Problem 2 (20 points)

A modified Brayton cycle with ideal intercooling and reheating is supplied with atmospheric air at 100 kPa. Use the air-standard assumption with variable specific heats* and the given design parameters in table 2.1 to complete the following tasks.

- **a)** (5 points) Draw the T-s diagram
- **b)** (10 points) Calculate q_{in} and w_{net} (kJ/kg) for the cycle.
- **c)** (5 points) Calculate the cycle's thermal efficiency

******If you solve this problem using constant specific heats, you will receive half credit for part* ***b*** *and zero credit for part* ***c***

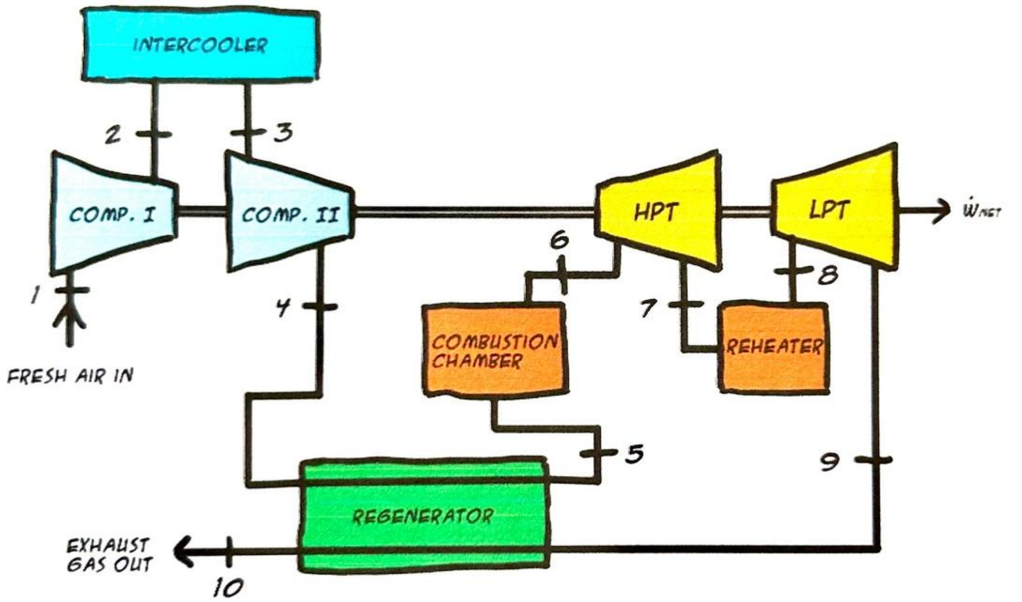

Figure 2.1: Modified Brayton Cycle

Overall Pressure ratio	9
Regenerator Effectiveness	80%
Compressors I and II	$\eta_C = 100\%$ and $T_{inlet} = 15°C$
HPT and LPT	$\eta_T = 100\%$ and $T_{inlet} = 750°C$

Table 2.1: Design Parameters

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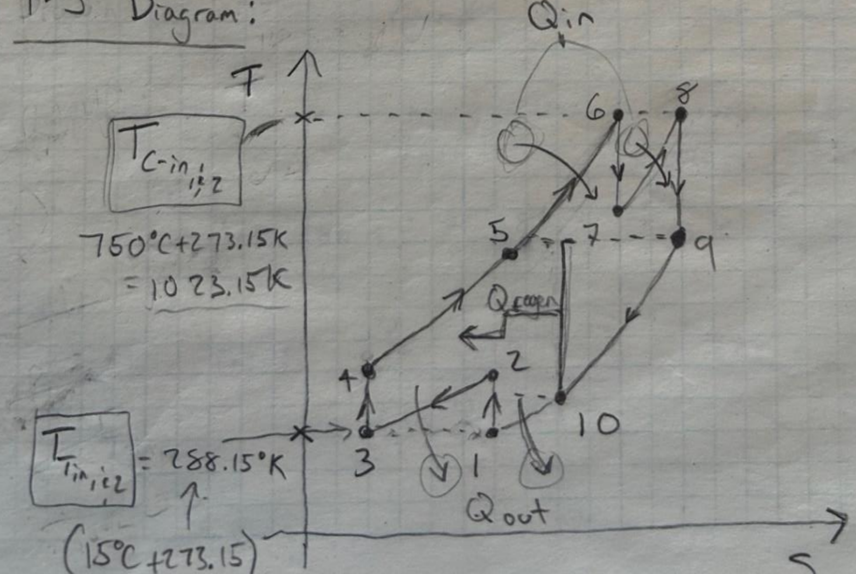
Problem #2:

Given:
- Brayton cycle w/ ideal intercooling & reheating.
- Atmospheric Air pressure is 100 kPa
- Pressure ratio, $r_p = \frac{9}{1}, \left(\frac{P_{c-out}}{P_{c-in}}\right), \left(\frac{P_{T-in}}{P_{T-out}}\right)$
- Regenerator effectiveness, $\eta_R = 0.80$
- Compressors 1 & 2 : $\eta_{c_{I,II}} = 1.0$ // $T_{c-in} = 15°C$
- Hi-PT and Lo-PT ; $\eta_{T_{I,II}} = 1.0$ // $T_{T-in} = 750°C$
- Use VARIABLE SPECIFIC HEATS

Purpose: We want to determine [a)] the proper drawing of the T-S diagram w/ reference to the schematic provided. [b)] The heat added to the system, q_{in} and the net work, W_{net} of the cycle. and [c)] to determine the cycle's thermal efficiency, η_{th}.

