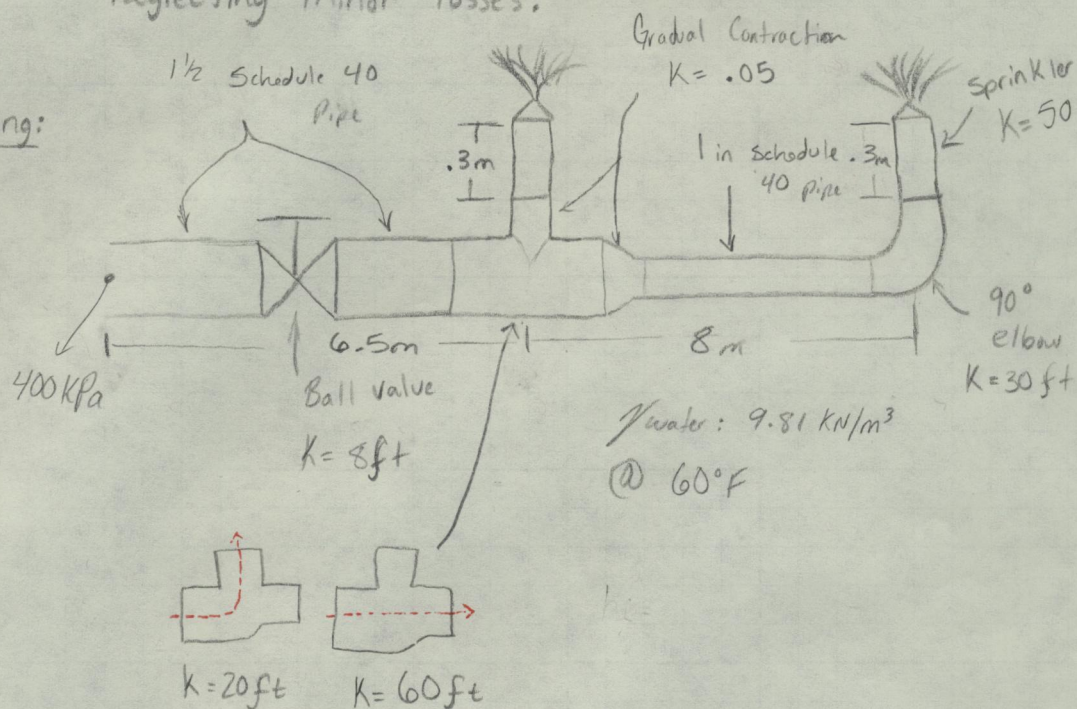


Purpose: The purpose of this problem is to determine the flow rate delivered to each sprinkler head without neglecting minor losses.

Drawing:



Sources:

Mott, R., Utener, J.A "Applied Fluids Mechanics"
7th edition, Pearson Education INC, (2015)

Design Considerations:

- Assume water temp is 15°C
- Assume K for Gate valve is same as K for ball valve
- Assume Gradual contraction $\Theta = 15^\circ - 40^\circ$ $K = .05$
- Incompressible Fluid
- Constant pressure

Data and Variables:

@ 15°C

$$\gamma_{\text{water}} = 9.81 \text{ kN/m}^3$$

$$h_L = f \frac{L}{D} \left(\frac{V^2}{2g} \right) \quad h_L = K \left(\frac{V^2}{2g} \right)$$

$$N_R = \frac{VD}{\nu_{\text{viscosity}}} \quad \frac{D}{\epsilon}$$

$$\epsilon = 4.6 \times 10^{-5} \quad \text{Viscosity} = 1.15 \times 10^{-6}$$

Sprinkler "K" = 50

90° elbow "K" = 30 ft

$$V = \frac{4Q}{\pi D^2}$$

$$f = \frac{.25}{\left[\log \left(\frac{\epsilon/D}{3.7} \right) \right]^2}$$

Valve "K" = 8 ft

Flow Through run "K" = 20 ft

Flow Through branch "K" = 60 ft

Procedure:

1. Calculate K values for pipes and modifications
2. Calculate energy losses
3. Calculate Flow rate (Q) from 1-A, A-B, and A-C
4. Calculate flow velocity

Materials:

- Water
- 2 gradual contractions
- 1 1/2 in Schedule 40 pipe
- T joint
- 1 in Schedule 40 pipe
- 90° elbow
- Ball valve
- 2 Sprinklers

