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Data and Variables:

Given in Problem:

$$T = 77^\circ F = 536.67^\circ R$$
$$Q = 250 gal/min = 0.557 ft^3/s$$

From Tables:

$$\gamma_m = 844.9 lb/ft^3$$
$$\gamma_e = 49.01 lb/ft^3$$
$$Diameter_p = 0.17225 ft$$
$$flowArea = 0.02333 ft^2$$
$$\epsilon = 1.5 * 10^{-4} (ft)$$
$$K_v = 1.37 * 10^{-5} (ft^2/s)$$

Materials:

* Ethyl Alcohol
* Mercury
* Air

Sources:

Mott, R., and Untener, J. Applied Fluid Mechanics. 7th Ed. 2015

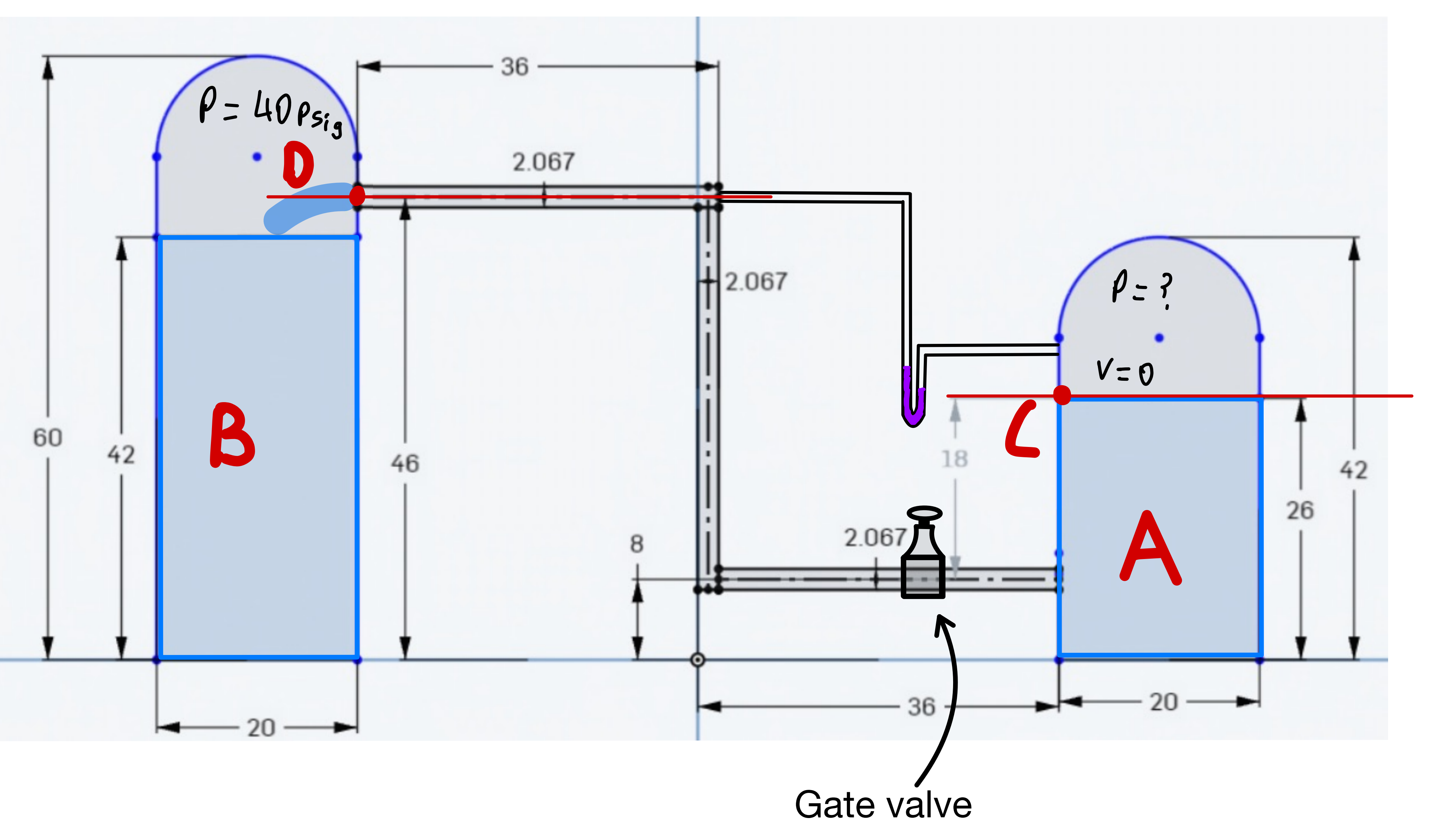
Purpose:
Find the required air pressure in tank A that is needed to deliver 0. 557 ft^3/s of ethyl alcohol to tank B in ***Figure 1***. Then find the height h of the mercury in the manometer. Do the calculations by hand and in an Excel spreadsheet. In part 2 find the pressure of tank A, such that the flow rate is zero. Then find the height h of the mercury in the manometer. In part 3 create a graph of Q vs P using the calculations from part 1. Then find an approximation of the flow rate when the air pressure is 75 psi

