Module 2 - Fundamentals of Risk Management

Unit 1: Fundamentals of Risk Management

Learning Outcomes

Upon completion this module, students will be able to:

• Define risk
• Distinguish quantitative and qualitative risk management
• Explain probability, value, and utility theories
• Discuss the extreme event analysis
• Explain the risk management process

Module Outline

• Definition of Risk
• Quantitate vs. Qualitative
• Basics of risk and decision theory
  • elements of probability theory
  • value function
  • GRADUATE utility function
  • GRADUATE extreme event analysis
• RM Process

Definition of Risk
What We Know About Risk

• The possibility of loss or injury
• The chance an unwanted event occurs
• In economics, this question is answered in terms of a person’s monetary perspective or value structure
• Requires human interpretation and value judgments specific to a situation

Exercise 1

• Name two risk events (preferably related to cyber) that if they happen would result in unwanted consequences?

What Does it Mean to Manage (and Assess) Risk?

• Risk management is a formal process used to continuously identify, analyze, and adjudicate events that, if they occur, have unwanted impacts on a system’s ability to achieve its outcome objectives (Garvey, 2008).

Objectives of Risk Management

Early and continuous identification, management, and resolution of risks such that engineering a system (including cyber element) is accomplished within cost, delivered on time, and meets performance requirements.
Why is Risk Management Important?

1. It fosters the early and continuous identification of risks
2. It enables risk-informed decision making and course-of-action planning
3. It enables identified risk events to be mapped to the various components of a system (including cyber components)
4. It helps identify where management should consider allocating limited (or competing) resources
5. It can be designed to provide management with situational awareness in terms of a system’s (including cyber components) risk status

Exercise 2

- Why EARLY?
- Why CONTINUOUS?

TARGET Case

(SANS Institute, 2014)
- In late 2013, attackers revealed a vendor portal and a list of HVAC and refrigeration companies providing equipment for TARGET.
- A malware attached email was sent to a refrigeration seller, Fazio Mechanical, two months before the credit card breach.
- "Malware installed on vendor machine may have been Citadel - a password stealing bot program that is derivative of the Zeus banking Trojan."
- The malware transmitted credentials to an online vendor portal.

TARGET Network
TARGET Case (cont’d)

• Next, the criminals accessed Target’s system via Fazio Mechanical’s credentials via a vendor portal.
• Once access was obtained to the necessary systems, malware was installed on point of sale systems.
• The software obtained credit card credentials from memory every time cards were swiped.
• The data was saved to a .dll file and stored in a temporary NetBIOS share over ports.
• “Data was moved to drop locations on hacked servers all over the world via FTP. Hackers retrieved the data from drop locations which hackers accessed to retrieve it.”
• Then, credit cards were sold in the black market.
• The cost of the breach was huge including Target, customers banks, and employees.
• Top employees lost their jobs including the CEO and CIO.

Risk Definition

Risk - an event that, if it occurs, adversely affects the ability of an engineering system (including cyber) to achieve its outcome objectives (Garvey, 2008).

Risk Defined

A general expression for measuring risk is given by (Garvey, 2008) as

\[ \text{Risk} = f(\text{Probability}, \text{Consequence}) \]

Uncertainty Defined

Uncertainty - an event is uncertain if there is indefiniteness about its outcome (Garvey, 2008).
Classification of Uncertainty

- **Epistemic** uncertainty refers to uncertainty about an event due to incomplete knowledge (Ayyub, 2001).
- **Aleatoric** uncertainty refers to inherent randomness associated with some events in the physical world (Ayyub, 2001).

Exercise 3

Pressures to meet cost, schedule, and technical performance are the practical realities in engineering today’s systems.

- Discuss ways **cyber practitioners (e.g., system administrators)** might lessen or guard against these pressures within and across these dimensions (10 minutes)

Exercise 3 (Cont.)

- Continuation of the previous example

Measurement scale

**Foundations of Risk and Decision Theory**
Why Do We Measure?

Measurement is the assignment of numbers to objects

Measurement Scales

- Three general types of measurement scales
  - Nominal
  - Ordinal
  - Cardinal
    - Interval - addition and subtraction permitted
    - Ratio - addition, subtraction, multiplication, and division permitted

Other Measurement Scales

- Other types for cyber practitioners
  - Natural or Constructed
  - Direct or Proxy

Nominal Scale

- A nominal scale is a measurement scale in which attributes are assigned a label (i.e., a name).
- Nominal data can be counted but no quantitative differences or preference ordering of the attributes are implied.
- Arithmetic operations are without meaning in a nominal scale.
Ordinal Scale

• An ordinal scale is a measurement scale in which attributes are assigned a number that represents order or rank.
• Arithmetic operations are without meaning in an ordinal scale.

1: Worst  2: Good  3: Very Good  4: Best

Cardinal Scale

• A cardinal scale is a measurement scale that is either interval scaled or ratio scaled.

Cardinal-Interval Scale (or Simply Interval)

• An interval scale is a measurement scale in which attributes are assigned numbers such that differences between them have meaning.
• However, the zero point on an interval scale is chosen for convenience and does not necessarily represent the absence of the attribute being measured.

Interval Scale

• Examples: Fahrenheit or Celsius temperature scales
• The zero point on these temperature scales does not mean the absence of temperature
• Because distances between numbers in an interval scale have meaning, addition and subtraction of interval scale numbers is permitted
• However, because the zero point is arbitrary, multiplication and division of interval scale numbers are not permitted.
Interval Scale Examples

- For example, we can say that 75 degrees Fahrenheit is 25 degrees hotter than 50 degrees Fahrenheit.
- But, we cannot say that 75 degrees Fahrenheit is 50% hotter than 50 degrees Fahrenheit.
- However, in an interval scale ratios of differences can be expressed meaningfully; for example, one difference can be one-half or twice or three times another difference.

Cardinal-Ratio Scale (or Simply Ratio)

- Ratio scale has a true zero representing a complete absence of the characteristic being measured by the attribute.

Ratio Scale

- Differences between the numbers on this scale reflect differences of the attribute.
- Ratios between the numbers on this scale reflect ratios of the attribute.
- All arithmetic operations are permitted on numbers that fall along a ratio scale.
- Examples of ratio scales include such measures as distance, weight, money, probability.

