Name: ________________________________________.

MET 330 Fluid Mechanics
Dr. Orlando Ayala
Spring 2024
Test 3

Take home – Due Friday April 12th, 2024, before midnight.

# READ FIRST

1. RELAX!!!! DO NOT OVERTHINK THE PROBLEMS!!!! There is nothing hidden. The test was designed for you to pass and get the maximum number of points, while learning at the same time. HINT: THINK BEFORE TRYING TO USE/FIND EQUATIONS (OR EVEN FIND SIMILAR PROBLEMS)

2. The total points on this test are one hundred (100). Ten (10) points are from your HW assignments, the other eighty (90) points will come from the problem solutions. I will not grade neither give you points for the technical writing, if you still want to present your test following the technical writing, you can follow the attached rubric.

3. There are 2 problems, but you are supposed to solve only one. It is your choice. If you solve both, I will grade only one and give you extra 10 points towards the 2nd test for the second problem you solve (if correctly solved). You need to tell me which problem you want me to grade towards this third test. The problem you pick to be graded is worth all 90 points.

4. What you turn in should be only your own work. You cannot discuss the exam with anyone, except me. Call me, skype me, text me, email me, come to my office, if you have any question.

5. I do not read minds. You should be explicit and organized in your answers. Use drawings/figures. If you make a mistake, do not erase it. Rather use that opportunity to explain why you think it is a mistake and show the way to correct the problem.

6. You have to turn in your test ON TIME and ONLY through CANVAS. You must submit your solution as a pdf file and the excel spreadsheet. For the ePortfolio (which is optional) you are supposed to upload this artifact to your Google drive.

7. Do not start at the last minute so you can handle anything that could happen. Late tests will not be accepted. Test submitted through email will not be accepted either.

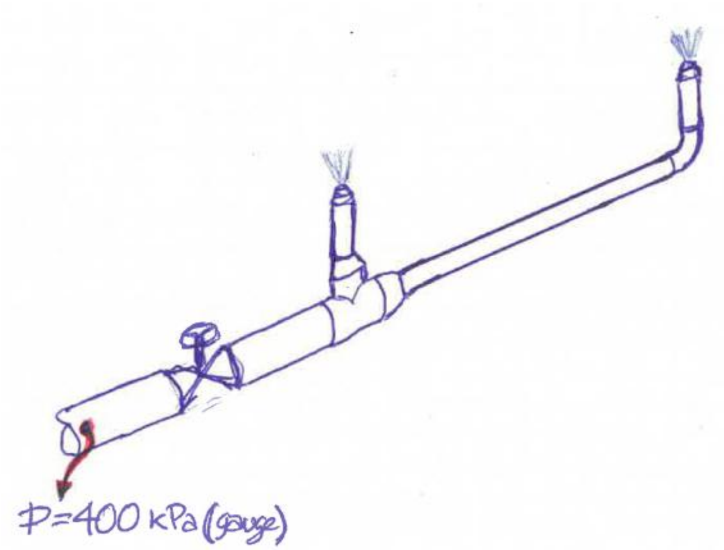
8. Cheating is completely wrong. The ODU Student Honor Pledge reads: "I pledge to support the honor system of Old Dominion University. I will refrain from any form of academic dishonesty or deception, such as cheating or plagiarism." By attending Old Dominion University you have accepted the responsibility to abide by this code. This is an institutional policy approved by the Board of Visitors. It is important to remind you the following part of the Honor Code:

    IX. PROHIBITED CONDUCT
    A. Academic Integrity violations, including:
    1. *Cheating*: Using unauthorized assistance, materials, study aids, or other information in any academic exercise (Examples of cheating include, but are not limited to, the following: using unapproved resources or assistance to complete an assignment, paper, project, quiz or exam; collaborating in violation of a faculty member's instructions; and submitting the same, or substantially the same, paper to more than one course for academic credit without first obtaining the approval of faculty).

**With that said, you are NOT authorized to use any online source of any type, unless is ODU related.**

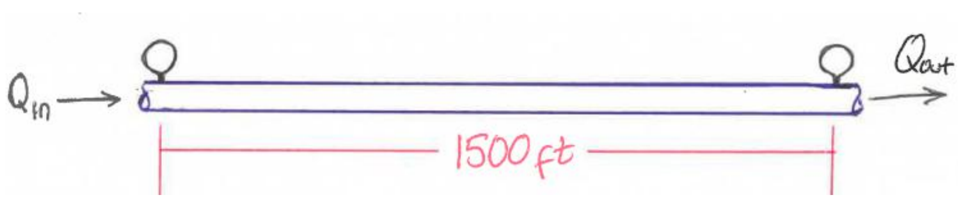
1) The system sketched in the figure is an automatic sprinfler system for a narrow plot of lawn. Water is supply by a main that guarantees a constant pressure of 400 kPa (gauge). The sprinkler pipeline is made of schedule-40 steel pipe. For a wide-open ball valve, determine the flow rate delivered to each sprinkler head. Do not neglect minor losses. The characteristic of the system is as follows:

   a. From point where pressure is 400 kPa to the T-joint: 1 ½ inches nominal pipe of 6.5 m.
   b. From T_joint to 1st sprinkler head: 1 inch nominal pipe of 0.3 m.
   c. From T_joint to 2nd sprinkler head: 1 inch nominal pipe of 8.3 m.
   d. K of the sprinkler head is 50.

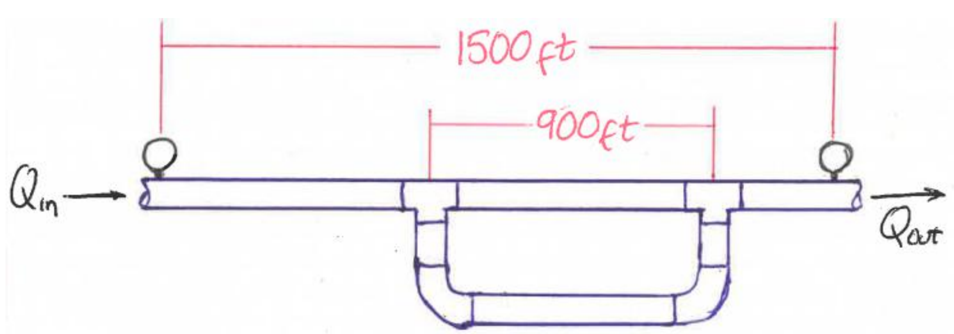



   Are the flows through each sprinkler the same? If not, what would you do to make them the same? How does the fluid velocity compare to the critical velocity (3 m/s)? If it is too far off, what would you do?

