HW2.1 Derive equations: 9, 11, 12, 34, 35, and 41 Ch: 5#4, 28, 29

9) $T(r) = \frac{g b^2}{4K}\left[1-\left(\frac{r}{b}\right)^2\right] + T_w$

heat flux anywhere: $q(r) = -K \frac{dT(r)}{dr}$

$$q(r) = \frac{g r}{2}$$

let $r = b$

11) $q(b) = \frac{g b}{2}$ W/m^2

12) $T_0 = \frac{g b^2}{4K} + T_w$

34) $\frac{1}{r}\frac{d}{dr}\left[r \frac{dT(r)}{dr}\right] + \frac{g}{k} = 0$ in $0 < r < b$

$\frac{dT(r)}{dr} = 0$ at $r = 0$

$-K \frac{dT(r)}{dr} = h_\infty [T(r) - T_\infty]$ at $r = b$

$$T(r) = \frac{g b^2}{4K}\left[1-\left(\frac{r}{b}\right)^2\right] + \frac{g b}{2h_\infty} + T_\infty$$

35) $T(r) = \frac{g b^2}{4K}\left[1-\left(\frac{r}{b}\right)^2\right] + \frac{g b}{2h_\infty} + T_\infty$

at $h_\infty \to \infty$

$$T(r) = \frac{g b^2}{4K}\left[1-\left(\frac{r}{b}\right)^2\right] + T_\infty$$

4.1) $\frac{1}{r^2}\frac{d}{dr}\left[r^2 \frac{dT(r)}{dr}\right] + \frac{g}{k} = 0$ in $0 < r < b$

integrate

$$r^2 \frac{dT}{dr} = -\frac{g}{3k} r^3 + C_1 \qquad C_1 = 0$$

$$T(r) = -\frac{g}{6k} r^2 + C_2$$

$$-\frac{gb}{3} + h_\infty\left(-\frac{gb^2}{6k} + C_2\right) = hT_\infty$$

$$C_2 = \frac{gb}{3h_\infty} + \frac{gb^2}{6k} + T_\infty$$

$$T(r) = \frac{bg^2}{6k}\left[1 - \left(\frac{r}{b}\right)^2\right] + \frac{bg}{3h_\infty} + T_\infty$$

5.4) Given: $L = .1m$ $k = 40\ w/m(°)$ $g_0 = 10^6\ w/m^3$ $T_\infty = 150°C$

Find: Plot temperature profile at $h = 200, 500, 500\ w/m^2(°)$

Solution:

$$\frac{d^2T}{dx^2} + \frac{g}{k} = 0$$

$$\frac{d^2t}{dx^2} = -\frac{g}{k}$$

integrate:

$$\frac{dt}{dx} = -\frac{g}{k} + C_1$$

$$t = -\frac{g}{k}\frac{x^2}{2} + C_1 x + C_2 \qquad \begin{matrix} x = 0 \\ x = L \end{matrix}$$

$$-k_A \frac{dt}{dx}\Big|_{x=L} = -\frac{g}{k} L$$

$$-\frac{h}{k}(T_2 - T_\infty) = -\frac{g}{k} 1$$

$$t(L) = T_\infty + \frac{g}{h} \cdot L$$

$$C_2 = T_\infty + \frac{g}{h} 1 + \frac{g}{2k} 1^2 \qquad t = T_\infty + \frac{g}{h} 1 + \frac{g}{2k} L^2$$

$x = 0 \quad t = t_{max}$

$T_{max} = T_a + \frac{q}{h} L + \frac{q}{2k} L^2$

at middle of wall

$x = \frac{L}{2}$

$t = T_a + \frac{q}{h} + \frac{q}{2k}\left(\frac{3}{4} L^2\right)$

$T_w = T_a + \frac{q}{h} L$

$T_{max} = T_a + \frac{q}{h} L + \frac{q}{2k} L^2$

$T_m = T_a + \frac{q}{h} + \frac{3q}{8k} L^2$

$T_w = T_a + \frac{q}{h} L$

5.26) Given: $I = 500\ A$ $D = 5mm$ $R = 5\times10^{-4}\ \Omega/m$ $T_\infty = 0°C$

$h = 50 W/m \cdot C$ $k = 60 W/m \cdot C$

Find: T_{center} and $T_{surface}$

Solution:

$$g = \frac{RI^2}{(\pi D^2/4)\cdot 1}\ W/m^3 = \frac{(5\times10^{-4}\ \Omega/m)(500A)}{\frac{\pi D^2}{4}}$$

$$g = 1273239.545\ W/m^3$$

$$T(r) = \frac{g b^2}{4k}\left[1 - \frac{r}{b}^2\right] + T_w$$

$$T(r) = \frac{1273239.545\ W/m^3(.0025)^2}{4(60 W/m \cdot C)}\left[1 - \frac{r}{(.0025)}^2\right] + 0$$