Exercise 4

Provide examples not yet mentioned in the discussion

- Nominal
- Ordinal
- Cardinal
  - Interval
  - Ratio
Basics of Risk and Decision Theory

Basics of Risk and Decision Theory - Overview

- Elements of probability theory
- Bayes’ Rule
- Value function
- GRADUATE Utility function
- GRADUATE Extreme event analysis

Elements of Probability Theory
Sample Space

• The set of all possible outcomes of an experiment
• Denoted by Ω (uppercase Greek omega)

Sample Points

• The individual outcomes of Ω
• Denoted by ω (lowercase Greek omega)

Event

• Any subset of the sample space.
• An event is simple (or elementary) if it consists of exactly one outcome
• An event is compound if it consists of more than one outcome.

Exercise 5

• Experiment: tossing of a die
• Sample space (Ω): ??
• Sample points (ω): ??
• Simple events: ??
• Compound events: ??
Union

• For any two events $A$ and $B$ of a sample space $\Omega$, the new event $A \cup B$ (which reads A union B) consists of all outcomes either in $A$ or in $B$ or in both $A$ and $B$.
• The event $A \cup B$ occurs if either $A$ or $B$ occurs.

Intersection

• For any two events $A$ and $B$ of a sample space $\Omega$, the new event $A \cap B$ (which reads A intersection B) consists of all outcomes that are in both $A$ and $B$.
• The event $A \cap B$ occurs only if both $A$ and $B$ occur.

Exercise 6

• Experiment: tossing of a die
• Event $A = \{1, 2, 3\}$
• Event $B = \{3, 4, 5\}$
• $A \cup B = ??$
• $A \cap B = ??$

Complement

• The complement of event $A$, denoted by $A^c$, consists of all outcomes in the sample space that are not in $A$.
• The event $A^c$ occurs if and only if $A$ does not occur.
Subset

• Event A is said to be a subset of event B if all the outcomes in A are also contained in B.
• This is written as $A \subseteq B$.

Exercise 7

• Experiment: tossing of a die
• $A = \{1, 2, 3\}$
• $A' = ??$
• $A \cup A' = ??$
• $A \cap A' = ??$
• $A \subseteq A'$, true or false?

Axiomatic Definition of Probability

In accordance with Kolmogorov’s axioms, probability is simply a numerical measure that satisfies the following:

**Axiom 1** $0 \leq P(A) \leq 1$ for any event A in $\Omega$

**Axiom 2** $P(\Omega) = 1$

**Axiom 3** For any sequence of mutually exclusive events $A_1, A_2, \ldots$ defined on $\Omega$

$$P \left( \bigcup_{i=1}^{\infty} A_i \right) = \sum_{i=1}^{\infty} P(A_i)$$

For any finite sequence of mutually exclusive events $A_1, A_2, \ldots, A_n$ defined on $\Omega$

$$P \left( \bigcup_{i=1}^{n} A_i \right) = \sum_{i=1}^{n} P(A_i)$$

Axiomatic Definition of Probability - Assumption

• It is assumed for each event $A_i$ in the sample space $\Omega$, there is a real number $P(A)$ that denotes the probability of $A$.
• These axioms define probability in a way that encompasses:
  • equally likely interpretation of probability
  • frequency interpretations of probability
Interpretations of Probability

• Equally likely interpretation
• Frequency interpretation
• Bayesian interpretation

Equally Likely Interpretation

• In this view, if a sample space Ω consists of a finite number of outcomes n, which are all equally likely to occur, then the probability of each simple event is 1/n.
• If an event A consists of m of these n outcomes, then the probability of event A is

\[ P(A) = \frac{m}{n} \]

Exercise 8

• Experiment: tossing of a die
• n = ?
• If A = \{1,2,3\}, then P(A) = ??

Frequency Interpretation

• In this view, the probability of an event is the limiting proportion of time the event occurs in a set of n repetitions of the experiment.
Frequency Interpretation (Cont.)

• In particular, we write this as
  \[ P(A) = \lim_{n \to \infty} \frac{n(A)}{n} \]

• It restricts events to those that can be subjected to repeated trials conducted under identical conditions.

Exercise 9

• Experiment: tossing of a die
  • If \( A = \{1,2,3\} \), then \( P(A) = ?? \)
  • What assumption have to be made to apply frequency interpretation of probability?

Conditional Probability

• The conditional probability of event \( A \) given event \( B \) has occurred is denoted by \( P(A|B) \).
  \[ P(A|B) = \frac{P(A \cap B)}{P(B)} \]

Independent Events

• Two events \( A \) and \( B \) are said to be independent if and only if \( P(A \cap B) = P(A)P(B) \), and dependent otherwise.
Mutually Independent Events

- Events $A_1, A_2, \ldots, A_n$ are (mutually) independent if and only if the probability of the intersection of any subset of these $n$ events is the product of their respective probabilities.

Random Variable

- A random variable is a real valued function defined over a sample space $\Omega$
- The sample space $\Omega$ is the domain of a random variable
- Traditionally, random variables are denoted by capital letters such as $X$.

Discrete or Continuous

- Random variables can be characterized as discrete or continuous
- A random variable is discrete if its set of possible values is finite or countably infinite
- A random variable is continuous if its set of possible values is uncountable

Cumulative Distribution Function

- The function that produces probabilities for events of the form $\{X \leq x\}$ is known as the cumulative distribution function (CDF)
- Formally, if $X$ is a discrete random variable, then its CDF is defined by

$$F_X(x) = P(X \leq x) = \sum_{t \leq x} p_X(t) \quad (-\infty < x < \infty)$$
Exercise 10

A sack contains 20 marbles exactly alike in size but different in color. Suppose the sack contains 5 blue marbles, 3 green marbles, 7 red marbles, 2 yellow marbles, and 3 black marbles. Picking a single marble from the sack and then replacing it, what is the probability of choosing the following:

a. Blue marble
b. Non-blue marble
c. Red or non-red marble
d. Relevance in RM?

Exercise 10 (Answer)

A sack contains 20 marbles exactly alike in size but different in color. Suppose the sack contains 5 blue marbles, 3 green marbles, 7 red marbles, 2 yellow marbles, and 3 black marbles. Picking a single marble from the sack and then replacing it, what is the probability of choosing the following:

a. \( \text{Pr}(\text{Blue marble}) = \frac{5}{20} \)
b. \( \text{Pr}(\text{Non-blue marble}) = \frac{15}{20} \)
c. \( \text{Pr}(\text{Red or non-red marble}) = \frac{20}{20} = 1 \)

Exercise 10 (Answer Cont.)

d. Relevance in RM?

