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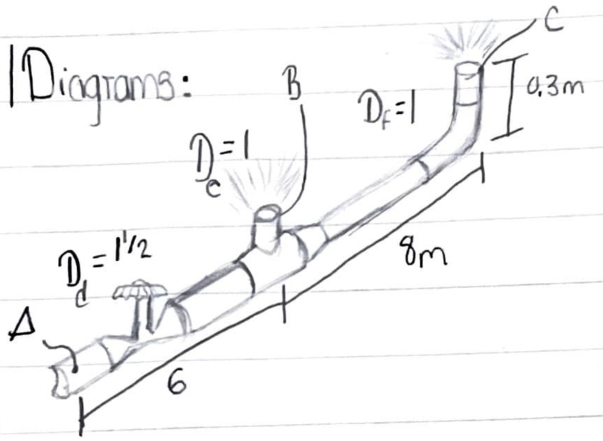
11/20/22

MET 330 Test 3

1) Purpose: Determine the flow rate through each of the sprinklers.

Drawings / Diagrams:



$P_A = 400$ kpa

Sources: Mott, R., Untener Joseph A. "Applied Fluid Mechanics," 7th edition. Pearson, 2015.

Design Considerations: Fluid is incompressible, properties are constant.

Data / Variables: From table F.1 Schedule 40

$D_d = 40.9$ mm $D_e = 26.6$ mm $D_f = 26.6$ mm

$\gamma_w = 9.78$ kN/m³

$P = 400$ kPa (gauge)

Procedure: Use bournoulli's equation for section AB as well as AC. Then use the law of conservation of mass to solve for the equation to complete the system.

Calculations: Bournoulli's:

$$P_A/\gamma + V_A^2/2g + Z_A = P_B/\gamma + V_B^2/2g + Z_B + h_{L\,A-B}$$

Apply 0's: $P_A/\gamma - Z_B = V_B^2 - V_A^2/2g + h_{L\,A-B}$

A B C D E F

Expand $h_{L\,A-B}$: $h_{L\,pipe\,a} + h_{L\,pipe\,b} + h_{L\,valve} + h_{L\,T\,Joint} + h_{L\,red} + h_{L\,sprinkler}$

A) $h_{L\,pipe\,a} = f_a\, L_a/D_a\, V_a^2/2g \rightarrow$ apply $V_a = Q_a/A_a = 4Q_a/\pi D_a^2$

$\rightarrow f_a\, L_a/D_a^5\, 8/\pi^2 g\, Q_a^2 = f_a\, 6.5m/(40.9\times10^{-3}m)\; 8/\pi^2\, 9.81\, m/s^2\, Q_a^2$

$\rightarrow \underline{h_{L\,pipe\,a} = 4,692,648.79\, f_a Q_a^2}$

B) $h_{L\,pipe\,b} = f_b\, L_b/D_b^5\, 8/\pi^2 g\, Q_b^2 = f_b\, 0.3m/(26.6\times10^{-3}m)^5\; 8/\pi^2\times9.81m/s^2\, Q_b^2$

$\rightarrow h_{L\,pipe\,b} = 1,861,376.84\, f_b Q_b^2$

C) $h_{L\,valve} = K_{valve}\, V_a^2/2g$; using $V_a = 4Q_a/\pi D_a^2$

$\rightarrow K_{valve}\, 1/2g\, 16/\pi^2 D_a^4\, Q_a^2 = 340\, f_{Ta}\, \dfrac{8}{\pi^2\times9.81m/s^2}\, \dfrac{1}{(40.9\times10^{-3}_{m})^4}\, Q_a^2$

$\rightarrow f_{Ta} = 0.020$ ~ Table 10.5 $\rightarrow h_{L\,valve} = 200,787.612\, Q_a^2$

Calculations (cont'd):

D) $h_{L\,Tee\,Joint} = k_{T\,Joint}\, V_a^2/2g = 60 f_{Ta}\, 8/\pi^2 g\; 1/D_a^4\, Q_a^2$
$= 60\,(0.02)(8/\pi^2 \times 9.81\ m/s^2 \times 1/40.9\times10^{-5}\ m^4)\, Q_a^2$