Design Considerations:

* Incompressible fluid
* Isothermal process
* Newtonian Fluid
* T = 77 °F
* Ethyl alcohol and Mercury do not mix
* Standard elbows

Drawings:

Figure 1:



Formulas:

$$Q = V \cdot A \qquad V_1 A_1 = V_2 A_2 \qquad \Delta P = \gamma \cdot h$$

$$h_A + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_R + h_L$$

Procedure and Calculations:

Part 1:

Let us find the velocity of the fluid as it exits the pipe into tank B

We have $Q = 0.557\ ft^3/s$ and $A = 0.02333\ ft^2$ using the formula

$Q = V \cdot A$

$$V_B = \frac{0.557\ ft^3/s}{0.02333\ ft^2} = 23.87\ ft/s$$

Next let us find the net height diffrence between the points C and D

$\Delta h = h_D - h_C \qquad \Delta h = 46ft - 26ft = 20\ ft$

Pipe Length: $\quad L = 36 + 38 + 36 = 110\ ft$

Find Relative roughness, Reynolds number, Relative friction, and friction.

$$Re = \frac{VD}{\nu} \qquad Re = \frac{23.87\ ft/s \cdot 0.17225\ ft}{1.37 \cdot 10^{-5}\ ft^2/s} = 3 \times 10^5$$

$$R_r = D/\varepsilon \qquad R_r = \frac{0.17225\ ft}{1.5 \times 10^{-4}\ ft} = 1148$$

$$f_T = \frac{0.25}{\left[\log\left(\frac{1}{3.7 \cdot R_r}\right)\right]^2} \qquad f_T = \frac{0.25}{\left[\log\left(\frac{1}{3.7 \cdot 1148}\right)\right]^2} \qquad f_T = 0.019$$

$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7 \cdot R_r} + \frac{5.74}{Re^{0.9}}\right)\right]^2} \qquad f = \frac{0.25}{\left[\log\left(\frac{1}{3.7 \cdot 1148} + \frac{5.74}{8.5}\right)\right]^2} \qquad f = 0.0202$$

Now find all the losses in the system Between C and D

$$h_L = \left(K_{ent}\frac{V^2}{2g}\right) + \left(8 f_T \frac{V^2}{2g}\right) + \left(f\frac{L}{D}\frac{V^2}{2g}\right) + 2\left(f_T \cdot 30 \frac{V^2}{2g}\right)$$

$$h_L = \left(0.5\cdot\frac{(23.87\ ft/s)^2}{2\cdot 32.2\ ft/s^2}\right) + \left(8\cdot 0.019\cdot\frac{(23.87\ ft/s)^2}{2\cdot 32.2\ ft/s^2}\right) + \left(0.0202\cdot\frac{110\ ft}{0.17225\ ft}\cdot\frac{(23.87\ ft/s)^2}{2\cdot 32.2\ ft/s^2}\right) + \left(30\cdot 0.019\cdot\frac{(23.87\ ft/s)^2}{2\cdot 32.2\ ft/s^2}\right)$$

$$h_L = 125\ ft.$$

Now Lets find the Pressure of Tank A using the formula:

$$\frac{P_A}{\gamma_e} + \cancel{\frac{V_A^2}{2g}} + Z_A = \frac{P_B}{\gamma_e} + \frac{V_B^2}{2g} + Z_B + h_L \qquad P_A = \gamma_e\left(\frac{P_B}{\gamma_e} + \frac{V_B^2}{2g} + (Z_B - Z_A) + h_L\right)$$

$$P_A = 49.01\ lb/ft^3\left(\frac{5760\ lb/ft^2}{49.01\ lb/ft^3} + \frac{(23.87\ ft/s)^2}{2(32.2\ ft/s^2)} + 20ft + 125ft\right) = 13296\ lb/ft^2$$