2) A horizontally laid 2 inches standard steel tubing is 1500 ft long and has water passing through it at a 65 gpm flow rate. Determine the corresponding pressure drop




   The pipe is modified by adding a loop made of 1 ½ inches standard steel tubing that is only 900 ft long. What is the expected increase in flow rate through the system for the same pressure as in the original pipe (the one you calculated)? Consider all minor losses.



## Problem solution rubric

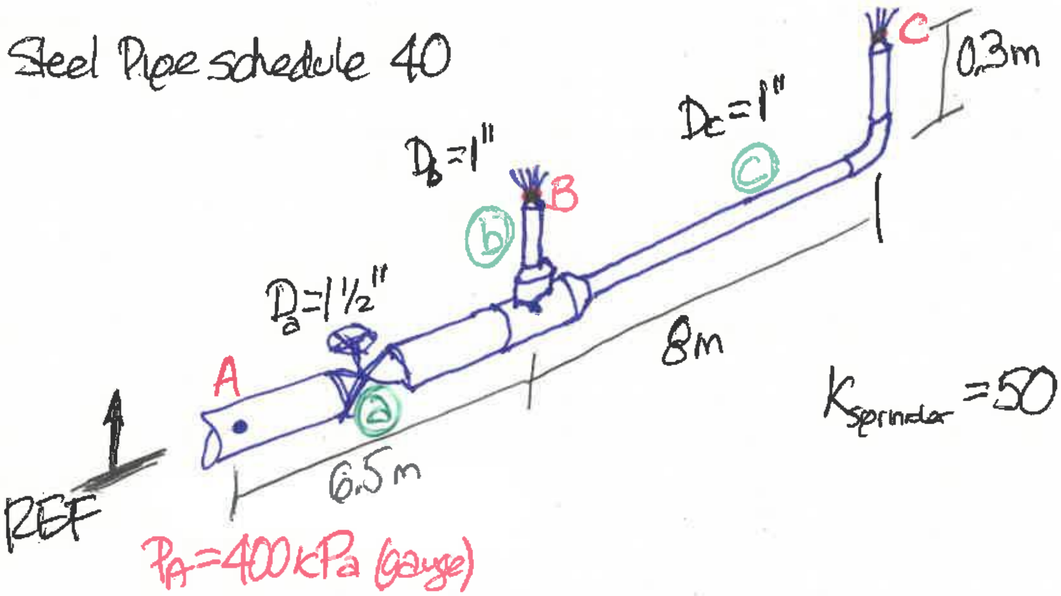
| | Exceeds Standard | Meets Standard | Approaches Standard | Needs Attention |
|---|---|---|---|---|
| | 4 | 3 | 2 | 1 |
| | **10 points** | **7 points** | **4 points** | **0 points** |
| **1. Purpose** 5% | The purpose of the section to be answered is clearly identified and stated. | The purpose of the section to be answered is identified, but is stated in a somewhat unclear manner. | The purpose of the section to be answered is partially identified, and is stated in a somewhat unclear manner. | The purpose of the section to be answered is erroneous or irrelevant. |
| **2. Drawings & Diagrams** 10% | Clear and accurate diagrams are included and make the section easier to understand. Diagrams are labeled neatly and accurately. | Diagrams are included and are labeled neatly and accurately. | Diagrams are included and are labeled. | Needed diagrams are missing OR are missing important labels. |
| **3. Sources** 5% | Several reputable background sources were used and cited correctly. | A few reputable background sources are used and cited correctly. | A few background sources are used and cited correctly, but some are not reputable sources. | Background sources are cited incorrectly. |
| **4. Design considerations (assumptions, safety, cost, etc)** 10% | Design is carried out with applicable assumptions and full attention to safety and cost, etc. | Design is generally carried out with assumptions and attention to safety, cost, etc. | Design is carried out with some assumptions and some attention to safety, cost, etc. | Assumptions, safety and cost were ignored in the design. |
| **5. Data and variables** 5% | All data and variables are clearly described with all relevant details. | All data and variables are clearly described with most relevant details. | Most data and variables are clearly described with most relevant details. | Data and variables are not described OR the majority lack sufficient detail. |
| **6. Procedure** 25% | Procedure is described in clear steps. The step description is in a complete and easy to understand short paragraph. | Procedure is described in clear steps but the step description is not in a complete short paragraph. | Procedure is described in clear steps. The step description is in a complete short paragraph but it is difficult to understand. | Procedure is not described in clear steps at all. |
| **7. Calculations** 20% | All calculations are shown and the results are correct and labeled appropriately. The units of all values are shown. | Some calculations are shown and the results are correct and labeled appropriately. | Some calculations are shown and the results labeled appropriately. | No calculations are shown OR results are inaccurate or mislabeled. |
| **8. Summary** 5% | Summary describes the design, the relevant information and some future implications. | Summary describes the design and some relevant information. | Summary describes the design. | No summary is written. |
| **9. Materials** 5% | All materials used in the design are clearly and accurately described. | Almost all materials used in the design are clearly and accurately described. | Most of the materials used in the design are clearly and accurately described. | Many materials are described inaccurately OR are not described at all. |
| **10. Analysis** 10% | The design is discussed and analyzed. Argumentative predictions are made about what might happen in case of change in the operation and how the design could be change. | The design is discussed and analyzed. Argumentative predictions are made about what might happen in case of change in the operation. | The design is discussed and analyzed. No argumentative predictions are made about what might happen in case of change in the operation and how the design could be change. | The design is not discussed and analyzed. |

(1) PURPOSE

A

Calculate the flow rates going through the sprinklers in the figure.

## DRAWINGS & DIAGRAMS




## SOURCES

Mott, R., Untener, J.A. "Applied Fluid Mechanics," 7th edition, Pearson Education Inc, (2015).