@ 1 1/2 in Schedule 40 pipe

$$A = 1.314 \times 10^{-3}$$

$$ID = 40.9 \text{ mm} = .0409 \text{ m} \quad @ \text{Appendix F}$$

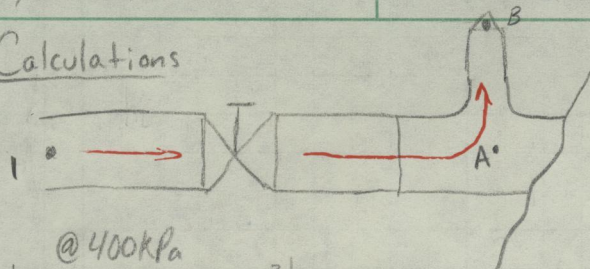
$$f_t = .020 \quad @ \text{Table 10.5}$$

@ 1 in schedule 40 pipe

$$A = 5.574 \times 10^{-4}$$

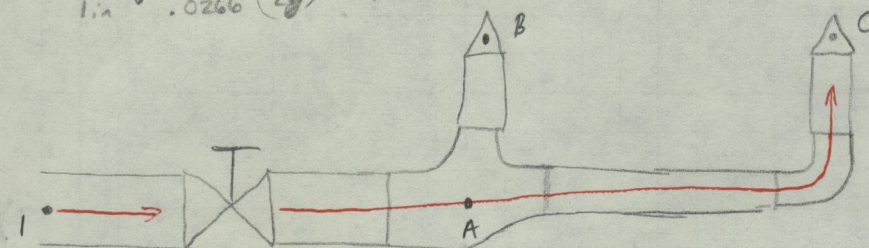
$$ID = 26.6 \text{ mm} = .0266 \text{ m} \quad @ \text{Appendix F}$$

$$f_t = .022 \quad @ \text{Table 10.5}$$

Calculations

@400kPa

- $h_{L \text{ pipe } 1/2} f \frac{6.5}{.0409} \left(\frac{V^2}{2g} \right) K = 3.18$
- $h_{L \text{ valve } 8ft} \left(\frac{V^2}{2g} \right) K = .16$
- $h_{L \text{ T-joint } 20ft} \left(\frac{V^2}{2g} \right) K = .4$
- $h_{L \text{ Sprinkler } 50} \left(\frac{V^2}{2g} \right) K = 50$
- $h_{L \text{ gradual } .05} \left(\frac{V^2}{2g} \right) K = .05$
- $h_{L \text{ pipe } 1in} f \frac{.3}{.0266} \left(\frac{V^2}{2g} \right) K = .25$



- $h_{L \text{ pipe } 1 1/2} f \frac{6.5}{.0409} \left(\frac{V^2}{2g} \right) K = 3.18$
- $h_{L \text{ valve } 8ft} \left(\frac{V^2}{2g} \right) K = .16$
- $h_{L \text{ T-joint } 60ft} \left(\frac{V^2}{2g} \right) K = 1.2$
- $h_{L \text{ grad } .05} \left(\frac{V^2}{2g} \right) K = .05$
- $h_{L \text{ pipe } 1in} f \frac{8.3}{.0266} \left(\frac{V^2}{2g} \right) K = 6.86$
- $h_{L \text{ elbow } 30ft} \left(\frac{V^2}{2g} \right) K = .66$
- $h_{L \text{ sprinkler } 50} \left(\frac{V^2}{2g} \right) K = 50$

Calculations:

$$h_{L1-B} = h_{L1-A} + h_{L A-B} \quad \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_{L1-B}$$

$$\frac{P_1}{\gamma} = h_{L1-B}$$

$$h_{L1-A} = \frac{8 L_{1-A} f_{1-A}}{\pi^2 g D^5} Q_{1-A}^2 + \frac{8}{\pi^2 g D^4} (K_{valve}) Q_{1-A}^2$$

$$h_{L1-A} = \frac{8(6.5)(.020)}{\pi^2 (9.81)(.0409)^5} Q_{1-A}^2 + \frac{8}{\pi^2 (9.81)(.0409)^4} (.16) Q_{1-A}^2$$

$$93693.69 Q_{1-A}^2 + 4723.25 Q_{1-A}^2$$

$$h_{L1-A} = 98416.94 Q_{1-A}^2$$

$$h_{L A-B} = \frac{8 L_{AB} f_{AB}}{\pi^2 g D^5} Q_{AB}^2 + \frac{8}{\pi^2 g D^4} (K_{TAB} + K_{Grnd\ cont} + K_{Sprinkler}) Q_{AB}^2$$