$\rightarrow h_{L\,T\,Joint} = 35{,}433.108\, Q_a^2$

E) $h_{L\,red} = k_{red}\, V_b^2/2g = k_{red}\, 8/\pi^2 g\; 1/D_b^4\, Q_b^2 = 0.14\, 8/\pi^2 \times 9.81\ m/s^2\, (26.6\times10^{-3}\ m)^4\, Q_b^2$

$\rightarrow h_{L\,red} = 23{,}105.891\, Q_b^2$

F) $h_{L\,sprinkler\,b} = k_{sprinkler}\, 8/\pi^2 g\; 1/D_b^4\, Q_b^2 = 50\, 8/\pi^2 \times 9.81\ m/s^2\; 1/(26.6\times10^{-3}\ m)^4\, Q_b^2$

$\rightarrow h_{L\,sprinkler\,b} = 8{,}252{,}103.98\, Q_b^2$

Next, apply energy losses to bernoulli's eq.

$P_A/\gamma - Z_B = V_B^2 - V_A^2/2g + 4{,}612{,}648.78\, f_a Q_a^2 + 1{,}861{,}376.839 f_b Q_b^2$
$+ 200{,}727.612\, Q_a^2 + 35{,}433.108\, Q_a^2 + 23{,}105.891\, Q_B + 8{,}252{,}103.98\, Q_b^2$

$V_B = 4Q_b/\pi D_b^2 = 4Q_b/\pi(26.6\times10^{-3}\ m)^2 = 1{,}799.479\, Q_b$

$V_A = 4Q_a/\pi D_a^2 = 4Q_a/\pi(40.9\times10^{-3}\ m)^2 = 761.138\, Q_a$

$V_B^2 - V_A^2/2g = \dfrac{165{,}042.079\, Q_b^2}{-29{,}527.576\, Q_a^2}$

Combine alike terms

Cont'd: $P_1/_\gamma - Z_B = 4{,}692{,}648.781 f_a Q_a^2 + 1{,}861{,}376.839 f_b Q_b^2$
$+ 206{,}693.144 Q_a^2 + 8{,}440{,}251.918 Q_b^2$

Left side values

$450kPa/9.789kN/m^3 = 4{,}692{,}648.781 f_a Q_a^2 + 1{,}861{,}376.839 f_b Q_b^2$
$+ 206{,}693.144 Q_a^2 + 8{,}440{,}251.958 Q_b^2$

Repeat process between A and C

$$P_A/_\gamma - Z_C = \frac{V_C^2 - V_A^2}{2g} + h_{L pipe a} + h_{L pipe c} + \cancel{h_{L valve}} + h_{L Trun} + h_{L red_c} + h_{L elbow} + h_{L sprinkler_c}$$

A B C D E

A) $h_{L pipe c} = f_c \, L_c/D_c^5 \, 8/\pi^2 g \, Q_c^2 = f_c \, 8.3m/(26.6\times10^{-3}m)^5 \, 8/\pi^2 \times 9.81 m/s^2 \, Q_c^2$

$\rightarrow h_{L pipe c} = 51{,}498{,}092.55 \, f_c Q_c^2$

B) $h_{L Trun} = K_{Trun} = 8/\pi^2 g \, 1/D_a 4 \, Q_a^2 = 20 f_{Ta} \, 8/\pi^2 g \, 1/D_a^4 \, Q_a^2 = 20 \times 0.02 \, 8/\pi^2 9.81 m/s^2$
$\times 1/(40.9\times10^{-3}m)^4 \, Q_a^2$

$\rightarrow h_{L trun} = 11{,}811.036 \, Q_a^2$

C) $h_{L redc} = K_{redc} \, 8/\pi^2 g \, 1/D_c^4 \, Q_c^2 = 0.14 \, 8/\pi^2 (9.81 m/s^2) \, 1/(26.6\times10^{-3}m)^4 \, Q_c^2$

$\rightarrow h_{L redc} = 23{,}105.891 \, Q_c^2$

D) $h_{L elbow} = K_{elbow} \, 8/\pi^2 g \, 1/D_c^4 \, Q_c^2 = 30 f_{Tc} \, 8/\pi^2 g \, 1/D_c^4 \, Q_c^2$ $\quad f_{Tc} = 0.022$