## DESIGN CONSIDERATIONS

- Constant properties
- Incompressible fluid
- Isothermal $T=25°C$
- Ball valve assumed to be a globe valve
- Reductions are gradual with angle 90°

## DATA & VARIABLES

- $D_a=40.9\,mm$
- $D_b=D_c=26.6\,mm$
- $\gamma_w=9.78\,kN/m^3$
- $\nu_w=8.94\times10^{-7}\,m^2/s$
- Tnpe given in figure
- $K_{red}\cong0.14$
- $K_{tee\,branch}=60f_T$
- $K_{tee\,run}=20f_T$
- $K_{elbow}=30f_T$
- $K_{valve}=340f_T$
- $\varepsilon=4.6\times10^{-5}\,m$

## PROCEDURE



I will, no doubt, use Bernoulli's equation. I will apply it from A to B and from A to C. That will give me two equation with 3 unknowns: $Q_a, Q_b$ & $Q_c$. I need one more equation for a complete system. This is conservation of mass: $Q_a = Q_b + Q_c$

## CALCULATIONS

Let us start with applying Bernoulli's from A to B:

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + \cancel{z_A}^{0} = \cancel{\frac{P_B}{\gamma}}^{0\,(gauge)} + \frac{V_B^2}{2g} + z_B + h_{L_{A-B}}$$

I will keep the term $V_B$ assuming it is the fluid velocity in pipe (b). Now,

$$\frac{P_A}{\gamma} - z_B = \frac{V_B^2 - V_A^2}{2g} + h_{L_{A-B}}$$

Looking into $h_{L_{A-B}}$

$$h_{L_{A-B}} = h_{L_{pipe_a}} + h_{L_{pipe_b}} + h_{L_{valve}} + h_{L_{tee\,branc}} + h_{L_{red_b}} + h_{L_{sprinkler_b}}$$

For each of those terms:

a) $h_{L_{pipe_a}} = f_a \frac{L_a}{D_a}\frac{V_a^2}{2g}$ ; using $V_a = \frac{Q_a}{A_a} = \frac{4Q_a}{\pi D_a^2}$

$$= f_a \frac{L_a}{D_a}\frac{1}{2g}\frac{16}{\pi^2 D_a^4} Q_a^2 = f_a \frac{L_a}{D_a^5}\frac{8}{\pi^2 g} Q_a^2 = f_a \frac{6.5\,m}{(40.9\times10^{-3}\,m)^5}\frac{8}{\pi^2\, 9.81\frac{m}{s^2}} Q_a^2$$

So,

C

$$h_{L_{pipe_a}} = 4{,}692{,}648.781\, f_a Q_a^2$$

b) $$h_{L_{pipe_b}} = f_b \frac{L_b}{D_b^5} \frac{8}{\pi^2 g} Q_b^2 = f_b \frac{0.3\,m}{(26.6\times10^{-3}\,m)^5} \frac{8}{\pi^2 \times 9.81\frac{m}{s^2}} Q_b^2$$

$$h_{L_{pipe_b}} = 1{,}861{,}376.839\, f_b Q_b^2$$

c) $$h_{L_{valve}} = K_{valve} \frac{V_a^2}{2g} \; ; \text{ using } V_a = \frac{4Q_a}{\pi D_a^2}$$

$$= K_{valve} \frac{1}{2g} \frac{16}{\pi^2 D_a^4} Q_a^2 = 340 f_{T_a} \frac{8}{\pi^2 \times 9.81\frac{m}{s^2}} \frac{1}{(40.9\times10^{-3}\,m)^4} Q_a^2$$

$f_{T_a} = 0.020$ using table 10.5

$$h_{L_{valve}} = 200{,}787.612\, Q_a^2$$

d) $$h_{L_{tee\,branch}} = K_{tee\,branch} \frac{V_a^2}{2g} = 60 f_{T_a} \frac{8}{\pi^2 g} \frac{1}{D_a^4} Q_a^2 = 60 \cdot 0.02 \cdot \frac{8}{\pi^2 \times 9.81\frac{m}{s^2}} \frac{1}{(40.9\times10^{-3}\,m)^4} Q_a^2$$

$$h_{L_{tee\,branch}} = 35{,}433.108\, Q_a^2$$

e) $$h_{L_{red_b}} = K_{red} \frac{V_b^2}{2g} = K_{red} \frac{8}{\pi^2 g} \frac{1}{D_b^4} Q_b^2 = 0.14 \frac{8}{\pi^2 \times 9.81\frac{m}{s^2}} \frac{1}{(26.6\times10^{-3}\,m)^4} Q_b^2$$

$$h_{L_{red_b}} = 23{,}105.891\, Q_b^2$$

f) $$h_{L_{sprinkler_b}} = K_{sprinkler} \frac{8}{\pi^2 g} \frac{1}{D_b^4} Q_b^2 = 50 \frac{8}{\pi^2 \times 9.81\frac{m}{s^2}} \frac{1}{(26.6\times10^{-3}\,m)^4} Q_b^2$$

$$h_{L_{sprinkler_b}} = 8{,}252{,}103.988\, Q_b^2$$

Substituing the energy losses into Bernoulli's equation: **D**

$$\frac{P_A}{\gamma}-Z_B=\frac{V_B^2-V_A^2}{2g}+4{,}692{,}648.781 f_a Q_a^2+1{,}861{,}376.839 f_b Q_b^2$$
$$+200{,}787.612 Q_a^2+35{,}433.108 Q_a^2+23{,}105.891 Q_b^2$$
$$+8{,}252{,}103.988 Q_b^2$$

For the terms $V_B$ & $V_A$:

$$V_B=\frac{4Q_b}{\pi D_b^2}=\frac{4Q_b}{\pi(266\times10^{-3}m)^2}=1{,}799.479 Q_b$$
$$V_A=\frac{4Q_a}{\pi D_a^2}=\frac{4Q_a}{\pi(40.9\times10^{-3}m)^2}=761.138 Q_a$$

$$\frac{V_B^2-V_A^2}{2g}=165{,}042.079 Q_b^2-29{,}527.576 Q_a^2$$

So; adding all similar terms:

$$\frac{P_A}{\gamma}-Z_B=4{,}692{,}648.781 f_a Q_a^2+1{,}861{,}376.839 f_b Q_b^2+206{,}693.144 Q_a^2+8{,}440{,}251.958 Q_b^2$$

