Problem 3 (20 points)

Using the flight conditions and engine design parameters in table 3.1 for a turbojet-powered aircraft, utilize the cold-air standard assumption to determine the following.

a) (5 points) The pressure rise generated by the diffuser (psia)
b) (10 points) The velocity of exhaust gases at the nozzle exit (ft/s)
c) (5 points) The thrust generated by the engine (lbf)

Although not required for this problem, drawing the cycle's T-s diagram may be helpful

Flight Conditions	
Altitude (See Table A-16E for the properties of air at high altitude)	26,000 ft
Aircraft Velocity	450 mph
Turbofan Engine Design Parameters	
Rate of heat supplied by combustion	32,000 Btu/s
Mass flow rate	100 lbm/s
Diffusor	$\eta_D = 100\%$
Compressor	$r_p = 12$ and $\eta_C = 75\%$
Turbine	$\eta_T = 100\%$
Nozzle	$\eta_N = 100\%$

Table 3.1: Flight and Engine Operating Conditions

Problem #3:

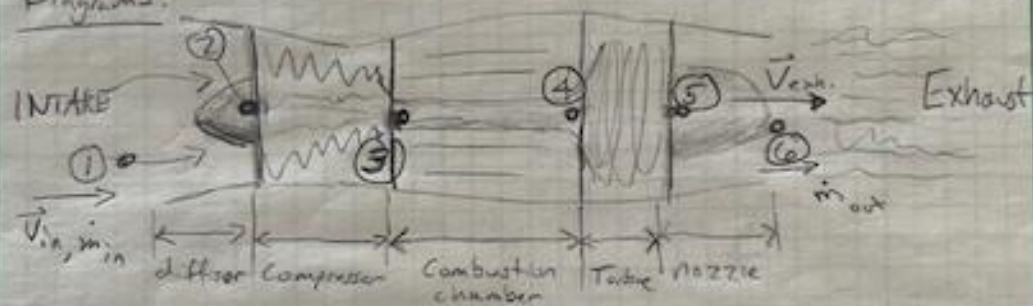
Purpose: We want to determine the following:

a) The pressure rise generated by the diffuser within the turbojet cycle (in psia).

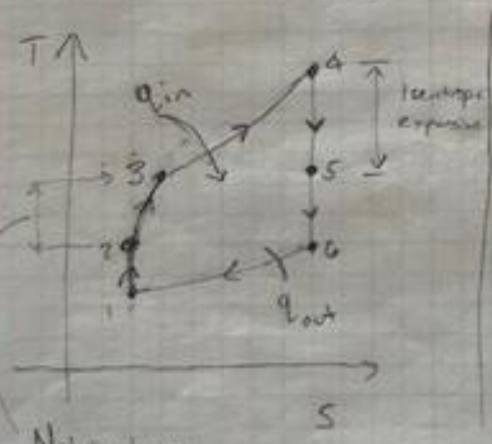
b) The velocity of exh. gases at the exit of the engines nozzle (in ft./s).

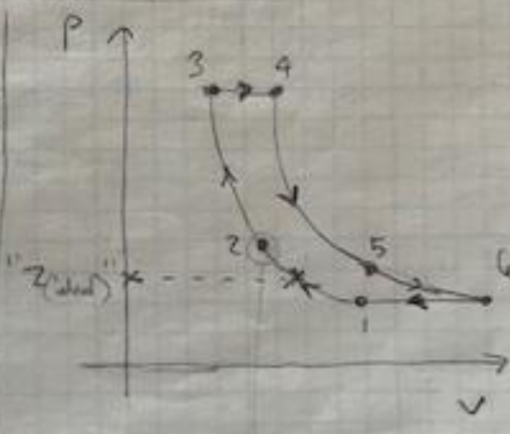
c) The overall thrust generated by the turbojet engine (in lb·force)

Diagrams:



Typical turbojet engine drawing simplified





Not isentropic since $\eta_c = 0.75$

• Since the compressor is not working isentropically the pressure input (stage 2) is slightly closer to output at stage 3.

Design Considerations:

- Although not stated in the problem, we will use constant specific heats to solve.
- Aircraft is at a higher altitude so properties of air will change accordingly @ 26,000 ft.

