22kPa}$$

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Bernoulli's Equation System 2 Branch 1:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_{L,1\to 2,branch1}$$

$$\frac{p_1}{\gamma} = \frac{p_2}{\gamma} + h_{L,1\to 2,branch1}$$

$$\Delta p = \gamma h_{L,1\to 2,branch1}$$

Energy Losses through branch 1:

$$h_{L,1\to 2,branch1} = \left(f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} \right) + \left(K_{tee,straight} \frac{V_1^2}{2g} \right) + \left(K_{reduction} \frac{V_1^2}{2g} \right) + \left(f_{B1} \frac{L_{B1}}{D_{B1}} \frac{V_{B1}^{\ 2}}{2g} \right) + \left(K_{tee,straight} \frac{V_{B1}^{\ 2}}{2g} \right)$$

$$h_{L,1\to 2,branch1} = \frac{V_1^2}{2g} \left[\left(f_1 \frac{L_1}{D_1} \right) + \left(K_{tee,straight} \right) + \left(K_{reduction} \right) \right] + \frac{V_{B1}^{\ 2}}{2g} \left[\left(f_{B1} \frac{L_{B1}}{D_{B1}} \right) + \left(K_{tee,straight} \right) \right]$$

$$h_{L,1\to 2,branch1} = Q_1^2 \left(\frac{8}{g\pi^2 D_1^{\ 2}} \right) \left[\left(f_1 \frac{L_1}{D_1} \right) + \left(K_{tee,straight} \right) + \left(K_{reduction} \right) \right] + Q_{B1}^{\ 2} \left(\frac{8}{g\pi^2 D_{B1}^{\ 2}} \right) \left[\left(f_{B1} \frac{L_{B1}}{D_{B1}} \right) + \left(K_{tee,straight} \right) \right]$$

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Combining and solving for Q_{B1}

$$\Delta p = \gamma h_{L,1\rightarrow 2,branch1}$$

$$\frac{\Delta p}{\gamma} = Q_1^2\left(\frac{8}{g\pi^2 D_1^2}\right)\left[\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,straight}\right)+\left(K_{reduction}\right)\right]+Q_2^2\left(\frac{8}{g\pi^2 D_2^2}\right)\left[\left(f_2\frac{L_2}{D_2}\right)+\left(K_{tee,straight}\right)\right]$$

$$\boxed{\sqrt{\frac{\frac{\Delta p}{\gamma}-Q_1^2\left(\frac{8}{g\pi^2 D_1^2}\right)\left[\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,straight}\right)+\left(K_{reduction}\right)\right]}{\left(\frac{8}{g\pi^2 D_{B1}^2}\right)\left[\left(f_{B1}\frac{L_{B1}}{D_{B1}}\right)+\left(K_{tee,straight}\right)\right]}}=Q_{B1}}$$

Bernoulli's Equation System 2 Branch 2:

$$\frac{\Delta p}{\gamma} = h_{L,1\rightarrow 2,branch2}$$

Energy Losses through branch 2:

$$h_{L,1\rightarrow 2,branch1} = \left(f_1\frac{L_1}{D_1}\frac{V_1^2}{2g}\right)+\left(K_{tee,turn}\frac{V_1^2}{2g}\right)+\left(K_{reduction}\frac{V_1^2}{2g}\right)+\left(f_{B2}\frac{L_{B2}}{D_{B2}}\frac{V_{B2}^2}{2g}\right)+\left(K_{tee,turn}\frac{V_{B2}^2}{2g}\right)+\left(2K_{elbow}\frac{V_{B2}^2}{2g}\right)$$

$$h_{L,1\rightarrow 2,branch1} = \frac{V_1^2}{2g}\left[\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,turn}\right)+\left(K_{reduction}\right)\right]+\frac{V_{B2}^2}{2g}\left[\left(f_{B2}\frac{L_{B2}}{D_{B2}}\right)+\left(K_{tee,turn}\right)+\left(2K_{elbow}\right)\right]$$

$$h_{L,1\rightarrow 2,branch1} = Q_1^2\left(\frac{8}{g\pi^2 D_1^2}\right)\left[\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,turn}\right)+\left(K_{reduction}\right)\right]+Q_{B2}^2\left(\frac{8}{g\pi^2 D_{B2}^2}\right)\left[\left(f_{B2}\frac{L_{B2}}{D_{B2}}\right)+\left(K_{tee,turn}\right)+\left(2K_{elbow}\right)\right.$$

Combine and solve for Q_{B2}:

$$\frac{\Delta p}{\gamma} = Q_1^2\left[\left(\frac{8}{g\pi^2 D_1^2}\right)\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,turn}\right)+\left(K_{reduction}\right)\right]+Q_{B2}^2\left[\left(\frac{8}{g\pi^2 D_{B2}^2}\right)\left(f_{B2}\frac{L_{B2}}{D_{B2}}\right)+\left(K_{tee,turn}\right)+\left(2K_{elbow}\right)\right]$$

$$\boxed{\sqrt{\frac{\frac{\Delta p}{\gamma}-Q_1^2\left(\frac{8}{g\pi^2 D_1^2}\right)\left(f_1\frac{L_1}{D_1}\right)+\left(K_{tee,turn}\right)+\left(K_{reduction}\right)}{\left(\frac{8}{g\pi^2 D_{B2}^2}\right)\left(f_{B2}\frac{L_{B2}}{D_{B2}}\right)+\left(K_{tee,turn}\right)+\left(2K_{elbow}\right)}}=Q_{B2}}$$

Due to conservation of mass:

$$\boxed{Q_1 = Q_{B1}+Q_{B2}}$$

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Iterations were completed within excel to determine the new flow rate as well as the flow rate through the individual branches:

$$Q_1 = 0.002333 \frac{m^3}{s}$$

$$Q_{b1} = 0.001206 \frac{m^3}{s}$$

$$Q_{b2} = 0.001127 \frac{m^3}{s}$$

Summary:

The first system is a simple straight pipe, in which the pressure drop was calculated to use in the more complicated second system. The second system is a parallel pipe system with two branches. Using the same pressure drop as system one, the new flow rate was determined.

Materials:

System 1:

- 1500 ft of 2in schedule 40 steel piping

System 2:

- 600ft of 2in schedule 40 steel piping
- 1800 ft of 1 ½ in schedule 40 steel piping
- 2 elbows
- 2 tees

Analysis:

The added branch in system two caused the flow rate to decrease, this is as expected due to there being more minor losses due to the fittings. The original flow rate was 0.0041m^3/s the new flow rate is 0.00233 m^3/s. This is a decrease of 0.00177 m^3/s. If the goal in the system change was to not have a decrease in flow rate than the pipe size should be increased.