T-S Diagram:



Design Considerations:

- Both compressors and both Turbines are working "ideally" and considered to produce both isentropic compression and expansions respectively. This is why the lines from stages 1-2, 3-4, 6-7, 8-9 were all perfectly vertical / linear. — these processes are considered adiabitic (no loss in heat/energy).
- The regenerator effectiveness, η_R, is 80% (0.80) and must be considered when making calculations.

Data Given:
- Air pressure = 100 kPa at stage 1
- $r_p = \frac{9}{1}$
- $\eta_R = 0.80 \,//\, \eta_T = 1.0 \,//\, \eta_c = 1.0$
- $T_{c_{in}} = 288.15°K \,/\, T_{T_{in}} = 1023.15°K$

Procedure: **Stage 1:** air @ 100 kPa // 288.15°K (15°C)

Interpolate for h_1: $\frac{h_1 - 285.14}{290.16 - 285.14} = \frac{288.15 - 285}{290 - 285}$

$\therefore h_{1 @ 288.15K} = 288.3 \text{ kJ/kg·K} = h_3$

Interpolate for P_{r_1}: $\frac{P_{r_1} - 1.1584}{1.2311 - 1.1584} = \frac{288.15 - 285}{290 - 285}$

$\therefore P_{r_1} @ 288.15K = 1.2042 = P_{r_3}$

Stage 2: using the known P_{r_1} value and r_p we must determine P_{r_2} to the interpolate for enthalpy @ stage 2 (h_2):

$$\frac{P_2}{P_1} = r_p = \frac{(9)}{(1)} = \frac{P_{r_2}}{(1.2042)} \rightarrow P_{r_2} = 10.8378$$

Interpolate for h_2 @ $P_{r_2} = 10.8378$

$$\frac{h_2 - 533.98}{544.35 - 533.98} = \frac{10.8378 - 10.37}{11.1 - 10.37}$$

$$h_2 @ P_{r_2} = 10.8378 = 540.63 \text{ kJ/kg}\cdot\text{K} = h_4$$

Stage 3 → 4: $h_3 = 288.3$ / $P_{r_3} = 1.2042$

$\therefore h_2 = h_4$ & $P_{r_2} = P_{r_4}$

Now, we must find enthalpies for Turbines & regen. h_{5-9}:

stage 5: $\eta_R = 0.80 = \dfrac{h_5 - h_4}{h_9 - h_4}$

↑ must find Value for h_9 before continuing

Now, $h_9 = 580.154$ (as found), we can solve for h_5

$$0.8 = \frac{h_5 - 540.63}{580.154 - 540.63} = \boxed{572.249 = h_5}$$

Stage 6:

$T_6 = 1023.15°K$

Interpolate for h_6 from $T_6 = 1023.15°K$

$$\frac{h_6 - 1068.89}{1091.85 - 1068.89} = \frac{1023.15 - 1020}{1040 - 1020}$$

$$h_{6 @ T_6} = 1072.51 \text{ kJ/kg·K} = h_8$$

Interpolate for P_{r_6} @ $T_6 = 1023.15$

$$\frac{P_{r_6} - 123.4}{133.3 - 123.4} = \frac{1023.15 - 1020}{1040 - 1020}$$

$$P_{r_6} = 124.959 = P_{r_8}$$

Stage 7:

$$\frac{P_{r_6}}{P_{r_7}} = r_p = \frac{9}{1} = \frac{124.959}{P_{r_7}}$$

$$\therefore P_{r_7} = 13.884 = P_{r_9}$$

Using P_{r_7}, find h_7 through interpolation;

$$\frac{h_7 - 575.59}{586.04 - 575.59} = \frac{13.884 - 13.50}{14.38 - 13.50}$$

$$h_7 = 580.154 = h_9$$

Stage 10: Determine value for h_{10}

Use conservation of energy derivation:

$$\Delta q_{in} = \Delta q_{out} \rightarrow h_5 - h_4 = h_9 - \boxed{h_{10}}$$

$$\therefore (572.249 - 540.63) = 580.154 - h_{10}$$

$$h_{10} = 548.535 \text{ kJ/kg·K}$$

Final Calculations:

b)

$$q_{in} = (h_6 - h_5) + (h_8 - h_7)$$

$$= (1072.51 - 572.249) + (1072.51 - 580.154)$$

$$\boxed{q_{in} = 992.65 \text{ kJ/kg·K}}$$

$W_{net} = W_T - W_C$; where: • $W_T = (h_6 - h_7) + (h_8 - h_9)$

• $W_C = (h_4 - h_3) + (h_2 - h_1)$

$W_T = (1072.51 - 580.154) \times 2 \rightarrow$ since $h_8 = h_6$ & $h_9 = h_7$

$W_T = 984.712$ kJ/kg·K

$W_C = (540.63 - 288.3) \times 2 \rightarrow$ since $h_4 = h_2$ & $h_3 = h_1$

$W_C = 504.66$ kJ/kg·K

$\therefore W_{net} = 984.712$ kJ/kg $- 504.66$ kJ/kg·K

c) Determine thermal efficiency, η_{th}

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{480.04 \text{ kJ/kg·K}}{992.65 \text{ kJ/kg·K}}$$

$$\boxed{\eta_{th} = 0.4836 \text{ or } 48.36\%}$$

Summary: We determined the following:

① overall heat added to system

$q_{in} = 992.65$ kJ/kg·K

② Total net work output by the system:

$w_{net} = 480.04$ kJ/kg·K

③ Overall system thermal efficiency:

$\eta_{th} = 48.36\%$