When estimating the chances of occurrence given possible ways things can turn out (desirable or not).

Bayes’ Rule
Bayes’ Rule - Equation

Degree of confidence or certainty based on knowledge of the phenomenon

\[ P(A_i|B) = \frac{P(A_i)P(B|A_i)}{\sum_{i=1}^{n} P(A_i)P(B|A_i)} \]

Total Probability Law

\[ P(B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3) + \ldots \]

\[ P(B) = \sum_{i=1}^{n} P(A_i)P(B|A_i) \]

Bayes’ and Total Prob. Together

- The conditional probability for each event Ai given event B has occurred is

\[ P(A_i|B) = \frac{P(A_i \cap B)}{P(B)} = \frac{P(A_i)P(B|A_i)}{P(B)} \]

Exercise 11: Monty Hall Paradox (1)

Hosts:
- Monty Hall (1963–91)
- Bob Hilton (1990)
- Wayne Brady (2009–present)

What do we do with new information?
Exercise 11: Monty Hall Paradox (2)

https://www.youtube.com/watch?v=mhlc7peGiGg
Pause at 1:30"

- What is the initial probability of the event "car behind door A?" B? or C?
- What is the new set of information?
- How do these probabilities change with the new set of information?
- How can we show these changes using Bayes' theorem?

Exercise 11: Monty Hall Paradox (3)

What is the initial probability of the event car behind door A?, B?, or C?

Given information: There is exactly one car behind one of the three doors
Designate X is the event "car behind door X", then

- \( \Pr(A) = \frac{1}{3} \)
- \( \Pr(B) = \frac{1}{3} \)
- \( \Pr(C) = \frac{1}{3} \)

Exercise 11: Monty Hall Paradox (4)

Let us assume you choose door C, without loosing generality

\( \Pr(C) = ? \)
\( \Pr(A \cup B) = ? \)

You chose door C.

Exercise 11: Monty Hall Paradox (5)

Monty Hall showed you that the car is not behind door B.
Exercise 11: Monty Hall Paradox (6)

What is the new set of information?

"Car is not behind door B."

• In fact Monty Hall will never show the contestant where is the car; he always opens the door without the car.

How do these probabilities change with the new set of information?

How can we show these changes using Bayes' theorem?

Let new information $Z = \text{event "Monty Hall opens door B"}$

That is, what are:

$P(A|Z) =$ ?

$P(B|Z) =$ ?

$P(C|Z) =$ ?

Monty Hall showed you that the car is not behind door B.

Exercise 11: Monty Hall Paradox (7)

$P(B|Z) = P(\text{car is behind door B}|\text{Monty Hall opens door B}) = 0$

...because he always opens the door without the car.

$P(A|Z) = P(Z|A)P(A) / P(Z)$

where

$P(Z|A) = P(\text{Monty Hall opens door B}|\text{car is behind door A})$

$P(A) = P(\text{car is behind door A})$

$P(Z) = P(\text{Monty Hall opens door B})$

$= (1 \times 1/3) / \frac{1}{2} = 2/3$

Monty Hall showed you that the car is not behind door B.

Exercise 11: Monty Hall Paradox (8)

$P(C|Z) = P(Z|C)P(C) / P(Z)$

where

$P(Z|C) = P(\text{Monty Hall opens door B}|\text{car is behind door C})$

$P(C) = P(\text{car is behind door C})$

$P(Z) = P(\text{Monty Hall opens door B})$

$= (1/2 \times 1/3) / \frac{1}{2} = 1/3$

Monty Hall showed you that the car is not behind door B.

Exercise 11: Monty Hall Paradox (9)

$P(A) = 1/3$

$P(B) = 1/3$

$P(C) = 1/3$

$P(A|Z) = 2/3$

$P(B|Z) = 0$

$P(C|Z) = 1/3$

Would you switch or stay with initial choice?
Exercise 11: Monty Hall Paradox (10)

\[ Pr(A) = \frac{1}{3} \]
\[ Pr(B) = \frac{1}{3} \]
\[ Pr(C) = \frac{1}{3} \]

\[ Pr(A | Z) = \frac{2}{3} \]
\[ Pr(B | Z) = 0 \]
\[ Pr(C | Z) = \frac{1}{3} \]

Are these still consistent with the axiomatic definition of probability?

Value of New Information in Risk Management

- Reducing uncertainty
- Express reduction in uncertainty by updating initial probabilities

Value of New Information in Cyber Risk Management

- Data traffic analysis as a sign of impending DDoS?
- Geo-political disturbances prior to state-sponsored hacking?
- Choosing among new encryption techniques, later on to be updated by new information on its success or failure?
- Updating likelihood of ZDV exploitation based on recent events?

Quality of New Information

Are all new information the same?

The quality of information matters
- How predictive is it of our event of interest?
Exercise 12: Cyber Intrusion Detection (1)

- Event of interest, A, is unauthorized access into a network
- Evidence, B, is intrusion detection technology setting off due to anomalous port traffic
- Assume: only two types of events, A or A'

Exercise 12: Cyber Intrusion Detection (2)

Case 1
- Chance there will be unauthorized access, \( p(A) = \frac{8}{25} \)
- Chance the intrusion detection technology setting off given there is unauthorized access, \( p(B|A) = \frac{2}{8} \)
- Chance the intrusion detection technology setting off given there is no unauthorized access, \( p(B|A') = \frac{4}{17} \)

Exercise 12: Cyber Intrusion Detection (3)

Case 2: same network + another type of intrusion detection technology
- \( p(A) = \frac{8}{25} \)
- More sensitive intrusion detection technology, \( p(B|A) = \frac{4}{8} \)
- Chance the intrusion detection technology setting off given there is no unauthorized access, \( p(B|A') = \frac{2}{17} \)

Exercise 12: Cyber Intrusion Detection (4)

- Use Bayes' Rule & Total Probability Theorem to compare Case 1 and Case 2 based on the evidences provided
- Try to use the included graph paper to accomplish this exercise
Exercise 12: Cyber Intrusion Detection (5)

This graph will be used in this exercise.