$\therefore T_{air}$ @ intake/stage 1 : $-33.6\,°F + 459.67\,R$

$\underline{T_1} = 426.07\,°R$

P_{air} @ 26,000 ft. = 5.22 psia = $\underline{P_1}$

$c_{p\,air}$ @ 26,000 ft ≈ 0.2401 BTU/lbm·°R

$K_{air} \approx 1.4$ // $F_{thrust} = \dfrac{\dot{m}_{air}(\vec{V}_{exit} - \vec{V}_{inlet})}{g}$

Data Given:
- Aircraft Velocity = 450 mph = V_1
- Rate of heat supplied by combustion, $\dot{Q}_{in}$ = 32,000 BTU/s
- mass flow rate; $\dot{m}$ = 100 lbm/s
- Diffuser efficiency = $\eta_D = 1.0$
- Compressor efficiency = $\eta_c = 0.75$ // Pressure ratio, $r_p = \frac{12}{1}$
- Turbine efficiency = $\eta_T = 1.0$
- Nozzle efficiency = $\eta_n = 1.0$

Procedure: must apply conservation of energy equation to solve for $\underline{T_2}$.

$$\Delta E = \Delta\left(KE + PE + U\right);$$ where "U" internal energy $= \dot{m}(u + P_v)$ & $(u+P_v) = \underline{\text{"}h\text{"}}$

$$= \dot{m}\left[(ke_1 + pe_1 + h_1) - (ke_2 + pe_2 - h_2)\right]$$

- Now, substitute $\underline{\emptyset \text{ for } \Delta E}$ since there is no change in energy, $\emptyset$ for PE_1 & PE_2 since potential energy is also $\emptyset$ and $\emptyset$ for KE_2.

$$\underline{ke_1 = \frac{(V_{intake})^2}{2}} \quad // \quad \dot{m} = 100\ lbm/s$$

$$\downarrow$$

$$V_{intake} = V_1 = 450\,\frac{mi}{hr.}\left(\frac{5280\ ft.}{1\ mile}\right)\left(\frac{1\ hr.}{3600\ sec.}\right)$$

$$\therefore \underline{V_1 = 660\ ft./s}$$

Substitute Values:

$$0 = \frac{(660\ ft/s)^2}{2}\ \boxed{+ h_1 - h_2}$$

$$h_2 - h_1 = 217{,}800\ ft^2/s^2\ ;\ \Delta q = \Delta h = \boxed{c_{p_{air}} \cdot \Delta T}$$

$$\frac{c_{p_{air}}(T_2 - T_1) = 217{,}800\ ft^2/s^2}{c_{p_{air}}} + T_1$$

$$T_2 = T_1 + \frac{217{,}800\ ft^2/s^2}{0.2401\ BTU/lbm\,R^\circ \left(\frac{25{,}037\ ft^2/s^2}{1\ BTU/lbm}\right)}$$

$$T_2 = 426.07^\circ R + 36.23^\circ R$$

$$\underline{\underline{T_2 = 462.30^\circ R}}$$

- Now that we have $\underline{T_2}$, we can find $\underline{P_2}$ since stage $1 \rightarrow 2$ is isentropic ($\underline{\eta_D = 1.0}$)

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$\left(\frac{462.30^\circ R}{426.07^\circ R}\right) = \left(\frac{P_2}{5.22\ psia}\right)^{\frac{0.4}{1.4}}$$

$$P_2 = \left(\frac{462.3}{426.07}\right)^{\frac{1.4}{0.4}} \times 5.22\ psia$$

$$\therefore P_2 = 6.946\ psia$$

Now, find P_{r_2}:

$P_{r_2} = P_{r_2}$ @ $T_2 = 462.3\ °R$

Interpolate:

$$\frac{P_{r_2} - 0.7913}{0.9182 - 0.7913} = \frac{462.3 - 460\ °R}{480\ °R - 460\ °R}$$

$$P_{r_2} = 0.8059$$

Answer to part a)

$$\Delta P = P_2 - P_1 = (6.946 - 5.22)\,psia$$

$$\boxed{\Delta P = 1.726\ psia}\ \boxed{a}$$

Stages 2→3: Non-isentropic compression

$$\eta_c = 0.75$$

$$\eta_c = \frac{\text{Ideal compressor work } (W_{is})}{\text{actual compressor work } (W_r)} = \frac{c_p(T_2 - T_{3_s})}{c_p(T_2 - T_3)}$$

Must determine T_{3_s}:

$$r_r = \frac{P_{r3_s}}{P_2} = \frac{12}{1} = \frac{P_{r3_s}}{0.8059} = P_{r_{3_s}} = 9.6707$$

Now, use $P_{r_{3_s}}$ to interpolate for T_{3_s}:

$$\frac{T_{3_s} - 920\,°R}{940 - 920\,°R} = \frac{9.6707 - 9.102}{9.834 - 9.102}$$

$$T_{3_s} = 935.54\,°R \text{ (Ideally)}$$

$$\eta_c = \frac{c_p(T_2 - T_{3_s})}{c_p(T_2 - T_3)}$$

$$0.75 = \frac{(462.30°R - 935.54°R)}{(462.30°R - T_3)}$$

$346.725 - 0.75 \cdot T_3 = -473.24$

-346.725

$-0.75 \cdot T_3 = -819.965$

$T_3 = 1093.29 \rightarrow \frac{T_?}{T_2} = \left(\frac{P_?}{P_?}\right)^{\frac{0.4}{1.4}}$