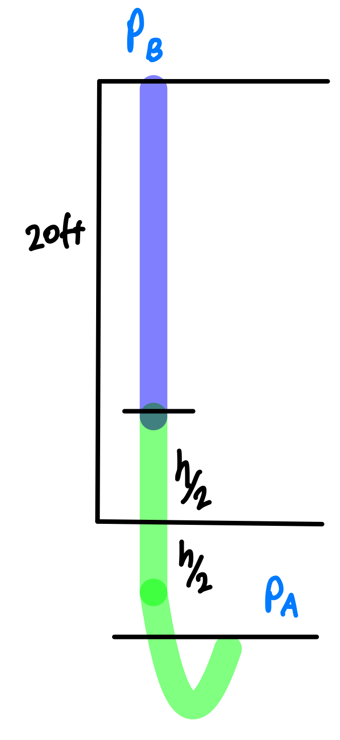
Convert to psi:

$$1\ lb/ft^2 \cdot \frac{1\ ft^2}{(12\ in)^2} = 0.00694\ psi \qquad 13296\ lb/ft^2 \cdot \frac{1\ ft^2}{(12\ in)^2} = \boxed{92.33\ psi}$$

Part 1A:

Find the height of the mercury:

Figure 2:



$Sg_m = 13.54$

$Sg_e = 0.787$

$$P_A - h\cdot(Sg_m) - (20 - h/2)\cdot(Sg_E) = P_B$$

$$92.33psi - (h\cdot 13.54) - ((20 - h/2)\cdot 0.787) = 40psi$$

$$\boxed{h = 2.78\ ft\ Hg \qquad h = 33.36\ in\ Hg}$$

Part 2:

There are two ways to find the pressure of tank A when $Q = 0$

Way 1: use the equation: $\Delta P = \gamma \cdot h$

$\Delta P = 49.01\ lb/ft^3 \cdot 20\ ft$ $\Delta P = 980.2\ lb/ft^2$

Convert to psi:

$1\ lb/ft^2 \cdot \frac{1\ ft^2}{(12\ in)^2} = 0.00694\ psi$ $980.2\ lb/ft^2 \cdot \frac{1\ ft^2}{(12\ in)^2} = \boxed{6.8\ psi}$

Then add 40 psi to get the pressure in tank A

$\boxed{46.8\ psi}$

Way 2: Per the formula: $Q = V \cdot A$ A cannot change

Therefore, for Q to equal zero, V must also equal zero.

Now lets find the pressure of Tank A using the formula:

$$\frac{P_A}{\gamma_e} + \cancel{\frac{V_A^2}{2g}} + Z_A = \frac{P_B}{\gamma_e} + \cancel{\frac{V_B^2}{2g}} + Z_B \qquad P_A = \gamma_e \left(\frac{P_B}{\gamma_e} + Z_B - Z_A \right)$$

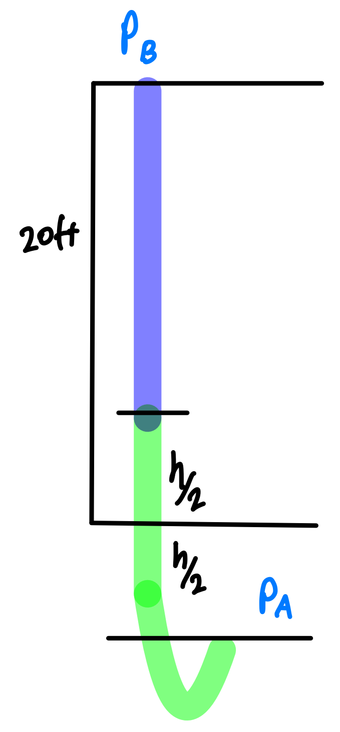
$$P_A = 49.01\ lb/ft^3 \left(\frac{5760\ lb/ft^2}{49.01\ lb/ft^3} + 20\ ft \right) = 6740\ lb/ft^2$$