Exercise 12: Case 1 (6)

33.03 p(A) = 1/4

1) B|A (p = 25/6)

Exercise 12: Case 1 (7)

33.03 p(A) = 1/4

1) B|A (p = 25/6)

Exercise 12: Case 1 (8)

33.03 p(A) = 1/4

1) B|A (p = 25/6)
Exercise 12: Case 1 (9)

\[
p(A) = \frac{3}{10} \quad p(B|A) = \frac{1}{4} \\
p(A') = \frac{7}{10} \quad p(B'|A') = \frac{3}{4}
\]

By the total probability theorem,

\[
p(A|B) = \frac{p(A) \cdot p(B|A)}{p(B)} = \frac{\frac{3}{10} \cdot \frac{1}{4}}{\frac{2}{5}} = \frac{3}{16} \\
p(A'|B) = \frac{p(A') \cdot p(B'|A')} {p(B')} = \frac{\frac{7}{10} \cdot \frac{3}{4}}{\frac{1}{4}} = \frac{21}{10} \\
p(B|A) = \frac{p(A) \cdot p(B|A)}{p(A)} = \frac{3}{3} = 1 \\
p(B'|A') = \frac{p(A') \cdot p(B'|A')}{p(A')} = \frac{\frac{7}{10} \cdot \frac{3}{4}}{\frac{7}{10}} = \frac{3}{4}
\]

Exercise 12: Case 1 (10)

\[
p(A) = \frac{2}{3} \quad p(B|A) = \frac{1}{2} \\
p(A') = \frac{1}{3} \quad p(B'|A') = \frac{1}{4}
\]

By the total probability theorem,

\[
p(A|B) = \frac{p(A) \cdot p(B|A)}{p(B)} = \frac{\frac{2}{3} \cdot \frac{1}{2}}{\frac{5}{6}} = \frac{2}{5} \\
p(A'|B) = \frac{p(A') \cdot p(B'|A')}{p(B')} = \frac{\frac{1}{3} \cdot \frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \\
p(B|A) = \frac{p(A) \cdot p(B|A)}{p(A)} = \frac{2}{2} = 1 \\
p(B'|A') = \frac{p(A') \cdot p(B'|A')}{p(A')} = \frac{\frac{1}{3} \cdot \frac{1}{4}}{\frac{1}{3}} = \frac{1}{4}
\]

Exercise 12: Case 2 (11)

\[
p(A) = \frac{6}{10} \quad p(B|A) = \frac{4}{3} \\
p(A') = \frac{4}{10} \quad p(B'|A') = \frac{2}{3}
\]

By the total probability theorem,

\[
p(A|B) = \frac{p(A) \cdot p(B|A)}{p(B)} = \frac{\frac{6}{10} \cdot \frac{4}{3}}{\frac{7}{5}} = \frac{4}{7} \\
p(A'|B) = \frac{p(A') \cdot p(B'|A')}{p(B')} = \frac{\frac{4}{10} \cdot \frac{2}{3}}{\frac{3}{5}} = \frac{4}{9} \\
p(B|A) = \frac{p(A) \cdot p(B|A)}{p(A)} = \frac{6}{6} = 1 \\
p(B'|A') = \frac{p(A') \cdot p(B'|A')}{p(A')} = \frac{\frac{4}{10} \cdot \frac{2}{3}}{\frac{4}{10}} = \frac{2}{3}
\]

Exercise 12: Case 2 (12)

\[
p(A) = \frac{8}{10} \quad p(B|A) = \frac{4}{5} \\
p(A') = \frac{2}{10} \quad p(B'|A') = \frac{2}{5}
\]

By the total probability theorem,

\[
p(A|B) = \frac{p(A) \cdot p(B|A)}{p(B)} = \frac{\frac{8}{10} \cdot \frac{4}{5}}{\frac{5}{4}} = \frac{32}{25} \\
p(A'|B) = \frac{p(A') \cdot p(B'|A')}{p(B')} = \frac{\frac{2}{10} \cdot \frac{2}{5}}{\frac{3}{5}} = \frac{4}{15} \\
p(B|A) = \frac{p(A) \cdot p(B|A)}{p(A)} = \frac{8}{8} = 1 \\
p(B'|A') = \frac{p(A') \cdot p(B'|A')}{p(A')} = \frac{\frac{2}{10} \cdot \frac{2}{5}}{\frac{2}{10}} = \frac{1}{5}
\]
Exercise 12: Case 2 (13)

\[ p(A) = \frac{6}{22}, \quad p(B | A) = \frac{5}{22}, \quad p(B | A') = \frac{14}{19} \]

\[ p(A | B) = \frac{4}{7} = 0.5714 \]

Exercise 12: Case 2 (14)

\[ p(A) = \frac{8}{25}, \quad p(B | A) = \frac{4}{8}, \quad p(B | A') = \frac{2}{17} \]

\[ p(A | B) = \frac{4}{7} = 0.5714 \]

Exercise 12: Case 2 (15)

\[ p(A) = \frac{8}{22}, \quad p(B | A) = \frac{4}{8}, \quad p(B | A') = \frac{2}{17} \]

\[ p(A | B) = \frac{4}{7} = 0.5714 \]

Exercise 12: Cases 1 & 2 (16)

\[ p(A | B) \]
Exercise 12: Analysis (17)

- Better intrusion detection technology does not have to mean more frequent alarms, \( P(B) \) are the same for both cases, rather each alarm becomes more predictive of a true cyber intrusion.

Exercise 12: Analysis (18)

- In general...
  ...when trying to reduce uncertainty, be selective of quality of evidences
  Evidences that may become part of the applications of RM tools and techniques.

Elements of Probability Theory - Definition

- Probability theory is the formalism to study chance.

Expected Value

\[ E[X] = \sum_{i=1}^{\infty} x_i p_i \quad \text{Discrete} \]
\[ E[X] = \int_{-\infty}^{\infty} x f(x) dx \quad \text{Continuous} \]
Exercise 13

A game of rolling a standard six-sided die...
• You pay $3K to roll the die
• You receive in $K the number on the face of the die after the roll

What is the expected value of this game? [1 minute]

Decision Theory

Decision theory is the formalism to study choice
• individuals
• groups (e.g., organizations, societies)

Exercise 14

A game of rolling a standard six-sided die...
• You pay $3K to roll the die
• You receive in $K the number on the face of the die after a roll

Is this game worth playing?

