Test 3

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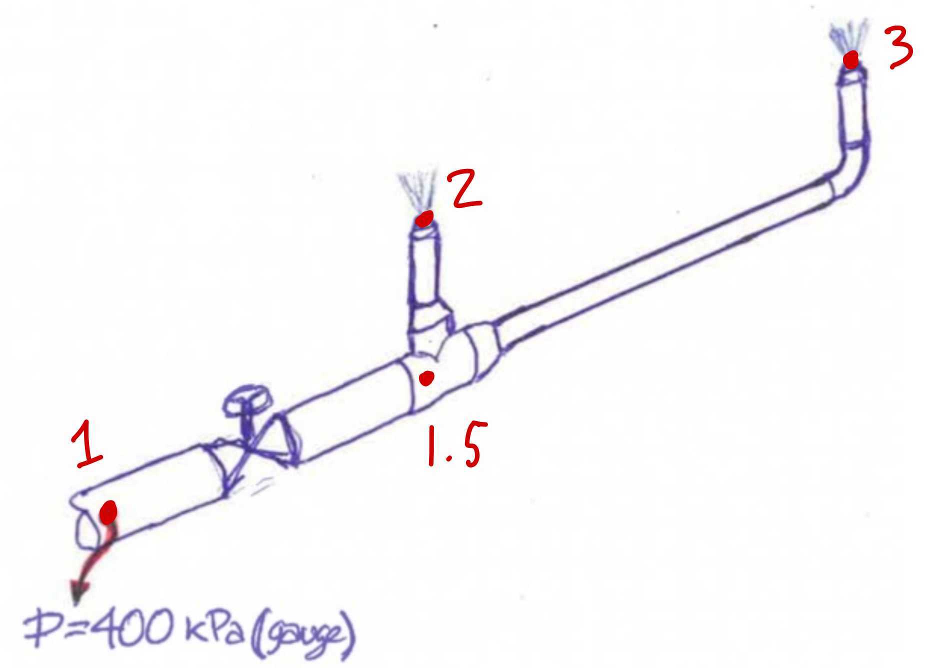
1) The system sketched in the figure is an automatic sprinfler system for a narrow plot of lawn. Water is supply by a main that guarantees a constant pressure of 400 kPa (gauge). The sprinkler pipeline is made of schedule-40 steel pipe. For a wide-open ball valve, determine the flow rate delivered to each sprinkler head. Do not neglect minor losses. The characteristic of the system is as follows:

   a. From point where pressure is 400 kPa to the T-joint: 1 ½ inches nominal pipe of 6.5 m.
   b. From T_joint to 1st sprinkler head: 1 inch nominal pipe of 0.3 m.
   c. From T_joint to 2nd sprinkler head: 1 inch nominal pipe of 8.3 m.
   d. K of the sprinkler head is 50.



Are the flows through each sprinkler the same? If not, what would you do to make them the same? How does the fluid velocity compare to the critical velocity (3 m/s)? If it is too far off, what would you do?

Sources of Loss to T-Junction: (1-1.5)

Pipe: $D = 40.9\,mm$, $A = 1.314 \times 10^{-3}\,m^2$ $L = 6.5\,m$

ball valve: wide open $K = 150 f_T$ [Table 10.4: check valve-Ball Type]

Sources of Loss to sprinkler head 1: (1.5-2)

T-Junction: $L_e/D = 60 f_T$

Reducer: $D_1/D_2 = 1.5$, $\emptyset = 70°$, $K = 0.1$ [Figure 10.11]

Pipe: $D = 26.6\,mm$, $A = 5.574 \times 10^{-4}\,m^2$, $L = 0.3\,m$

Sprinkler nozzle: $K = 50$

Sources of Loss to sprinkler head 2: (1.5-3)

T-Junction: $L_e/D = 20 f_T$

Reducer: $D_1/D_2 = 1.5$, $\emptyset = 70°$, $K = 0.1$

Pipe: $D = 26.6\,mm$, $A = 5.574 \times 10^{-4}\,m^2$, $L = 8.3\,m$

standard Elbow : $L_e/D = 30\, f_T$

Sprinkler nozzle : $K = 60$

Data / variables

$\varepsilon = 4.6\times10^{-5}$

$V_c = 3\ m/s$

$P_1 = 400\ kPa$

$P_{3/4} = 101\ kPa$

$z_2 = 0.3\ m$

$f_{T_1} = 0.020$

$f_{T_2} = 0.022$

Table 10.5

$$\cancel{h_A} + \cancel{\frac{V_1^2}{2g}} + \frac{P_1}{\gamma} + z_1 = \cancel{\frac{V_2^2}{2g}} + \frac{P_2}{\gamma} + z_2 + h_L$$

$$\frac{P_1 - P_2}{\gamma} + z_2 = h_L \qquad h_L = f \cdot \frac{L}{D} \cdot \frac{V^2}{2g}$$

$$h_{L(1-2)} = \left(f \cdot \frac{L_1}{D_1} + K_V \cdot f_T\right) \frac{8Q_1^2}{\pi^2 g D_1^4}$$

$$h_{L(2-3)} = \left(f \cdot \frac{L_2}{D_2} + K_T \cdot f_T + K_N\right) \frac{8Q_2^2}{\pi^2 g D_2^4}$$

$$h_{L(2-4)} = \left(f \cdot \frac{L_3}{D_2} + K_T \cdot f_T + K_E \cdot f_T + K_N\right) \frac{8Q_4^2}{\pi^2 g D_3^4}$$