$h_{L elbow} = 30 \times 0.022 \, 8/\pi^2 \times 9.81 m/s^2 \, (1/26.6\times10^{-3}m)^4 \, Q_c^2$ (Table 10.5)

$h_{L elbow} = 108{,}977.773 \, Q_c^2$

Calculations (cont'd):

e) $h_{L\,sprinkler_C} = 8{,}252{,}103.988\,Q_c^2$

Sub into bernoulli's

$$40.599\,m = 165{,}042.079\,Q_c^2 + 29{,}527.576\,Q_a^2 + 4{,}692{,}648.781\,f_a Q_a^2 + 51{,}498{,}092.55\,f_c Q_c^2 + 200{,}787.612\,Q_a^2 + 11{,}811.036\,Q_a^2 + 23{,}105.891\,Q_c^2 + 108{,}927.173\,Q_c^2 + 8{,}252{,}103.988\,Q_c^2$$

Simplify

$$40.599\,m = 4{,}692{,}648.781\,f_a Q_a^2 + 51{,}498{,}092.55\,f_c Q_c^2 + 183{,}071.072\,Q_a^2 + 8{,}549{,}179.731\,Q_c^2$$

Remaining is the equation $Q_a = Q_b = Q_c$. To solve, use class process.

1) Assume Q_b (unknown)
2) Find Q_b using first equation
3) Find Q_c using second equation
4) Use Q_b, Q_c to find Q_a with third equation.
5) repeat if new Q_a matches old, if not, repeat.

To be done in excel (per example)

Calculations (cont'd):

$$Q_b = \left[\frac{40.599 - 4{,}692{,}648.781 f_a Q_a^2 - 206{,}693.144 Q_a^2}{1{,}861{,}376.839 f_b + 8{,}440{,}251.958}\right]^{1/2}$$

$$Q_c = \left[\frac{40.599 - 4{,}692{,}648.781 f_a Q_a^2 - 183{,}071.072 Q_a^2}{51{,}498{,}092.55 f_c + 8{,}549{,}179.731}\right]^{1/2}$$

To be completed in excel.

Summary: Assuming the data from excel, the flow rate of the first sprinkler should be slightly greater than the second, as it travels less distance to experience less loss.

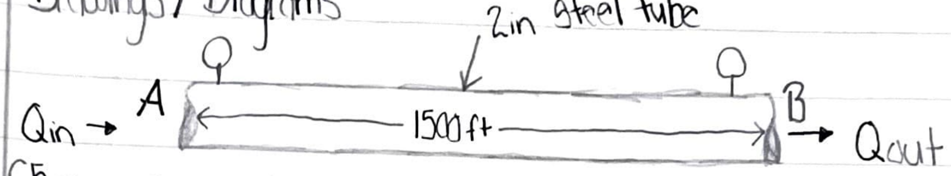
Materials: water, Sch 40 steel pipe.

Analysis: The system should not be modified, as the flow rates in the pipes have been assumed to be very similar.

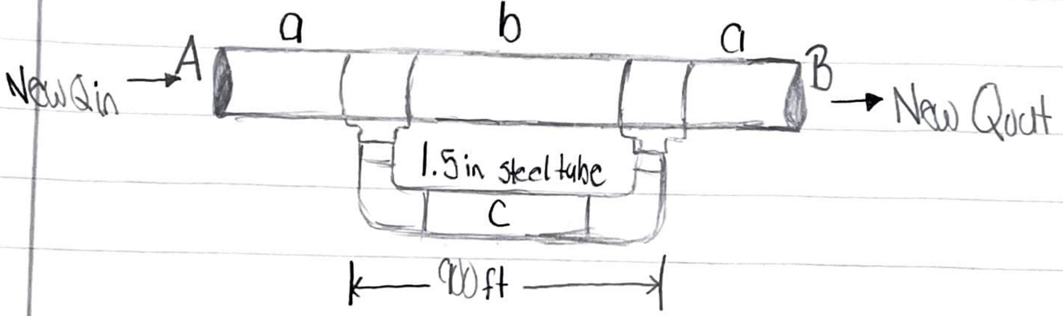
2)

Purpose: Compute pressure drop in a single pipe, then when an additonal loop is added, find the new flow rate in the system.