Substituing values on the left hand side:

$$\frac{400\,kPa}{9.79\,\frac{kN}{m^3}}-0.3m=$$

eq1 $$\boxed{40.599_m=4{,}692{,}648.781 f_a Q_a^2+1{,}861{,}376.839 f_b Q_b^2+206{,}693.144 Q_a^2+8{,}440{,}251.958 Q_b^2}$$

Doing the same with Bernoulli's between A & C:

$$\frac{P_A}{\gamma}-Z_C=\frac{V_C^2-V_A^3}{2g}+h_{pipe_a}+h_{pipe_c}+h_{L_{valve}}+h_{L_{tee\,run}}+h_{L_{red_C}}+h_{L_{elbow}}+h_{L_{sprinkler_C}}$$

E

The terms $h_{L_{pipes}}$ & $h_{L_{valve}}$ were already obtained, For the others:

a) $h_{L_{pipe_c}} = f_c \frac{L_c}{D_c^5} \frac{8}{\pi^2 g} Q_c^2 = f_c \frac{83m}{(266\times10^{-3}m)^5} \frac{8}{\pi^2 9.81\frac{m}{s^2}} Q_c^2$

$$h_{L_{pipe_c}} = 51{,}498{,}092.55\, f_c Q_c^2$$

b) $h_{L_{tee\,run}} = K_{tee\,run} \frac{8}{\pi^2 g} \frac{1}{D_a^4} Q_a^2 = 20 f_{T_a} \frac{8}{\pi^2 g} \frac{1}{D_a^4} Q_a^2 = 20\times0.02 \frac{8}{\pi^2\times9.81\frac{m}{s^2}} \frac{1}{(40.9\times10^{-3}m)^4} Q_a^2$

$$h_{L_{tee\,run}} = 11{,}811.036\, Q_a^2$$

c) $h_{L_{red_c}} = K_{red} \frac{8}{\pi^2 g} \frac{1}{D_c^4} Q_c^2 = 0.14 \frac{8}{\pi^2 9.81\frac{m}{s^2}} \frac{1}{(266\times10^{-3}m)^4} Q_c^2$

$$h_{L_{red_c}} = 23{,}105.891\, Q_c^2$$

d) $h_{L_{elbow}} = K_{elbow} \frac{8}{\pi^2 g} \frac{1}{D_c^4} Q_c^2 = 30 f_{T_c} \frac{8}{\pi^2 g} \frac{1}{D_c^4} Q_c^2$

$f_{T_c} = 0.022$ using table 10.5

$$h_{L_{elbow}} = 30\times0.022 \frac{8}{\pi^2\times9.81\frac{m}{s^2}} \frac{1}{(266\times10^{-3}m)^4} Q_c^2$$

$$h_{L_{elbow}} = 108{,}927.773\, Q_c^2$$

e) $h_{L_{service_c}} = 8{,}252{,}103.988\, Q_c^2$

Putting all terms into Bernoulli's between A & C: F

$$40.599_m = 165{,}042.079\,Q_c^2 - 29{,}527.576\,Q_a^2$$
$$+ 4{,}692{,}648.781\,f_a Q_a^2 + 51{,}498{,}092.55\,f_c Q_c^2$$
$$+ 200{,}787.612\,Q_a^2 + 11{,}811.036\,Q_a^2 + 23{,}105.891\,Q_c^2$$
$$+ 108{,}927.773\,Q_c^2 + 8{,}252{,}103.988\,Q_c^2$$

Simplifying:

(eq2) $$40.599_m = 4{,}692{,}648.781\,f_a Q_a^2 + 51{,}498{,}092.55\,f_c Q_c^2 + 183{,}071.072\,Q_a^2 + 8{,}549{,}179.731\,Q_c^2$$

So far I have 2 equations and 3 unknowns ($Q_a$, $Q_b$ & $Q_c$). The other equation is:

(eq3) $$Q_a = Q_b + Q_c$$

To solve the system, I will do the following:

1) Assume $Q_a$
2) Solve for $Q_b$ using eq1 (requires iteration because $f_b$ depends on $Q_b$)
3) Solve for $Q_c$ using eq 2 (requires iteration because $f_c$ depends on $Q_c$)
4) With the values of $Q_b$ & $Q_c$, calculate $Q_a$ using eq3.
5) If the new $Q_a$ is "equal" to the previous $Q_a$, stop. If not go back to step 1).

I will use EXCEL to help me with calculations.

For steps 2) & 3) I need to manipulate eq1 & eq2: G

$$Q_b = \left[\frac{40.599 - 4{,}692{,}648.781 f_a Q_a^2 - 206{,}693.144 Q_a^2}{1{,}861{,}376.839 f_b + 8{,}440{,}251.958}\right]^{1/2}$$

$$Q_c = \left[\frac{40.599 - 4{,}692{,}648.781 f_a Q_a^2 - 183{,}071.072 Q_a^2}{51{,}498{,}092.55 f_c + 8{,}549{,}179.731}\right]^{1/2}$$

| | | |
|---|---|---|
| Da= | 4.09E-02 | m |
| Db= | 2.66E-02 | m |
| Dc= | 2.66E-02 | m |
| e= | 4.60E-05 | m |
| D/e_a= | 889.1304 | |
| D/e_b= | 578.2609 | |
| D/e_c= | 578.2609 | |
| nu= | 8.94E-07 | m2/s |