Economics

Economics studies the behavior of individuals, groups, and organizations, when they manage or use scarce resources to achieve goals
Exercise 15

A game of rolling a standard six-sided die...
• you pay $3K to play the game
• you receive in $K the number on the face of the die after a roll

<table>
<thead>
<tr>
<th>you pay</th>
<th>possible outcomes of a roll</th>
<th>you receive</th>
<th>net receipt</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3,000</td>
<td>1</td>
<td>$1,000</td>
<td>$2,000</td>
</tr>
<tr>
<td>$3,000</td>
<td>2</td>
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<tr>
<td>$3,000</td>
<td>6</td>
<td>$6,000</td>
<td>$3,000</td>
</tr>
</tbody>
</table>

Will you play this game?

Exercise 16

A game of rolling a standard six-sided die...
• you pay $3M to play the game
• you receive in $M the number on the face of the die after a roll

Will you still play this game?

Probability, Decision Theory, and Economics

Together, they provide the formalism to study risk

Value Function
Value function - Definition

A real-valued mathematical function defined over an evaluation criterion (or attribute) that represents an option's measure of "goodness" over the levels of the criterion.

Exponential Value Function

\[ V_e(x) = \begin{cases} 
1 - e^{-(x-x_{\text{min}})/\rho} & \text{if } \rho \neq \infty \\
1 - e^{-(x_{\text{max}}-x_{\text{min}})/\rho} & \text{if } \rho = \infty \\
\frac{x-x_{\text{min}}}{x_{\text{max}}-x_{\text{min}}} & \text{if } \rho = \infty 
\end{cases} \]

Risk Based on Value Function – Risk Score

- Suppose the risk (or risk score) of event E can be given by

\[ \text{Risk Score}(E) = RS(E) = u_1 V_{Probability}(E) + u_2 V_{Impact}(E) \]
Risk Based on Value Function

- $V_{\text{probability}}(E)$ is a value function for the risk event's occurrence probability
- $V_{\text{impact}}(E)$ is a value function for the severity of the risk event's impact, if it occurs.
- The coefficients $u_1$ and $u_2$ are nonnegative weights such that $0 \leq u_1 \leq 1$, $0 \leq u_2 \leq 1$, and $u_1 + u_2 = 1$

$$Risk \ Score(E) = RS(E) = u_1 V_{\text{probability}}(E) + u_2 V_{\text{impact}}(E)$$

Certainty Equivalent of a Lottery & Risk Preference

Certainty Equivalent of a Lottery - Definition

- A certainty equivalent of lottery $X$ is an amount $x_{CE}$ such that the decision maker is indifferent between $X$ and the amount $x_{CE}$ for certain.

"Indifferent"

Similar to:
- equally preferred
- would not matter which one
- indistinguishable
- equal merits
Certainty Equivalent of a Lottery – Expected Value

- What is the expected value of this lottery?

\[ L_X \equiv X = \begin{cases} \text{Win$500 with probability 0.60} \\ \text{Lose$150 with probability 0.40} \end{cases} \]

Certainty Equivalent of a Lottery - Example

- For example, what amount of dollars would you be willing to receive with certainty that makes you indifferent between that amount and engaging in lottery X, given as follows:

\[ L_X \equiv X = \begin{cases} \text{Win$500 with probability 0.60} \\ \text{Lose$150 with probability 0.40} \end{cases} \]

Certainty Equivalent of a Lottery

- If you would be indifferent between receiving ___ dollars with certainty and engaging in the lottery, then we say your certainty equivalent for this lottery is ___.

Certainty Equivalent of a Lottery – Risk Averse

- A person is a risk averse if he or she is willing to accept, with certainty, an amount of money less than the expected amount that might be received if the decision were made to participate in the lottery (or gamble).
Certainty Equivalent of a Lottery – Risk Seeking

- People are considered to be risk takers or risk seeking if their certainty equivalent is greater than the expected value of the lottery.
- People are considered risk neutral if their certainty equivalent is equal to the expected value of the lottery.

Utility Function

The St. Petersburg Paradox (1)

Peter proposes a lottery of tossing a coin and continuing to do so until it lands ‘head’

The St. Petersburg Paradox (2)

- If the lottery pays:
  - $1 if 'head' on the 1st throw
  - $2 if 'head' on the 2nd throw
  - $4 if 'head' on the 3rd throw
  - $8 if 'head' on the 4th throw
  ...and so on...
- So that with each additional throw the payoff is doubled
The St. Petersburg Paradox (3)

If Peter proposes a lottery that pays:
$1 if 'head' on the 1st throw
$2 if 'head' on the 2nd throw
$4 if 'head' on the 3rd throw
$8 if 'head' on the 4th throw
...and so on...

How much would Paul be willing to pay to play in this lottery?

The St. Petersburg Paradox (4)

If Peter proposes a lottery that pays:
$1 if 'head' on the 1st throw
$2 if 'head' on the 2nd throw
$4 if 'head' on the 3rd throw
$8 if 'head' on the 4th throw
...and so on...

How much would Paul be willing to pay to play in this lottery?

...looking at the payoff

The St. Petersburg Paradox (5)

If Peter proposes a lottery that pays:
$1 if 'head' on the 1st throw
$2 if 'head' on the 2nd throw
$4 if 'head' on the 3rd throw
$8 if 'head' on the 4th throw
...and so on...

How much would Paul be willing to pay to play in this lottery?

...looking at the payoff and chances

The St. Petersburg Paradox (6)

If Peter proposes a lottery that pays:
$1 if 'head' on the 1st throw
$2 if 'head' on the 2nd throw
$4 if 'head' on the 3rd throw
$8 if 'head' on the 4th throw
...and so on...

How much would Paul be willing to pay to play in this lottery?

Common method to consider payoff and chances together:

**Expected value of the lottery**
The St. Petersburg Paradox (7)

The expected value of this lottery is:

\[ E(L_X) \equiv E(X) = \frac{1}{2} (1) + \frac{1}{4} (2) + \frac{1}{8} (4) + \frac{1}{16} (8) + \cdots = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots = \infty \]

The St. Petersburg Paradox (8)

How much would Paul be willing to pay to play in this lottery?

- A player should expect an infinite gain from this lottery.
- If the expected value is used as the decision rule to enter this game, then a player should be willing to pay any very large (but finite) entry fee to play with the expectation of an infinite monetary gain.