Drawings/Diagrams



65 gpm = 0.00409 m³/s



Sources: Mott, R., Untener. Joseph A. "Applied Fluid Mechanics", 7th edition. Pearson, 2015.

Design Considerations: Fluid is incompressible, pipe diameter in addition is reduced.

Per table G.1

Data/Variables: $D_a = 47.50$ mm $D_b = 47.50$ $D_c = 34.80$

$\gamma_w = 9.78\ kN/m^3$ $\varepsilon = 1.5 \times 10^{-6} m$

Procedure: For the pipe without the addition, use bernoulli's to find pressure drop from A to B. For the addition, the same equation will determine losses through the rest of the system. To finish the problem, the conservation of mass will be considered.

Calculations: $P_a/\gamma + V_A^2/2g + Z_A = P_B/\gamma + V_B^2/2g + Z_b + h_{L\,A-B}$

$$\frac{P_A - P_B}{\gamma} = h_{L\,A-B}$$

Integrate energy loss equation:

$$\frac{P_A - P_B}{\gamma} = f_a \frac{L_a}{L_a} \frac{V_a^2}{2g} = f_a \frac{L_a}{D_a^5} \frac{8}{\pi^2 g} Q_a^2$$

To find f_a:

$$D/\varepsilon = 47.50\times10^{-3}\,m / 1.5\times10^{-6}\,m = 31{,}666.67$$

$$Re = VD/\nu = 4Q/\pi D\nu = 4(0.00409\,m^3/s)/\pi(47.50\times10^{-3}\,m)\,8.94\times10^{-7}\,m^2/s)$$
$$= 122{,}631.57$$

Refer to moody chart: $f_a \cong 0.0171$

Substitute values into equation:

$$\frac{P_A - P_B}{\gamma} = 0.0171\ 1500(0.3048\,m/ft) / (47.50\times10^{-3}\,m)^5 \times 8/\pi^2\times9.81\,m/s^2$$
$$\times 0.00409\,m^3/s^2$$

Calculations (Cont'd)

$P_A - P_B / \gamma = 44.689\,m \;|\; P_A - P_B = 9.78\,kN/m^3 (44.689\,m)$

$P_A - P_B = 437.06\,kpa$

Apply bernoulli's to get equations for the remaining branches of pipe.

$P_A - P_B / \gamma$ for h_{LAB_b} and h_{LAB_c}

Compute energy losses.

$$h_{LAB_b} = \overset{a}{h_{La}} + \overset{b}{h_{Lb}} + \overset{c}{2h_{L\,Trun}}$$

a) $h_{La} = f_a \, L_a / D_a^5 \; 8/\pi^2 g \; Q_a^2 = f_a \; [1500ft - 900ft](0.3048\,m/ft) / 47.55 \times 10^{-3}\,m^5 \times 8/\pi^2 \times 9.81\,m/s^2 \; Q_a^2$

$h_{La} = 62,491,276.95 \, f_a \, Q_a^2$

b) $h_{Lb} = f_b \, L_b / D_b^5 \; 8/\pi^2 g \; Q_b^2 = f_b \; 900ft(0.3048\,m/ft) / (47.50 \times 10^{-3}\,m)^5$

$h_{Lb} = 93,736,915.42 \, f_b \, Q_b^2$

c) $h_{L\,Trun} = k_{trun} \; 8/\pi^2 g \; 1/D_a^4 \; Q_a^2 = 20 f_{Ta} \; 8/\pi^2 g \; 1/D_a^4 \; Q_a^2$

$D_a/\varepsilon = 31,666.67$ Moody: $f_{Ta} \cong 0.0095$

Calculations cont'd: $= 20(0.0095)\left(\frac{8}{\pi^2(9.81\,m/s^2)}\right)\left(\frac{1}{(47.50\times10^{-3}m)^4}\right)Q_a^2$

$h_{L\,Trun} = 3{,}083.90\,Q_a^2$

Place results in bernoulli's:

① $44.689\,m = 62{,}491{,}276.956\,Q_a^2 + 93{,}736{,}915.42\,f_b Q_b^2 + 3083.90\,Q_a^2$

a b c d e

For c; $h_{L\,AB_c} = h_{L_a} + h_{L_c} + 2h_{L\,Tbranch} + 2h_{L\,elbow} + h_{L\,red} + h_{L\,exp}$

a) $h_{L_c} = f_c \frac{L_c}{D_c^5}\frac{8}{\pi^2 g}Q_c^2 = f_c\, 900(0.3048\,m/ft)/(34.80\times10^{-3}m)^5 \times \frac{8}{\pi^2\times9.81\,m/s^2}Q_c^2$

$h_{Lc} = 444{,}101{,}752.10\,f_c Q_c^2$

b) $h_{L\,tbranch} = K_{Tbranch}\frac{8}{\pi^2 g}\frac{1}{D_a^4}Q_a^2 = 60\,f_{Ta}\frac{8}{\pi^2\times g}\frac{1}{D_a^4}Q_a^2$

$h_{L\,tbranch} = 60\times0.0095\,\frac{8}{\pi^2\times9.81\,m/s^2}\,\frac{1}{(47.50\times10^{-3}m)^4}Q_a^2$

$h_{L\,Tbranch} = 9{,}251.70\,Q_a^2$

c) $h_{L\,elbow} = K_{elbow}\frac{8}{\pi^2 g}\frac{1}{D_c^4}Q_c^2 = 30\,f_{Tc}\frac{8}{\pi^2 g}\frac{1}{D_c^4}Q_c^2$

$D_c/\varepsilon = 34.80\times10^{-3}m / 1.5\times10^{-6}m = 23{,}200$ | Moody's: $f_{Tc} \cong 0.01$

$= 30(0.011)\frac{8}{\pi^2\times9.81}\frac{1}{(34.80\times10^{-3}m)^4}Q_c^2$

$h_{L\,elbow} = 18{,}591.66\,Q_c^2$

d) $h_{L red} = k_{red} \frac{8}{\pi^2 g} \frac{1}{D_c^4} Q_c^2 = 0.11 \frac{8}{\pi^2 \times 9.81 \, m/s^2} \frac{1}{(34.80 \times 10^{-3} \, m)^4} Q_c^2$

$h_{L red} = 6{,}197.22 Q_c^2$

e) $h_{L exp} = k_{exp} \frac{8}{\pi^2 g} \frac{1}{D_c^4} Q_c^2 = 0.46 \frac{8}{\pi^2 \times 9.81 \, m/s^2} \frac{1}{(34.80 \times 10^{-3} m)^4} Q_c^2$

$h_{L exp} = 25{,}915.65 Q_c^2$

Substitute into bournoulli's for c

$$44.689 \, m = 62{,}491{,}276.95 f_a Q_a^2 + 444{,}101{,}752.16 f_c Q_c^2 + 9{,}251.70 Q_a^2 + 18{,}591.66 Q_c^2 + 6{,}197.22 Q_c^2 + 25{,}915.65 Q_c^2$$

② $$44.689 m = 62{,}491{,}276.95 f_a Q_a^2 + 444{,}101{,}752.10 f_c Q_c^2 + 9{,}251.70 Q_a^2 + 50{,}704.53 Q_c^2$$

③ $$Q_a = Q_b + Q_c$$

To complete system,

1) assume Q_a
2) Solve for Q_b (f_b dependent) ①
3) Solve for Q_c (f_c dependent) ②
4) Find Q_a using Q_b and Q_c ③
5) If Q_a matches, stop. If it matches, continue.

} To be done in excel.

calculations (cont'd)

$$Q_b = \left[\frac{44.689 - 62{,}491{,}276.95 f_a Q_a^2 - 3{,}083.90 Q_a^2}{93{,}736{,}915.42 f_b}\right]^{1/2}$$

$$Q_c = \left[\frac{44.689 - 62{,}491{,}276.95 f_a Q_a^2 - 9{,}251.70 Q_a^2}{444{,}101{,}752.10 f_c + 50{,}704.53}\right]^{1/2}$$

Summary: The pressure drop across the single pipe is 437.06 kpa. It is assumed that the flow rate with an addition so slight would incorporate a small increase.

Materials: Water, Sch 40 Steel pipe

Analysis: It is assumed that after modifying the system, the flow rate ncrease would very small, and a larger pipe would be needed for a larger difference.