Convert to psi:

$1\ lb/ft^2 \cdot \frac{1\ ft^2}{(12\ in)^2} = 0.00694\ psi$ $6740\ lb/ft^2 \cdot \frac{1\ ft^2}{(12\ in)^2} = \boxed{46.8\ psi}$

Part 2A:

Find the height of the Mercury in the manometer.



$$\Delta P = -\gamma_e \cdot h_1 - \gamma_m \cdot h_2$$

$$h_1 = 20\ ft - h_2/2\ ft$$

$$980.2\ lb/ft^2 = -(49.01\ lb/ft^3 \cdot (20 - h/2)_{ft}) - (844.9\ lb/ft^3 \cdot h\ ft)$$

$$\boxed{\begin{array}{l} h_2 = -2.39\ ft\ Hg \\ h_2 = -28.66\ in\ Hg \end{array}}$$

Summary:

The air pressure in tank A that was required to produce a flow rate of 250 gal/min. was P= 92.33 psig. with a Manometer reading of 33.36 in Hg towards tank B. For a flow rate of zero the pressure was P= 46.8 psig. with a Manometer reading of. 28.66 in Hg towards tank A

Analysis:

The first pressure was not that far from the second, which shows that the original flow rate was close to the static pressure required to keep the system from reversing flow. To make the system more efficient the height differential between the tanks should be reduced. Additionally, if tank B was open to the atmosphere you could reduce the required pressure in tank A by about 25 psi.

Part 3:

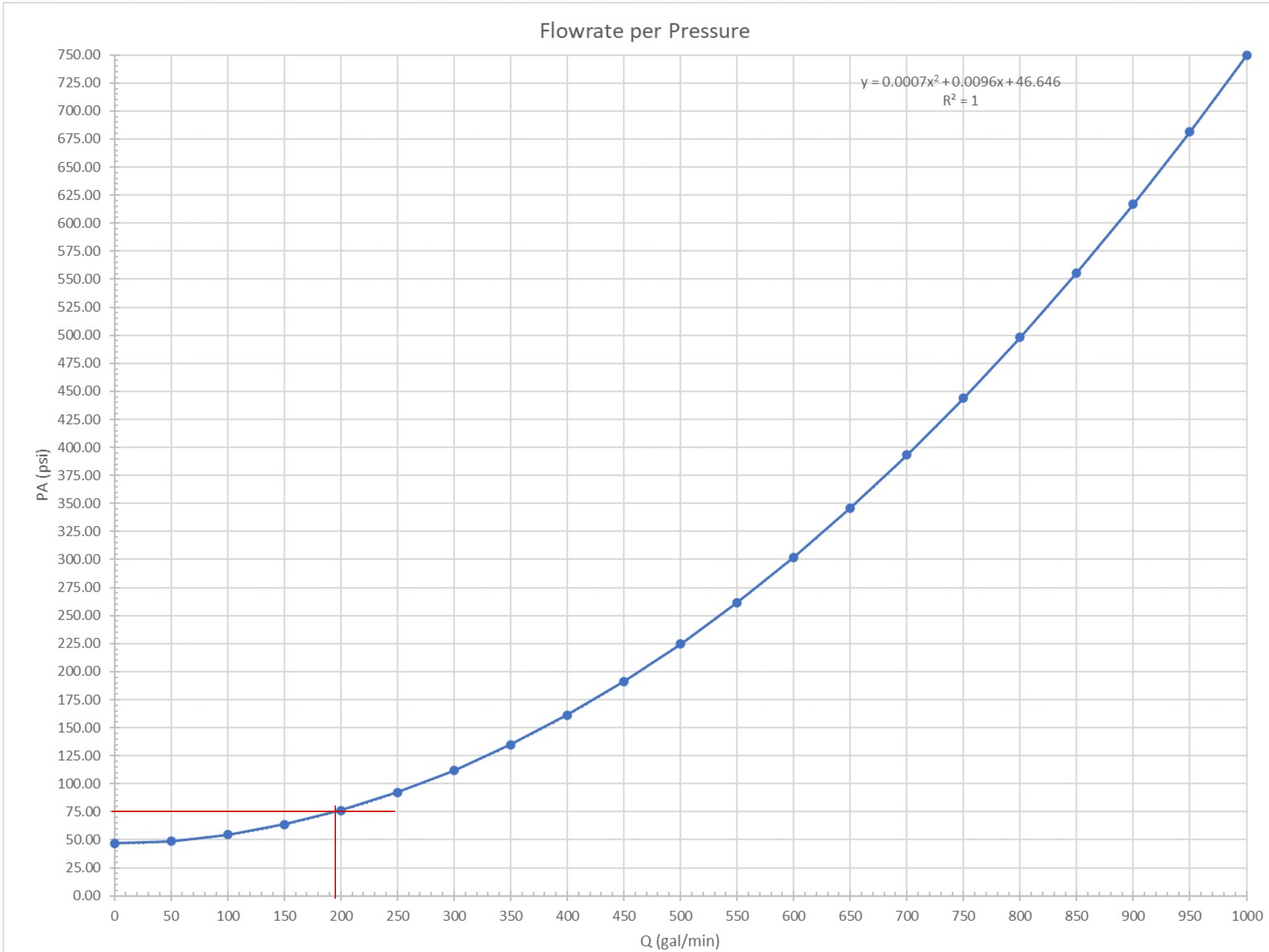
When making the excel spreadsheet I essentially just copied the work I did by hand, but for convenience I also calculated the velocity head as it was in so many of the formulas, and I wanted to reduce the chance of making an error. When graphing the flowrate to pressure curve I just copied and pasted the work from the first part and then filled in the series from 0 to 1000 with 50 gal/min increments. For the zero value entry I used 0.00001 to prevent divide by zero errors. I checked to make sure that the value was the same as the one found by hand. Then to further analyze the data I changed the h to 0 to see how much that would change the required pressure. I also checked the effects of having tank B keep a pressure of 14.7 psi.