The St. Petersburg Paradox (9)

Paradox: “an argument that apparently derives self-contradictory conclusions by valid deduction from acceptable premises” (Merriam-Webster)

Exercise 17

What are paradoxical about the St. Petersburg Paradox? (5 minutes)
Exercise 17 (Possible Responses)

What are paradoxical about the St. Petersburg Paradox?
- Paul may not have enough resources to pay to play
- Peter may not have enough resources to pay infinite payoff
- the chances of Paul winning large payoff may be very very small
- ...others

Exercise 18

What are possible remedies to this paradox of the expected value of the lottery? [5 minutes]

Exercise 18 (Sample Responses)

What are possible remedies to this paradox of the expected value of the lottery?
- To make the expected value finite
- To zero out very small chances (aka Nikolaus Bernoulli’s remedy)
- To consider additional amounts of money are possibly meaningless after some point (aka Gabriel Cramer’s remedy)
- Distinguish the utility—desirability or satisfaction —of a payoff from its dollar amount (aka Daniel Bernoulli’s remedy)
Daniel Bernoulli (1700-1782)

Formed the idea that valuing monetary loss or gain from a gamble or lottery should be measured in the context of a player's personal circumstance and existing wealth.

• Rather not only the monetary value

Daniel Bernoulli (1700-1782) - Quote

A gain of one thousand ducats is more significant to a pauper than to a rich man though both gain the same amount.

- Bernoulli, 1738

Daniel Bernoulli (1700-1782) Utility

• Measurement of the value of a risk needs consideration of the utility of whatever gain accrues to the individual or, conversely, how much profit is required to yield a given utility.
Utility - Definition

- A utility is a measure of worth, satisfaction, or preference an outcome has for an individual.
- It is a dimensionless number that is sometimes referred to as a “util.”

Bernoulli's Log Utility Function

- $U(W) = \log(W)$
- $U'(W) = 1/W$

Bernoulli's log utility function for wealth $W$ reflects decreasing marginal utility with increasing wealth

Progression of Utility Function

- 1738: Bernoulli's utility function...
- 1944: John von Neumann and Oskar Morgenstern formalized the axioms of expected utility theory
- 1976: R. L. Keeney and H. Raiffa developed preference theory
- 1979: Dyer and Sarin introduced the concept of a measurable value function which enable use of difference between any two levels (or scores) within a criterion (or attribute)

Families of Risk Attitude or Utility Functions

- Risk averse
- Risk neutral
- Risk seeking

Risk aversion as $\alpha$ increased from $\alpha = 0$ to $\alpha = 10$
Utility Function – Risk Averse

A risk-averse person is willing to accept, with certainty, an amount of money less than the expected winnings that might be received from a lottery.

- Exhibits a property known as diminishing marginal utility.

Utility Function – Increasing vs. Decreasing Preference

Concave utility functions are always associated with a risk-averse person.

Increasing Preference

Same x: Risk Neutral
Same x: Risk Neutral vs. Averse

Same x: Risk Neutral vs. Averse vs. Seeking

Same U: Risk Neutral

Same U: Risk Neutral vs. Averse
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Exercise 19

Choice A
• $10,000 with certainty

Choice B
Participate in a lottery with two possible outcomes
• Receive $100,000 with probability $p$
• Receive nothing with probability $(1-p)$

What $p$ would lead you to choose B?

Exercise 19 Answer

\[ \sum Choice B(p) \geq Choice A \]
\[ \sum Choice B(p) \geq 10 K + 1 \]
\[ \sum Choice B(p) \geq 10 K \]
\[ $100 K (p) + $0 (1 - p) \geq 10 K \]
\[ p \geq 0.1 \]

Decreasing Preference

Decreasing Preference - Same x: Risk Neutral
Decreasing Preference - Same x: Risk Neutral vs. Averse

Decreasing Preference - Same x: Risk Neutral vs. Averse vs. Seeking

Decreasing Preference - Same U: Risk Neutral

Decreasing Preference - Same U: Risk Neutral vs. Averse
Exercise 20

- Choice A
  - Pay cyber insurance premium of $10,000 with certainty

- Choice B
  - Take the risk of
  - Incurring $100,000 cost of malware incident with probability $p$
  - Incur nothing with probability $(1-p)$

What $p$ would lead you to choose B?

Exercise 20 Answer

$$
\sum Choice\ B(p) \leq Choice\ A \\
\sum Choice\ B(p) \leq 10K + 1 \\
\sum Choice\ B(p) \leq 10K \\
$100K (p) + 0 (1 - p) \leq 10K \\
p \leq 0.1
$$

Exercise 21

- Choice A
  - Pay cyber insurance premium of $100,000 with certainty

- Choice B
  - Take the risk of
  - Incurring $1,000,000 cost of malware incident with probability $p$
  - Incur nothing with probability $(1-p)$

What $p$ would lead you to choose B?
Utility Theory From a Systems Perspective

“behavior of supposedly rational players to obtain maximal gains and minimal losses by appropriate strategies against other players”

- Bertalanffy, 1968

vNM Expected Utility Theory
von Neumann and Morgenstern

John von Neumann and Oskar Morgenstern (1944) extended Bernoulli’s ideas to a set of axioms known as the axioms of expected utility theory.

vNM Expected Utility Theory – Principal Axioms

Four principal axioms
1. Completeness Axiom
2. Transitivity Axiom
3. Continuity Axiom
4. Independence (Substitution) Axiom

vNM Expected Utility Theory- Completeness

Completeness Axiom
- Given lottery A and lottery B, a person can state that A is strictly preferred to B (i.e., A > B) or B is strictly preferred to A (i.e., B < A), or the person is indifferent to them (i.e., A ∼ B)
vNM Expected Utility Theory - Transitivity

Transitivity Axiom
• If a person prefers lottery A more than lottery B and lottery B more than lottery C, then lottery A is preferred to lottery C.
  
  If A>B and B>C, then A>C

vNM Expected Utility Theory - Continuity

Continuity Axiom
• If a person prefers lottery A more than lottery B and lottery B more than lottery C, then there is a probability p that this person is indifferent between receiving lottery B with certainty and receiving a compound lottery with probability p of receiving lottery A and probability (1− p) of receiving lottery C.

vNM Expected Utility Theory - Independence

Independence (Substitution) Axiom
• If a person prefers lottery A more than lottery B, then a compound lottery that produces lottery A with probability p and lottery C with probability (1− p) is preferred to a compound lottery that produces lottery B with probability p and lottery C with probability (1− p).