ITERATION 1

| Qa (m3/s) | Va (m/s) | Re_a | fa |
|---|---|---|---|
| 0.001 | 0.76113817 | 3.48E+04 | 0.025818 |

| fb | Qb (m3/s) | Vb | Re_b | fb | %error-fb |
|---|---|---|---|---|---|
| 0.01 | 0.00218193 | 3.93E+00 | 1.17E+05 | 0.024353 | -143.53% |
| 0.024353 | 0.00217849 | 3.92E+00 | 1.17E+05 | 0.024356 | -0.01% |
| 0.024356 | 0.00217849 | 3.92E+00 | 1.17E+05 | 0.024356 | 0.00% |
| 0.024356 | 0.00217849 | 3.92E+00 | 1.17E+05 | 0.024356 | 0.00% |

| fc | Qc (m3/s) | Vc | Re_c | fc | %error-fc |
|---|---|---|---|---|---|
| 0.01 | 0.00210844 | 3.79E+00 | 1.13E+05 | 0.024405 | -144.05% |
| 0.024405 | 0.00202711 | 3.65E+00 | 1.09E+05 | 0.024467 | -0.25% |
| 0.024467 | 0.00202679 | 3.65E+00 | 1.09E+05 | 0.024467 | 0.00% |
| 0.024467 | 0.00202679 | 3.65E+00 | 1.09E+05 | 0.024467 | 0.00% |

| NEW Qa (m3/s) Qb+Qc (m3/s) | %error-Qa |
|---|---|
| 0.00420528 | -320.53% |

ITERATION 2

| Qa (m3/s) | Va (m/s) | Re_a | fa |
|---|---|---|---|
| 0.00420528 | 3.20079751 | 1.46E+05 | 0.022095 |

| fb | Qb (m3/s) | Vb | Re_b | fb | %error-fb |
|---|---|---|---|---|---|
| 0.01 | 0.00203733 | 3.67E+00 | 1.09E+05 | 0.024459 | -144.59% |
| 0.024459 | 0.00203409 | 3.66E+00 | 1.09E+05 | 0.024461 | -0.01% |
| 0.024461 | 0.00203409 | 3.66E+00 | 1.09E+05 | 0.024461 | 0.00% |
| 0.024461 | 0.00203409 | 3.66E+00 | 1.09E+05 | 0.024461 | 0.00% |

| fc | Qc (m3/s) | Vc | Re_c | fc | %error-fc |
|---|---|---|---|---|---|
| 0.01 | 0.0019798 | 3.56E+00 | 1.06E+05 | 0.024505 | -145.05% |
| 0.024505 | 0.00190294 | 3.42E+00 | 1.02E+05 | 0.024570 | -0.26% |
| 0.024570 | 0.00190262 | 3.42E+00 | 1.02E+05 | 0.024570 | 0.00% |
| 0.024570 | 0.00190262 | 3.42E+00 | 1.02E+05 | 0.024570 | 0.00% |

| NEW Qa (m3/s) Qb+Qc (m3/s) | %error-Qa |
|---|---|
| 0.00393671 | 6.39% |

ITERATION 3

| Qa (m3/s) | Va (m/s) | Re_a | fa |
|---|---|---|---|
| 0.00393671 | 2.99637956 | 1.37E+05 | 0.022198 |

| fb | Qb (m3/s) | Vb | Re_b | fb | %error-fb |
|---|---|---|---|---|---|
| 0.01 | 0.00205671 | 3.70E+00 | 1.10E+05 | 0.024444 | -144.44% |
| 0.024444 | 0.00205345 | 3.70E+00 | 1.10E+05 | 0.024447 | -0.01% |
| 0.024447 | 0.00205345 | 3.70E+00 | 1.10E+05 | 0.024447 | 0.00% |
| 0.024447 | 0.00205345 | 3.70E+00 | 1.10E+05 | 0.024447 | 0.00% |

| fc | Qc (m3/s) | Vc | Re_c | fc | %error-fc |
|---|---|---|---|---|---|
| 0.01 | 0.00199699 | 3.59E+00 | 1.07E+05 | 0.024491 | -144.91% |
| 0.024491 | 0.00191953 | 3.45E+00 | 1.03E+05 | 0.024555 | -0.26% |
| 0.024555 | 0.00191921 | 3.45E+00 | 1.03E+05 | 0.024556 | 0.00% |
| 0.024556 | 0.00191921 | 3.45E+00 | 1.03E+05 | 0.024556 | 0.00% |

| NEW Qa (m3/s) Qb+Qc (m3/s) | %error-Qa |
|---|---|
| 0.00397265 | -0.91% |

ITERATION 4

| Qa (m3/s) | Va (m/s) | Re_a | fa |
|---|---|---|---|

| fb | Qb (m3/s) | Vb | Re_b | fb | %error-fb |
|---|---|---|---|---|---|

| | | | |
|---|---|---|---|
| 0.00397265 | 3.02373716 | 1.38E+05 | 0.022184 |

| | | | | | |
|---|---|---|---|---|---|
| 0.01 | 0.0020542 | 3.70E+00 | 1.10E+05 | 0.024446 | -144.46% |
| 0.024446 | 0.00205094 | 3.69E+00 | 1.10E+05 | 0.024448 | -0.01% |
| 0.024448 | 0.00205094 | 3.69E+00 | 1.10E+05 | 0.024448 | 0.00% |
| 0.024448 | 0.00205094 | 3.69E+00 | 1.10E+05 | 0.024448 | 0.00% |

| fc | Qc (m3/s) | Vc | Re_c | fc | %error-fc |
|---|---|---|---|---|---|
| 0.01 | 0.00199476 | 3.59E+00 | 1.07E+05 | 0.024493 | -144.93% |
| 0.024493 | 0.00191738 | 3.45E+00 | 1.03E+05 | 0.024557 | -0.26% |
| 0.024557 | 0.00191706 | 3.45E+00 | 1.03E+05 | 0.024557 | 0.00% |
| 0.024557 | 0.00191706 | 3.45E+00 | 1.03E+05 | 0.024557 | 0.00% |

| NEW Qa (m3/s) Qb+Qc (m3/s) | %error-Qa |
|---|---|
| 0.003968 | 0.12% |

ITERATION 5

| Qa (m3/s) | Va (m/s) | Re_a | fa |
|---|---|---|---|
| 0.003968 | 3.02019282 | 1.38E+05 | 0.022185 |

| fb | Qb (m3/s) | Vb | Re_b | fb | %error-fb |
|---|---|---|---|---|---|
| 0.01 | 0.00205452 | 3.70E+00 | 1.10E+05 | 0.024446 | -144.46% |
| 0.024446 | 0.00205127 | 3.69E+00 | 1.10E+05 | 0.024448 | -0.01% |
| 0.024448 | 0.00205127 | 3.69E+00 | 1.10E+05 | 0.024448 | 0.00% |
| 0.024448 | 0.00205127 | 3.69E+00 | 1.10E+05 | 0.024448 | 0.00% |