$$h_{L(1-3)} = h_{L(1-2)} + h_{L(2-3)}$$

$$h_{L(1-4)} = h_{L(1-2)} + h_{L(2-4)}$$

$$Q_2 = \sqrt{\frac{\frac{\Delta P}{\gamma} - \left(f_1 \cdot \frac{L_1}{D_1} + K_V \cdot fT_1\right) \frac{8 Q_1^2}{g \pi^2 D_1^4}}{\left(f \cdot \frac{L_2}{D_2} + K_T \cdot fT + K_N + K_R\right) \frac{8}{\pi^2 g D_2^4}}}$$

$$Q_3 = \sqrt{\frac{\frac{\Delta P}{\gamma} - \left(f_1 \cdot \frac{L_1}{D_1} + K_V \cdot fT_1\right) \frac{8 Q_1^2}{g \pi^2 D_1^4}}{\left(f \cdot \frac{L_3}{D_2} + K_T \cdot fT + K_E \cdot fT + K_N + K_R\right) \frac{8}{\pi^2 g D_3^4}}}$$

$$Q_1 = Q_2 + Q_3$$

$$\boxed{\begin{aligned} Q_1 &= 4.03 \times 10^{-3} \ m^3/s \\ Q_2 &= 2.09 \times 10^{-3} \ m^3/s \\ Q_3 &= 1.94 \times 10^{-3} \ m^3/s \\ V_1 &= 3.07 \ m/s \\ V_2 &= 3.75 \ m/s \\ V_3 &= 3.48 \ m/s \end{aligned}}$$

Data / Variables		
$\mathcal{E}$	4.60E-05	
P_1	400	kPa
P_3&4	101	kPa
fT_1	0.02	
fT_2	0.022	
z	0.3	m
g	9.81	m/s^2
v	4.80E-06	m^2/s
γ	9.79	kN/m^3
T	20	°C

Fitting Losses		
K	value	K * fT
ball_valve	150	3
T-Junction ^	60	1.32
T-Junction >	20	0.44
Elbow	30	0.66
Nozzle	50	
Reduction	0.1	

Pipe Data		
D_1	0.0409	m
A_1	1.31E-03	m^2
L_1	6.5	m
D_2&3	0.0266	m
A_2&3	5.57E-04	m^2
L_2	0.3	m
L_3	8.3	m
D_1/$\mathcal{E}$	889.1304	
D_2/$\mathcal{E}$	578.2609	

Iteration	Q_1 Guess	Re #_1	Re #_2	Re #_3	f_1	f_2	f_3	Numer.	Denom. 2	Denom. 3	Q_2	Q_3	% Diff Q_1	% Diff f_2	% Diff f_3
1	1.00E-02	6.49E+04	3.24E+04	3.24E+04	2.38E-02	1.00E-02	1.00E-02	10.526	8.55E+06	8.97E+06	1.11E-03	1.08E-03	-78.07%	-	-
2	2.19E-03	1.42E+04	7.19E+03	7.03E+03	3.04E-02	3.65E-02	3.67E-02	39.746	8.60E+06	1.03E+07	2.15E-03	1.96E-03	87.48%	15.4%	14.0%
3	4.11E-03	2.67E+04	1.39E+04	1.27E+04	2.70E-02	3.16E-02	3.22E-02	37.223	8.59E+06	1.01E+07	2.08E-03	1.92E-03	-2.63%	-0.6%	-0.4%
4	4.00E-03	2.60E+04	1.35E+04	1.25E+04	2.71E-02	3.18E-02	3.23E-02	37.402	8.60E+06	1.01E+07	2.09E-03	1.93E-03	0.21%	0.0%	0.0%
5	4.01E-03	2.60E+04	1.35E+04	1.25E+04	2.71E-02	3.18E-02	3.23E-02	37.388	8.60E+06	1.01E+07	2.09E-03	1.93E-03	-0.02%	0.0%	0.0%
6	4.01E-03	2.60E+04	1.35E+04	1.25E+04	2.71E-02	3.18E-02	3.23E-02	37.389	8.60E+06	1.01E+07	2.09E-03	1.93E-03	0.00%	0.0%	0.0%
7	4.01E-03	2.60E+04	1.35E+04	1.25E+04	2.71E-02	3.18E-02	3.23E-02	37.389	8.60E+06	1.01E+07	2.09E-03	1.93E-03	0.00%	-	-

Q_1	4.01E-03	m^3/s	V_1	3.0525	m/s
Q_2	2.09E-03	m^3/s	V_2	3.7418	m/s
Q_3	1.93E-03	m^3/s	V_3	3.4542	m/s

For my Excel Spreadsheet I used a novel iteration method, where I fed the previous answers to the next line essentially sub-iterating within the main iteration. [For example, the Reynolds numbers were calculated from the previous Q values.] After the initial guess it runs by itself until the percent difference is zero.

The Flowrate through each of the nozzles are not the same, with a difference of 7.7% This could be fixed by either reducing the losses from point 1 to point 3 or increasing the losses between 1 and 2. Increasing the pipe size between the T-Junction and point 3 would allow for less energy losses while also reducing the velocity so it doesn't exceed the critical velocity. However, the easiest way would be to add another valve that could be adjusted to provide the optimal energy loss. Which would be roughly K_valve = 8