Utility Function

• A utility function is a real-valued mathematical function that relates outcomes along the horizontal axis to measures of worth or utility along the vertical axis.
Expected Utility of a Lottery

The expected utility of lottery $L_x$ with possible outcomes $\{x_1, x_2, x_3, \ldots, x_n\}$ is

$$E(U(x)) = E(U(X)) = p_1U(x_1) + p_2U(x_2) + p_3U(x_3) + \cdots + p_nU(x_n)$$

where $p_i$ is the probability $L_x$ produces $x_i$ (for $i = 1, \ldots, n$) and $U(x_i)$ is the cardinal utility associated to each outcome $x_i$.

Exercise 22 A1

We need to create John’s “preference function” to answer Q1.

A1: $U(0) = 0$

$$U(x) = 10\sqrt{x}, \text{where } x \text{ is in dollars thousand (K)}.$$

Exercise 22 Q1

- John Doe is a rational person whose satisfaction or preference for various amounts of money that can be expressed:

Q1. How much satisfaction does $0 bring to John?

$$U(x) = 10\sqrt{x}, \text{where } x \text{ is in dollars thousand (K)}.$$

Exercise 22 Q2

Q2. How much satisfaction does $100K bring to John?

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Exercise 22 A2

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</table>

Q2. How much satisfaction does $100K bring to John?

A2: \( U(100) = 1 \)

Exercise 22 A2 (Cont.)

- If we limit the range of \( U(x) \) between 0 and 100, then we can use this function to represent John’s utility, i.e. \( U(x) \) becomes his utility function.
- Note that a preference function can be construed as a utility function within specific constraints, e.g. between 0 and 100 (or 1)

Exercise 22 Q3-4

Q3. How do we define utility?
Q4. How do define utility function?

Exercise 22 A3

A3: Utility is a measure of worth, satisfaction, or preference an outcome has for an individual. It is a dimensionless number that is sometimes referred to as a “util.”
Exercise 22 A4

A4: A utility function is a real-valued mathematical function that relates outcomes along the horizontal axis to measures of worth or utils along the vertical axis.

Exercise 22 A4 (Cont.)

A4: the vertical axis of a cardinal utility function is an interval scale and usually runs between 0 and 1 or between 0 to 100 utils (as shown in Figure 3.23). With this convention, the utility of the least preferred outcome is assigned 0 utils and the utility of the most preferred outcome is assigned 100 utils. Higher preferred outcomes have higher "utils" than lower preferred outcomes.

Exercise 22 Q5

Consider John's utility function:

Q5. How does his utility function look like?

\[ U(x) = 10\sqrt{x}, \text{ where } x \text{ is in dollars thousand (K)}. \]

Exercise 22 A5

Utility for John Doe

<table>
<thead>
<tr>
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<tr>
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</tbody>
</table>
Exercise 22 Q6

Q6. What does this graph show about John's satisfaction with money?

Exercise 22 A6

Q6. What does this graph show about John's satisfaction with money?

A6: increasing and concave

Exercise 22 Q7

Q7. What does this graph show about John's incremental satisfaction?

Exercise 22 A7

• A7: incremental satisfaction decreases with increasing x

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<th>ΔU(x)</th>
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<tr>
<td>100</td>
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</tbody>
</table>
Exercise 22 Q8-9

Q8. What is John's satisfaction in winning $80K?
Q9. ...in winning $10K?

Exercise 22 A8-9

A8: U(80) = 89.4
A9: U(10) = 31.6

Exercise 22 Q10

Q10. If John is thinking of buying a new firewall (e.g. new firmware and software) with uncertain net economic benefits as shown below. How much will be his long-term average satisfaction, i.e. expected utility of this lottery?

\[
\text{Lottery } X = \begin{cases} 
\text{Win $80K with probability 0.60} \\
\text{Win $10K with probability 0.40} 
\end{cases}
\]

Exercise 22 A10

A10: Table top simulation

\[
\text{Lottery } X = \begin{cases} 
\text{Win $80K with probability 0.60} \\
\text{Win $10K with probability 0.40} 
\end{cases}
\]

<table>
<thead>
<tr>
<th>Game</th>
<th>x</th>
<th>U(x)</th>
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<tbody>
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<td>$80K</td>
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<td>...</td>
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</table>
Exercise 22 A10 (Cont.)

A10: Continued...

\[
\text{Lottery } X = \begin{cases} 
\text{Win }$80K \text{ with probability 0.60} \\
\text{Win }$10K \text{ with probability 0.40} 
\end{cases}
\]

\[
U(80) = 89.4 \\
U(10) = 31.6 \\
E[U(x)] = 0.4 \times U(10) + 0.6 \times U(80) \\
E[U(x)] = 0.4 \times 31.6 + 0.6 \times 89.4 \\
E[U(x)] = 66.3
\]

Exercise 22 Q11

Q11. For John, what certain amount would give him satisfaction equal to this lottery, i.e. what is the certainty equivalent of this lottery?

\[
\text{Lottery } X = \begin{cases} 
\text{Win }$80K \text{ with probability 0.60} \\
\text{Win }$10K \text{ with probability 0.40} 
\end{cases}
\]

Exercise 22 A11

A11:

\[
E[U(x)] = 66.3 \\
U(x_{eq})=E[U(x)] \\
x_{eq} = $43.93K
\]
Exercise 22 A11 (Cont.)

\[ E[U(x)] = 66.3 \]
\[ U(x_c) = E[U(x)] \]
\[ x_c = 43.93K \]

That is, based on John's utility function, his long-term average satisfaction with this lottery - of buying a new firewall - is equal to receiving $43.93K, with certainty.

Exercise 22 A11 Discussion

As such, he would be indifferent between playing the lottery of buying a new firewall and receiving $43.93K, with certainty.

Exercise 22 Q12

• Paul, (John's brother) is also considering the same firewall technology
• Paul figured out that he has a utility function given by
  \[ U(x) = 0.8257(x - 80) + 89.4, \text{ if } 0 < x < 100 \]
  \[ U(x) = 0, \text{ if } x = 0 \]
  \[ U(x) = 100, \text{ if } x = 100 \]

Q12. How does Paul's utility function compare with John's?
Exercise 22 Q13-14

Q13. What is Paul's satisfaction in getting $80K of benefits with this new firewall?
Q14. ...in $10K of benefits?

\[ \text{Lottery } X = \begin{cases} \text{Win }$80K \text{ with probability 0.60} \\ \text{Win }$10K \text{ with probability 0.40} \end{cases} \]

Exercise 22 A13-14

A13: \( U(80) = 89.4 \)
A14: \( U(10) = 31.6 \)

Exercise 22 Q15

* Q15. If Paul played this lottery repeatedly, how much will be his long-term average satisfaction, i.e. expected utility of the lottery?

\[ \text{Lottery } X = \begin{cases} \text{Win }$80K \text{ with probability 0.60} \\ \text{Win }$10K \text{ with probability 0.40} \end{cases} \]

Exercise 22 A15

A15: Table top simulation

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<thead>
<tr>
<th>Game</th>
<th>( x )</th>
<th>( U(x) )</th>
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<td>...</td>
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</table>
Exercise 22 A15 (Cont.)