| fc | Qc (m3/s) | Vc | Re_c | fc | %error-fc |
|---|---|---|---|---|---|
| 0.01 | 0.00199505 | 3.59E+00 | 1.07E+05 | 0.024492 | -144.92% |
| 0.024492 | 0.00191766 | 3.45E+00 | 1.03E+05 | 0.024557 | -0.26% |
| 0.024557 | 0.00191734 | 3.45E+00 | 1.03E+05 | 0.024557 | 0.00% |
| 0.024557 | 0.00191734 | 3.45E+00 | 1.03E+05 | 0.024557 | 0.00% |

| NEW Qa (m3/s) Qb+Qc (m3/s) | %error-Qa |
|---|---|
| 0.0039686 | -0.02% |

ITERATION 6

| Qa (m3/s) | Va (m/s) | Re_a | fa |
|---|---|---|---|
| 0.0039686 | 3.02065403 | 1.38E+05 | 0.022185 |

| fb | Qb (m3/s) | Vb | Re_b | fb | %error-fb |
|---|---|---|---|---|---|
| 0.01 | 0.00205448 | 3.70E+00 | 1.10E+05 | 0.024446 | -144.46% |
| 0.024446 | 0.00205122 | 3.69E+00 | 1.10E+05 | 0.024448 | -0.01% |
| 0.024448 | 0.00205122 | 3.69E+00 | 1.10E+05 | 0.024448 | 0.00% |
| 0.024448 | 0.00205122 | 3.69E+00 | 1.10E+05 | 0.024448 | 0.00% |

| fc | Qc (m3/s) | Vc | Re_c | fc | %error-fc |
|---|---|---|---|---|---|
| 0.01 | 0.00199501 | 3.59E+00 | 1.07E+05 | 0.024492 | -144.92% |
| 0.024492 | 0.00191763 | 3.45E+00 | 1.03E+05 | 0.024557 | -0.26% |
| 0.024557 | 0.0019173 | 3.45E+00 | 1.03E+05 | 0.024557 | 0.00% |
| 0.024557 | 0.0019173 | 3.45E+00 | 1.03E+05 | 0.024557 | 0.00% |

| NEW Qa (m3/s) Qb+Qc (m3/s) | %error-Qa |
|---|---|
| 0.00396852 | 0.00% |

## SUMMARY

The flow rate through the 1st sprinkler is 0.00205 $m^3/s$ and the one through the 2nd is 0.00192 $m^3/s$. the total flow rate is 0.00397 $m^3/s$.

## MATERIALS

Water

## ANALYSIS

a) The flows through the sprinklers are very similar. There is only a difference of 6.77%. There is no need to modify the system.

b) The fluid velocities in the pipes varies from 3.02 m/s to 3.69 m/s (for 1st sprinkler). We should use a larger pipe in path ⓑ but that will increase the flow rate towards the 1st sprinkler. We could either leave pipe ⓑ as it is (after all is a small section), or increase its size but use a valve in pipe ⑥ to control/reduce flow (less likely...).

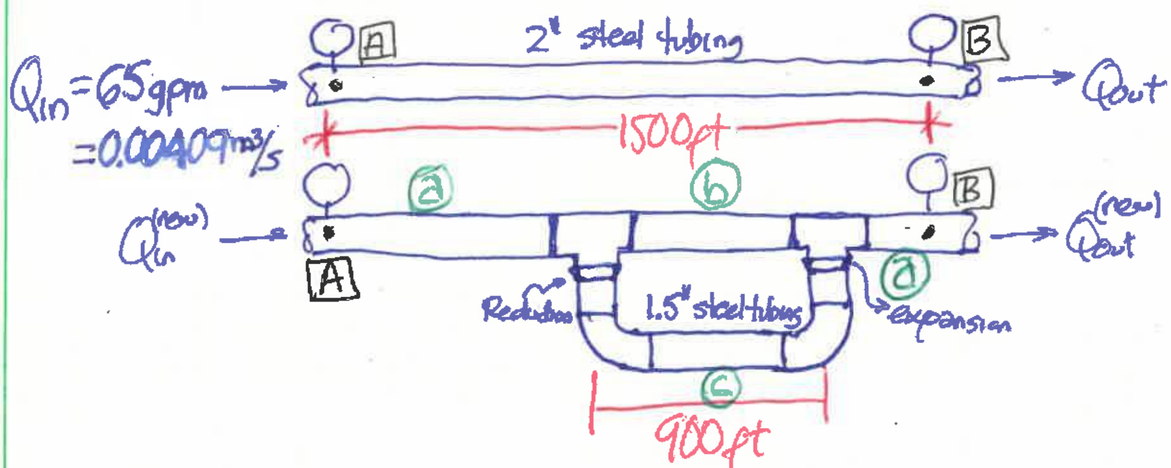
## (2) PURPOSE

[J]

Compute the pressure drop in a single straight pipe as show in the figure.

By modifying the single straight pipe with an additional loop and with the same previous pressure drop, calculate the new total flow rate through the new system.

## DRAWINGS & DIAGRAMS




## SOURCES

Mott, R, Untener, J.A, "Applied Fluid Mechanics", 7th edition Pearson Education Inc, (2015)

## DESIGN CONSIDERATIONS

- Constant properties
- Incompressible fluid
- Isothermal T=25°C
- Reduction is gradual with angle 90°
- Tubing wall thickness t=1.65 mm
- Expansion is gradual with angle 60°

## DATA & VARIABLES

- $D_a = 47.50$ mm (same per $D_b$)
- $D_c = 34.80$ mm
- $\gamma_w = 9.78$ kN/m³
- $\nu_w = 8.94\times10^{-7}$ m²/s
- Info in figure
- $K_{red} \cong 0.11$
- $K_{exp} \cong 0.46$
- $\varepsilon = 1.5\times10^{-6}$ m
- $K_{elbow} = 30 f_T$
- $K_{tee\,branch} = 60 f_T$
- $K_{tee\,run} = 20 f_T$

## PROCEDURE

K

By simply applying Bernoulli's between point A & B in the single pipe, we can compute the pressure drop.