Procedure and Calculations:

Q (gal/min)	250	Given Flowrate
Q (ft^3/s)	=B4/448.83	Convert the flowrate to (ft^3 / s)
D (ft)	0.17225	Given pipe Diameter
A (ft^2)	0.0233333	Given pipe Area
h (ft)	20	Given Height difference
γe (lb/ft^3)	49.01	Given specific weight of Ethyl Alcohol
γm (lb/ft^3)	844.9	Given specific weight of Mercury
ε (ft)	=1.5*10^-4	Given Roughness
Re	=((B14*B6)/(1.37*10^-5))	Reynolds Number
Rr	=B6/B11	Relative Roughness
VB (ft/s)	=B5/B7	Velocity B
Velocity head (ft)	=(B14^2)/(2*32.2)	Velocity Head
f	=(0.25/(LOG10((1/(3.7*B13))+(5.74/(B12^0.9)))^2))	Friction in the pipe
fT	=(0.25/(LOG10((1/(3.7*B13)))^2))	Relative Friction
hL (ft)	=(0.5*B15) + (8*B17*B15) + (B16*(110/B6)*B15) + (30*B17*B15)	Head Loss
PA (lb/ft^2)	=B9*((B3/B9)+(B15)+B8+B18)	Pressure at A
PA (psi)	=B19/144	Convert the Pressure to (psi)
ΔP (in Hg)	=ABS((2*B20-2*B9*B8)/(B9+2*B10))*12	Convert the Pressure to (in Hg)

Figure 3:

Using my Excel spreadsheet from Part 1 I created the following graph:



Summary:

The graph was created using data from 0 to 1000 in 50 gal/min. intervals ensuring a professional looking graph. When the pressure is 75 psi the flow rate is approximately equal to 196 gal/min.

Analysis:

For the standard graph series the curve is a second order polynomial with an initial y value of 46.8 psi. The other two curves follow the same equation with just their initial y values reduced. While the height difference between the two tanks makes a difference to the required pressure provided by the pump, changing the pressure in tank B makes the most significant reduction in the required pressure.