A15: Continuation

\[ U(80) = 89.4 \]
\[ U(10) = 31.6 \]

\[ E[U(x)] = 0.4 \times U(10) + 0.6 \times U(80) \]
\[ E[U(x)] = 0.4 \times 31.6 + 0.6 \times 89.4 \]
\[ E[U(x)] = 66.3 \, \text{the same with John!} \]

Exercise 22 Q16

Q16. For Paul, what certain amount would give him satisfaction equal to this lottery, i.e. what is the certainty equivalent of this lottery?

Exercise 22 A16

A16: \( E[U(x)] = 66.3, x_{ce} = \$52K \)

Exercise 22 A16 (Cont.)

\[ E[U(x)] = 66.3, x_{ce} = \$52K \]

That is, based on Paul's utility function, his long-term average satisfaction with this lottery is equal to receiving $52K, with certainty.

As such, he would be indifferent between these two.
Exercise 22 Q17-18

Q17. How does Paul’s certainty equivalent compare to John’s?
Q18. Who is more risk averse?

Exercise 22 A17

A17:
For John, $x_{se} = $43.93K
For Paul, $x_{se} = $52K

Based on John’s and Paul’s utility functions, John will agree to a lower certain payoff than will Paul.

Exercise 22 A18

• A18: John is more risk averse

Exercise 22 Q19-21

Q19. How do we describe someone who is risk neutral?
Q20. Who is risk neutral, John or Paul?
Q21. How does risk neutrality relate to $E[x]$ and $U(x)$?
Exercise 22 A19

A19: People are considered risk neutral if their certainty equivalent is equal to the expected value of the lottery.

Exercise 22 A20-21

A20: \[ E[\text{lottery}] = \$80K \times 0.6 + \$10K \times 0.4 = \$52K. \] Recall that Paul's certainty equivalent: \[ x_c = \$52K. \] Thus, Paul is risk neutral because his certainty equivalent \( x_c \) is equal to the expected value of the lottery \( E[\text{lottery}] \).

A21: For risk neutral utility function \( U(x) \), \[ U(E[\text{lottery}]) = E[U(x)]. \]

Lotteries and Risk Preference

- Lottery = any engineering activity, including those related to risks
  - have cost and benefits
  - uncertain outcome
- EMSE may not be the decision maker, only the facilitator
  - providing information (probabilistic or deterministic)
  - providing estimate measures of value or utility of options

Extreme Event Analysis
Extreme Events and Rare Events

- **Extreme** events refer to phenomena that have relatively extreme high or low degree of magnitude
- **Rare** events refer to phenomena that have relatively very low frequency of occurrence

Extreme But Not Rare

![Diagram](image)

Rare But Not Extreme

![Image](image)

Extreme and Rare Events

- Prolonged drought
- Thousand-year flood (Ohio River flood of 1937)
- Catastrophic earthquake (2011 Tohoku, 9.03 Mw)
Extreme and Rare

- In most well-engineered systems, undesirable events of great magnitude (i.e. extremes) are rare
- Examples: bridge failure, space shuttle disaster

Mental Exercise 23: Extreme and Rare

More extreme events $\Rightarrow$ More damage, why?

Mental Exercise 23: Extreme and Rare (Answer)

The way we design and build systems
- Very dependable systems
- High degree of dependence on a particular system
- The way we make decisions
- Resource constraints

More extreme events $\Rightarrow$ More damage, why?

Mental Exercise 24: Extreme and Rare

More extreme events $\Rightarrow$ Higher Risk?
Mental Exercise 24: Extreme and Rare (Answer)

How do I measure risk?

Exercise 25

- Provide two real examples of an extreme cyber risk event and three examples of a rare cyber risk event.

Importance to Risk Management

- These are the situations where you have the most to lose or to win, e.g. stock market crash, war, natural disasters

Risk Management Process
Common Risk Management Steps

Step 1: Risk Identification

- Traditional Engineering Management and Systems Engineering (EMSE): What can go wrong?
- Nontraditional EMSE: What can go wrong
  - within my system?
  - outside my system?
  - due to dependencies (i.e. dependency-induced risks)?

Step 2: Risk Impact (Consequence) Assessment

Traditional EMSE: What are the costs (including monetary cost, schedule, and technical performance objectives)?

Nontraditional EMSE: Direct costs, plus its impact on my portfolio and ultimately for the enterprise through dependencies

Step 3: Risk Prioritization Analysis

- Traditional EMSE: Which risk events are most important?
- Nontraditional EMSE:
  - What are the relative importance of risk events based on their effects on my capabilities?
  - Which capabilities are most important?
    - Including the roll-up from previous nontraditional steps
Step 4: Risk Mitigation Planning and Progress Monitoring

Traditional EMSE: How effective are my RM decisions (i.e. monetary cost, schedule, or technical performance objectives)?

Nontraditional EMSE: ...including a capability's ability to achieve its outcome objectives for the portfolio and ultimately for the enterprise through dependencies

Comparison

<table>
<thead>
<tr>
<th>Traditional EMSE</th>
<th>Nontraditional EMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>What will be the overall impact to the projects and the larger systems (e.g. program, mission, etc.)?</td>
<td></td>
</tr>
</tbody>
</table>

Exercise 26

Systems analysis
1. For the purpose of RM, how similar and different is a cyber risk event from a transportation risk event (e.g. Sony hack vs Lac Megantic derailment)?
2. For the purpose of RM, how similar and different is a cyber risk event from a wide-area blackout (e.g. Sony hack vs 2003 blackout)?

Exercise 27

Critique of your current RM framework
1. Identify at least 5 reasons why your current RM framework will not work for cyber risk events?
2. Re-build your RM framework so it can be applicable to cyber-intensive systems
Summary

References