For the second part, I will apply Bernoulli between same points. However, this time, the Bernoulli's equation will produce two separate equations. One considering energy losses through branch (b) and another considering energy losses through branch (c). The number of unknowns is 3: $Q_a$, $Q_b$, and $Q_c$. The third equation is the conservation of mass: $Q_a = Q_b + Q_c$.

## CALCULATIONS

Let us start with the single pipe

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{L_{A-B}}$$

$$\frac{P_A - P_B}{\gamma} = h_{L_{A-B}}$$

and including energy losses equation:

$$\frac{P_A - P_B}{\gamma} = f_a \frac{L_a}{D_a} \frac{V_a^2}{2g} = f_a \frac{L_a}{D_a^5} \frac{8}{\pi^2 g} Q_a^2$$

we need $f_a$. For that we need $D/\varepsilon$ and Re

$$\frac{D}{\varepsilon} = \frac{47.50 \times 10^{-3} m}{1.5 \times 10^{-6} m} = 31{,}666.67 \qquad Re = \frac{VD}{\nu} = \frac{4Q}{\pi D \nu} = \frac{4 \times 0.00409\, m^3/s}{\pi (47.50 \times 10^{-3} m) 8.94 \times 10^{-7} m^2/s}$$

$$Re = 122{,}631.57$$

L

Reading from Moody chart:

$$f_a \cong 0.0171$$

Substituting all values in the equation:

$$\frac{P_A - P_A}{\gamma} = 0.0171 \frac{1500\,ft\,(0.3048\,m/ft)}{(47.50 \times 10^{-3}\,m)^5} \frac{8}{\pi^2 \times 9.81\frac{m}{s^2}} \times \left(0.00409\frac{m^3}{s}\right)^2$$

$$\frac{P_A - P_B}{\gamma} = 44.689\,m$$

$$P_A - P_B = 9.78\,\frac{kN}{m^3} \times 44.689\,m$$

$$\boxed{P_A - P_B = 437.06\,kPa}$$

For the second part of the problem, when applying Bernoulli we got same equation:

$$\frac{P_A - P_B}{\gamma} = h_{L_{AB}}$$

However, since for the second part we have to branches to go from A to B, the Bernoulli's equation produces two equations:

$$\frac{P_A - P_B}{\gamma} = h_{L_{AB\,\text{Branch (b)}}} \quad \text{and} \quad \frac{P_A - P_B}{\gamma} = h_{L_{AB\,\text{Branch (c)}}}$$

Let us get the energy losses for branch ⓑ: [M]

$$h_{L_{AB\,Branch\,ⓑ}} = h_{L_{ⓐ}} + h_{L_{ⓑ}} + 2h_{L_{tee\,run}}$$

For each of the terms:

a) $h_{L_{ⓐ}} = f_a \frac{L_a}{D_a^5} \frac{8}{\pi^2 g} Q_a^2 = f_a \frac{[1500\,ft - 900\,ft](0.3048\,m/ft)}{(47.50 \times 10^{-3}\,m)^5} \frac{8}{\pi^2 \times 9.81\frac{m}{s^2}} Q_a^2$

$$h_{L_{ⓐ}} = 62{,}491{,}276.95 f_a Q_a^2$$

b) $h_{L_{ⓑ}} = f_b \frac{L_b}{D_b^5} \frac{8}{\pi^2 g} Q_b^2 = f_b \frac{900\,ft(0.3048\,m/ft)}{(47.50 \times 10^{-3}\,m)^5} \frac{8}{\pi^2 \times 9.81\frac{m}{s^2}} Q_b^2$

$$h_{L_{ⓑ}} = 93{,}736{,}915.42 f_b Q_b^2$$

c) $h_{L_{tee\,run}} = K_{tee\,run} \frac{8}{\pi^2 g} \frac{1}{D_a^4} Q_a^2 = 20 f_{T_a} \frac{8}{\pi^2 g} \frac{1}{D_a^4} Q_a^2$

For $f_{T_a}$, we need relative roughness: $\frac{D_a}{\varepsilon} = 31{,}666.67$

Reading from Moody: $f_{T_a} \cong 0.0095$

(we cannot use table 10.5)

$$= 20 \times 0.0095 \frac{8}{\pi^2 \times 9.81\frac{m}{s^2}} \cdot \frac{1}{(47.50 \times 10^{-3}\,m)^4} Q_a^2$$

$$h_{L_{tee\,run}} = 3{,}083.90\, Q_a^2$$

Putting everything into Bernoulli for branch ⓑ

(eq.) $$\boxed{44.689\,m = 62{,}491{,}276.95 f_a Q_a^2 + 93{,}736{,}915.42 f_b Q_b^2 + 3{,}083.90\, Q_a^2}$$

As for using branch ©, the energy losses are: **N**

$$h_{L_{AB\ Branch\ ©}} = h_{L_{ⓐ}} + h_{L_{©}} + 2h_{L_{tee\ branch}} + 2h_{L_{elbow}} + h_{L_{red}} + h_{L_{exp}}$$

For each of the terms ($h_{L_{ⓐ}}$ is the same):

a) $h_{L_{©}} = f_c \frac{L_c}{D_c^5} \frac{8}{\pi^2 g} Q_c^2 = f_c \frac{900\,ft\,(0.3048\,m/ft)}{(34.80\times10^{-3}m)^5} \frac{8}{\pi^2 \times 9.81\frac{m}{s^2}} Q_c^2$

$$h_{L_{©}} = 444{,}101{,}752.10 f_c Q_c^2$$

b) $h_{L_{tee\ branch}} = K_{tee\ branch} \frac{8}{\pi^2 g} \frac{1}{D_a^4} Q_a^2 = 60 f_{T_a} \frac{8}{\pi^2 \times g} \frac{1}{D_a^4} Q_a^2$

$$= 60 \times 0.0095 \frac{8}{\pi^2 \times 9.81\frac{m}{s^2}} \frac{1}{(47.50\times10^{-3}m)^4} Q_a^2$$

$$h_{L_{tee\ branch}} = 9{,}251.70\, Q_a^2$$

c) $h_{L_{elbow}} = K_{elbow} \frac{8}{\pi^2 g} \frac{1}{D_c^4} Q_c^2 = 30 f_{T_c} \frac{8}{\pi^2 g} \frac{1}{D_c^4} Q_c^2$