$$h_{L AB} = \frac{8(.3)(.022)}{\pi^2 (9.81)(.0266)^5} Q_{AB}^2 + \frac{8}{\pi^2 (9.81)(.0266)^4} (50.45) Q_{AB}^2$$

$$40930.23 Q_{AB}^2 + 8321649.49 Q_{AB}^2$$

$$h_{LAB} = 8362579.72 Q_{AB}^2$$

$$\frac{P_1}{\gamma} = h_{L1-B} \quad \frac{400 \text{ kPa}}{9.81 \text{ kN/m}^3} = 40.77 \text{ m} = 98416.94 Q_{1-A}^2 + 8362579.72 Q_{AB}^2 \quad \text{I}$$

$$h_{L1-C} = h_{L1-A} + h_{LAC} \quad \frac{P_1}{\gamma} = h_{L1-C}$$

$$h_{LAC} = \frac{8 L_{AC} f_{AC}}{\pi^2 g D^5} Q_{AC}^2 + \frac{8}{\pi^2 g D^4} (K_{TAC} + K_{elbow} + K_{Sprinkler}) Q_{AC}^2$$

$$h_{LAC} = \frac{8(8.3)(.022)}{\pi^2 (9.81)(.0266)^5} Q_{AC}^2 + \frac{8}{\pi^2 (9.81)(.0266)^4} (51.86) Q_{AC}^2$$

$$1132403 Q_{AC}^2 + 8554226.8 Q_{AC}^2$$

$$h_{LAC} = 9686629.9 Q_{AC}^2$$

$$40.77 \text{ m} = 98416.94 Q_{1-A}^2 + 9686629.9 Q_{AC}^2 \quad \text{III}$$

Calculations:

$$Q_{I-A} = Q_{AB} + Q_{AC} \quad \text{III}$$

$$\frac{8362579.72}{8362579.72} Q_{AB}^2 = \frac{9686629.9}{8362579.72} Q_{AC}^2$$

$$Q_{AB} = 1.076 Q_{AC} \quad \text{IV}$$

$$40.77m = 98416.94 (Q_{AB} + Q_{AC})^2 + 9686629.9 Q_{AC}^2$$

$$40.77m = 98416.94 (1.076 Q_{AC} + Q_{AC})^2 + 9686629.9 Q_{AC}^2$$

$$40.77m = 98416.94 (2.076 Q_{AC})^2 + 9686629.9 Q_{AC}^2$$

$$40.77m = 424157.33 Q_{AC}^2 + 9686629.9 Q_{AC}^2$$

$$40.77m = 10110787.23 Q_{AC}^2$$

$$\frac{10110787.23}{10110787.23}$$

$$Q_{AC} = \sqrt{4.03 \times 10^{-6}}$$

$$Q_{AC} = 2.01 \times 10^{-3}$$

$$Q_{AB} = 1.076 Q_{AC}$$

$$Q_{AB} = 1.076 (2.01 \times 10^{-3})$$

$$Q_{AB} = 2.16 \times 10^{-3}$$

$$Q_{I-A} = Q_{AB} + Q_{AC}$$

$$Q_{I-A} = (2.16 \times 10^{-3}) + (2.01 \times 10^{-3})$$

$$Q_{I-A} = 4.17 \times 10^{-3}$$

Comparing velocity to critical velocity.

$$V = \frac{4Q}{\pi D^2}$$

$$V_{AB} = \frac{4(2.16 \times 10^{-3})}{\pi (.0266)^2}$$

$$V_{AB} = 3.89 \text{ m/s}$$

$$V_{AC} = \frac{4(2.01 \times 10^{-3})}{\pi (.0266)^2}$$

$$V_{AC} = 3.62 \text{ m/s}$$

Summary:

Using the K values provided in the book for each connection in the piping system I was able to calculate the energy losses throughout the system. After calculating my losses I then manipulated Bernoulli's equation to find the flow rates from points I-A, A-B, and A-C.

Once I knew the flow rate of the system I was able to calculate the flow velocity and compare it to the critical velocity (3m/s). In this design the velocities were larger than the critical velocity, so to fix this issue the pipe sizes would need to be increased. Comparing the flow rate at each sprinkler, Q_{AB} was slightly higher than Q_{AC} but still acceptable. If I were to modify the system to make them the same, I would increase the energy losses in pipe A-B.

Analysis:

Analyzing the system, I conclude that the pipe sizes should be increased to adjust the flow velocity. Also the system flow rate at each sprinkler are similar enough to work effectively.