$\underline{P_3} = 141.28 \text{ psia} = \underline{P_4}$

Now, since $\eta_c = 0.75$; $\eta_T = 1.0$; $\eta_n = 1.0$

$\eta_c = 0.75 = \frac{W_{C\,isentropic}}{W_{C\,ACTUAL}} = W_{c-isentropic} = 0.75 \cdot W_{C\,ACTUAL}$

$W_{C_{isent}} = W_{T\,isent} \rightarrow \therefore \frac{0.75 \cdot W_{C\,ACTUAL} = W_T}{0.75}$

$\boxed{W_{C_{ACTUAL}} = \frac{W_T}{0.75}}$

AND $\boxed{W_T = W_N}$

States 3→4: $\cdot \dot{Q}_{in} = 32{,}000$ BTU/s (Given)

$\cdot \dot{m} = 100$ lbm/s (Given)

$$\Delta q_{in} = \frac{32{,}000 \text{ BTU/s}}{100 \text{ lbm/s}} = 320 \text{ BTU/lbm}$$

$$\Delta q_{in} = c_p (T_4 - T_3)$$

$$320 \text{ BTU/lbm} = 0.2401 \text{ BTU/lbm}\cdot R \, (T_4 - 1093.29°R)$$

$$320 \text{ BTU/lbm} = 0.2401 \cdot T_4 - 262.499 \text{ BTU/lbm}$$

$+$

$$T_4 = 2426.07°R$$

$P_{r_4 @ T_4 = 2426.07°R} = 384.75$ (interpolate)

States 4→5: Since we set $W_c = \frac{W_T}{0.75}$)

$$\therefore \cancel{c_p}(T_3 - T_2) = \frac{\cancel{c_p}(T_4 - T_5)}{0.75}$$

$$(1093.29°R - 462.30°R) = \frac{(2426.07°R - T_5)}{0.75}$$

$$473.2425 = (2426.07°R - T_5)$$

$$-2426.07°R$$

$$T_5 = 1952.83°R$$

Now, since $\eta_T = \eta_N = 1.0$

$$\therefore \frac{P_4}{P_6} = \frac{P_{r_4}}{P_{r_6}} \text{ ; where } \frac{P_4}{P_6} = r_p$$

$$(12) = \frac{384.75}{P_{r_6}} \longrightarrow P_{r_6} = 32.06$$

Find T_6 @ $P_{r_6} = 32.06$ through interpolation:

$$\frac{T_6 - 1280}{1320 - 1280} = \frac{32.06 - 30.55}{34.31 - 30.55}$$

$$\underline{\underline{T_6 = 1296.09\ °R}}$$

Now, determine velocity of exhaust at nozzle exit (stage 6) using 1st law of thermodynamics.

$$\Delta E = \dot{m}\left(\Delta q + \Delta ke + \cancel{\Delta pe}\right) + \cancel{\dot{Q}} - \cancel{\dot{W}}$$

$pe = 0$

$\Delta q = \Delta h = \underline{c_p \cdot \Delta T}$

$V_5 =$ negligable $\therefore \underline{ke_5 = 0}$

$$\dot{m}\left(c_p\left(T_6 - T_5\right)\right) + \dot{m}\left(\frac{V_6^2}{2}\right) = 0$$

$$\cancel{\dot{m}}\left(c_p\left(T_6 - T_5\right)\right) = -\cancel{\dot{m}}\left(\frac{V_6^2}{2}\right)$$

$$2\left(c_p\left(T_5 - T_6\right)\right) = \left(\frac{V_6^2}{\cancel{2}}\right) \times 2$$

$$V_6 = \sqrt{2 \cdot c_p\left(T_5 - T_6\right)}$$

$$V_6 = \sqrt{2 \cdot 0.2401\ \text{BTU}/\text{lbm}\cdot\cancel{R}\left(1952.83\,\cancel{R} - 1296.01\,\cancel{R}\right)}$$

$$= \sqrt{315.367\ \text{BTU}/\text{lbm}\left(\frac{25{,}037\ \text{ft}^2/\text{s}^2}{1\ \text{BTU}/\text{lbm}}\right)}$$

$$V_6 = \sqrt{7895832.26\ \text{ft}^{\cancel{2}}/\text{s}^{\cancel{2}}}$$

$$\boxed{V_6 = 2809.95\ \text{ft}/\text{s}}$$

b)

c) Finally, determine thrust produced by the turbojet engine:

$$F_{thrust} = \frac{\dot{m}\left(\vec{V}_{exit} - \vec{V}_{inlet}\right)}{g}$$

$$= 100\ lbm/s\left(2809.95\ ft/s - 660\ ft/s\right)$$

$$= 100\ lbm/s\left(2149.95\ ft/s\right)$$

$$= \frac{214995.24\ \frac{lb \cdot \cancel{ft}}{\cancel{s}^2}}{32.1741\ \cancel{ft/s^2}}$$

$$\boxed{F_{thrust} = 6,682.25\ lbf}$$

$\boxed{c}$