For $f_{T_c} \rightarrow \frac{D_c}{\varepsilon} = \frac{34.80\times10^{-3}m}{1.5\times10^{-6}m} = 23{,}200.00$

From Moody: $f_{T_c} \cong 0.011$

$$= 30 \times 0.011 \frac{8}{\pi^2 \times 9.81\frac{m}{s^2}} \frac{1}{(34.80\times10^{-3}m)^4} Q_c^2$$

$$h_{L_{elbow}} = 18{,}591.66\, Q_c^2$$

d) $h_{L_{red}} = K_{red} \frac{8}{\pi^2 g} \frac{1}{D_c^4} Q_c^2 = 0.11 \frac{8}{\pi^2 \times 9.81\frac{m}{s^2}} \frac{1}{(34.80\times10^{-3}m)^4} Q_c^2$

$$h_{L_{red}} = 6{,}197.22\, Q_c^2$$

e) $h_{L_{exp}} = K_{exp} \frac{8}{\pi^2 g} \frac{1}{D_c^4} Q_c^2 = 0.46 \frac{8}{\pi^2 \times 9.81 \frac{m}{s^2}} \frac{1}{(34.80 \times 10^{-3} m)^4} Q_c^2$ **0**

$$h_{L_{exp}} = 25{,}915.65\, Q_c^2$$

Putting everything into Bernoulli for branch ⓒ:

$$44.689\,m = 62{,}491{,}276.95 f_a Q_a^2 + 444{,}101{,}752.10 f_c Q_c^2$$
$$+ 9{,}251.70 Q_a^2 + 18{,}591.66 Q_c^2 + 6{,}197.22 Q_c^2$$
$$+ 25{,}915.65 Q_c^2$$

So,

(eq2) $$44.689\,m = 62{,}491{,}276.95 f_a Q_a^2 + 444{,}101{,}752.10 f_c Q_c^2 + 9{,}251.70 Q_a^2 + 50{,}704.53 Q_c^2$$

eq1 & eq2 combine with

(eq3) $$Q_a = Q_b + Q_c$$

give us 3 equations with 3 unknowns.
To solve the system, I will do the following:

1) Assume $Q_a$
2) Solve for $Q_b$ using eq1 (requires iteration because $f_b$ depends on $Q_b$)
3) Solve for $Q_c$ using eq2 (requires iteration because $f_c$ depends on $Q_c$)
4) With $Q_b$ & $Q_c$, calculate $Q_a$ using eq3.
5) If new $Q_a$ is "equal" to previous $Q_a$, stop. If not go back to step 1).

I will use EXCEL to help with calculations

For steps 2) & 3) I need to manipulate eq1 & eq2: **P**

$$Q_b = \left[ \frac{44.689 - 62{,}491{,}276.95 f_a Q_a^2 - 3{,}083.90 Q_a^2}{93{,}736{,}915.42 f_b} \right]^{1/2}$$

$$Q_c = \left[ \frac{44.689 - 62{,}491{,}276.95 f_a Q_a^2 - 9{,}251.70 Q_a^2}{444{,}101{,}752.10 f_c + 50{,}704.53} \right]^{1/2}$$

| | | |
|---|---|---|
| Da= | 4.75E-02 | m |
| Db= | 4.75E-02 | m |
| Dc= | 3.48E-02 | m |
| e= | 4.60E-05 | m |
| D/e_a= | 1032.6087 | |
| D/e_b= | 1032.6087 | |
| D/e_c= | 756.5217 | |
| nu= | 8.94E-07 | m2/s |

| | Qa (m3/s) | Va (m/s) | Re_a | fa |
|---|---|---|---|---|
| | 0.0043297 | 2.44E+00 | 1.30E+05 | 0.021725 |

I manipulated this
value of Qa until
I saw a small %error-Qa

| fb | Qb (m3/s) | Vb | Re_b | fb | %error-fb |
|---|---|---|---|---|---|
| 0.01 | 0.00452354 | 2.55E+00 | 1.36E+05 | 0.021648 | -116.48% |
| 0.021648 | 0.00307443 | 1.73E+00 | 9.22E+04 | 0.022410 | -3.52% |
| 0.022410 | 0.00302176 | 1.71E+00 | 9.06E+04 | 0.022449 | -0.17% |
| 0.022449 | 0.00301914 | 1.70E+00 | 9.05E+04 | 0.022451 | -0.01% |
| 0.022451 | 0.00301900 | 1.70E+00 | 9.05E+04 | 0.022451 | 0.00% |

| fc | Qc (m3/s) | Vc | Re_c | fc | %error-fc |
|---|---|---|---|---|---|
| 0.01 | 0.00206022 | 2.17E+00 | 8.43E+04 | 0.023742 | -137.42% |
| 0.023742 | 0.00134146 | 1.41E+00 | 5.49E+04 | 0.024803 | -4.47% |
| 0.024803 | 0.00131259 | 1.38E+00 | 5.37E+04 | 0.024865 | -0.25% |
| 0.024865 | 0.00131095 | 1.38E+00 | 5.37E+04 | 0.024869 | -0.01% |
| 0.024869 | 0.00131086 | 1.38E+00 | 5.36E+04 | 0.024869 | 0.00% |

| NEW Qa (m3/s) Qb+Qc (m3/s) | %error-Qa |
|---|---|
| 0.00432986 | 0.00% |

R

## SUMMARY

The pressure drop in the single pipeline of 1500 ft is 437.06 kPa, for a flow rate of 65 gpm.
The new flow rate of the system after modifying it with a loop of 1½ inches steel tube of 900 ft is 68.63 gpm.
This is just a 5.58 % increase

## MATERIALS

Water

## ANALYSIS

- The increase in flow rate after the modification is not high. If we want to increase the flow rate we need a larger diameter pipe for the new loop.

- Looking at the pipe velocities, the are all below 3 m/s. To make the velocities closer to this value, we should reduce the pipe diameters.