

HW 31 Ch. 2: 15, 26 Ch. 6: 10, 14

2.15) Given: Grp. $k = 0.6 \text{ W/m}\cdot^\circ\text{C}$ $\rho = 1100 \text{ kg/m}^3$ $C = 4200 \text{ J/kg}\cdot^\circ\text{C}$

$$d = 1 \text{ cm}$$

$$h = 20 \text{ W/m}^2\cdot^\circ\text{C}$$

$$T(t) = 10^\circ\text{C}$$

$$T_i = 20^\circ\text{C}$$

$$T_\infty = 5^\circ\text{C}$$

Find: $t = ?$

Solution:

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-mt}$$

$$\frac{V}{A} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3}$$

$$r = \frac{d}{2} = \frac{0.01 \text{ m}}{2} = 0.005 \text{ m}$$

$$m = \frac{hA}{\rho C V} = \frac{20 \text{ W/m}^2\cdot^\circ\text{C}}{(1100 \text{ kg/m}^3)(4200 \text{ J/kg}\cdot^\circ\text{C})(0.005 \text{ m})} = 0.002597 \text{ s}^{-1} \text{ or } 1/385 \text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = \frac{10^\circ\text{C} - 5^\circ\text{C}}{20^\circ\text{C} - 5^\circ\text{C}} = e^{(-t/385)}$$

$$\frac{5}{15} = e^{(-t/385)}$$

$$3 = e^{(t/385)}$$

$$t = \ln(3) 385$$

$$t = 422.9 \text{ sec}$$

or

$$t \approx 7 \text{ min}$$

2.26)

Given:

$$T_i = 200^\circ\text{C}$$

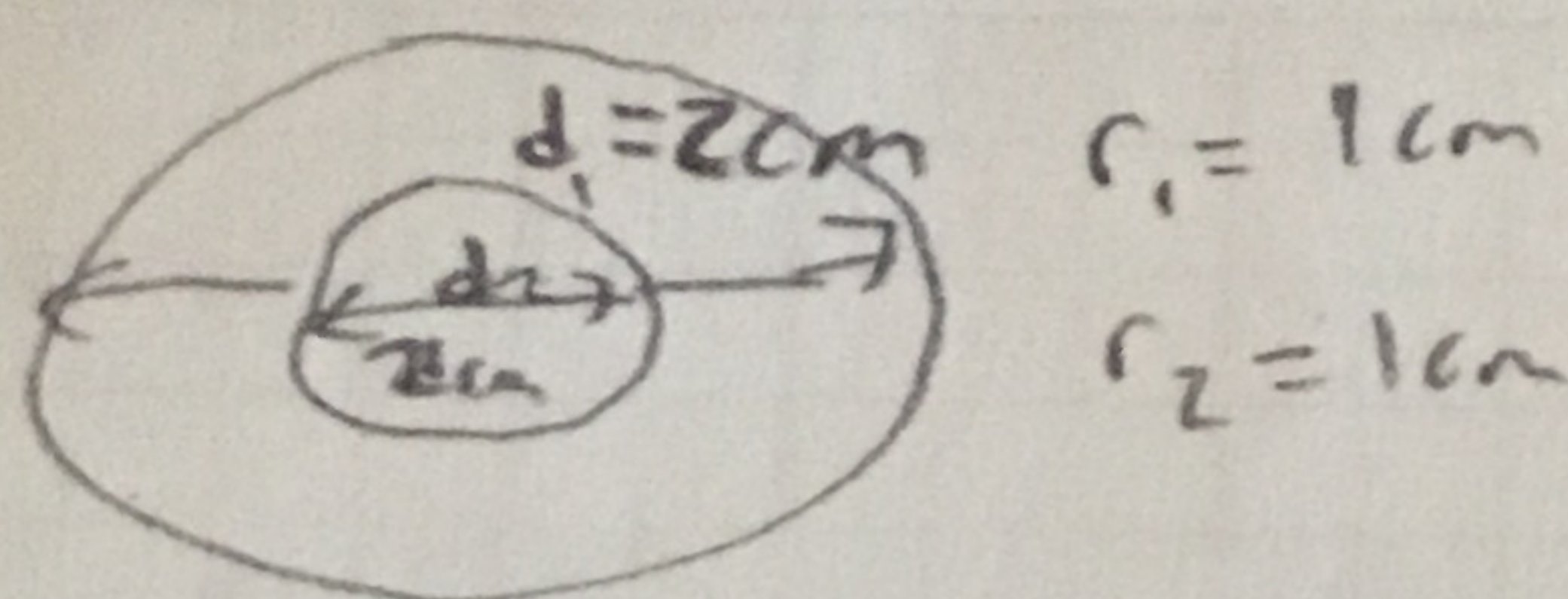
$$h = 10 \text{ W/m}^2\text{C}$$

$$\text{Material Clay: } K = 1.28 \text{ W/mC} \quad \rho = 1458 \text{ kg/m}^3$$

$$T_\infty = 25^\circ\text{C}$$

$$t = 10 \text{ min} \approx 600 \text{ sec}$$

$$c = 880 \text{ J/kgC}$$



Find: $T(r) = ?$

Solution:

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-mt}$$

$$\frac{V}{A} = \frac{\frac{4}{3}\pi R^3}{2\pi R^2} = \frac{\frac{4}{3}\pi (0.1 \text{ m})^3}{2\pi (0.1 \text{ m})^2} = 0.0667$$

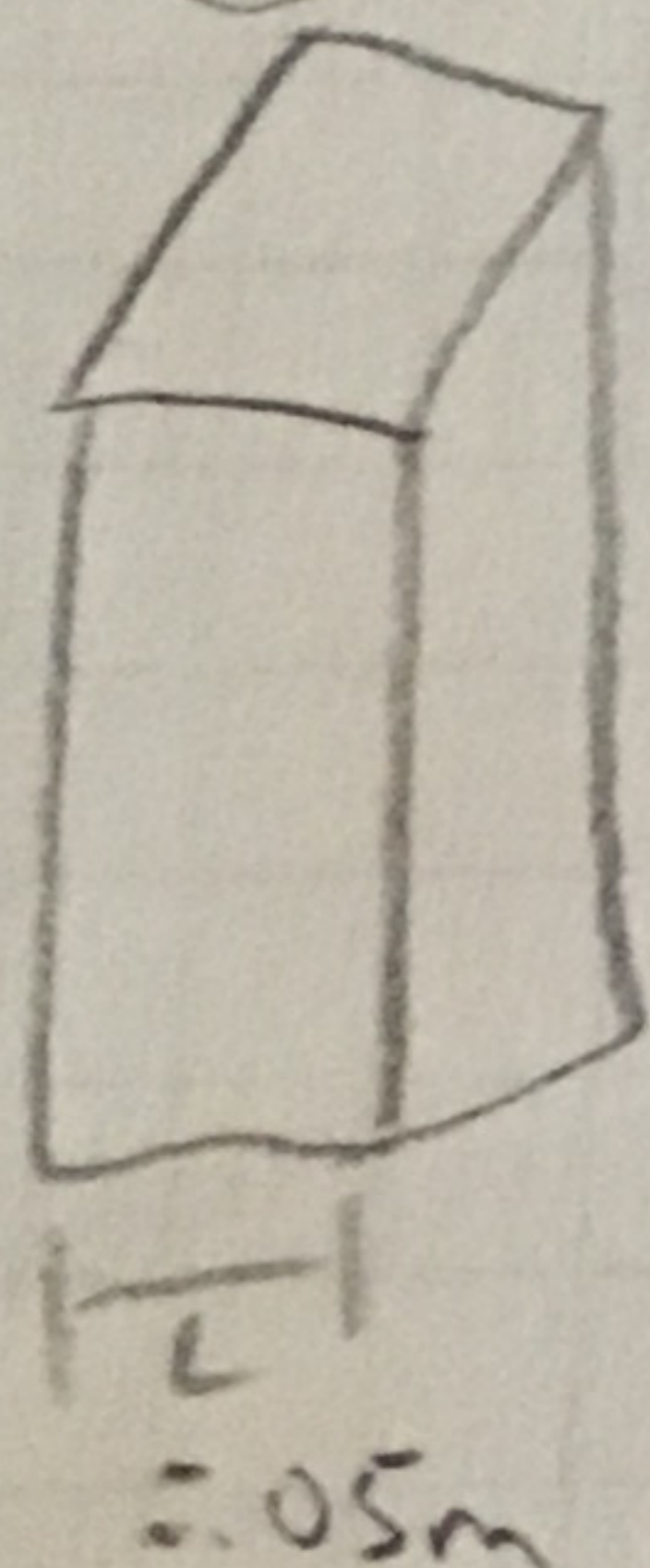
$$m = \frac{hA}{\rho V c} = \frac{h}{\rho c \frac{V}{A}} = \frac{10 \text{ W/m}^2\text{C}}{(1458 \text{ kg/m}^3)(880 \text{ J/kgC})(0.0667)} = \frac{1}{855.8 \text{ s}}$$

$$\frac{T(t) - 25^\circ\text{C}}{200^\circ\text{C} - 25^\circ\text{C}} = e^{(-t/855.8)}$$

$$T(t) - 25^\circ\text{C} = e^{(-600 \text{ sec}/855.8)} \cdot 175^\circ\text{C}$$

$$T(t) - 25^\circ\text{C} = 86.8^\circ\text{C}$$

$$T(t) = 111.8^\circ\text{C}$$



$$\frac{V}{A} = \frac{A \cdot L}{2A} = \frac{L}{2} = \frac{0.05 \text{ m}}{2} = 0.025$$

$$m = \frac{hA}{\rho V c} = \frac{h}{\rho c \frac{V}{A}} = \frac{10 \text{ W/m}^2\text{C}}{(1458 \text{ kg/m}^3)(880 \text{ J/kgC})(0.025)} = \frac{1}{3207.6 \text{ s}}$$

$$\frac{T(t) - 25^\circ\text{C}}{200^\circ\text{C} - 25^\circ\text{C}} = e^{(-600 \text{ sec}/3207.6)}$$

$$T(t) - 25^\circ\text{C} = e^{(-600/3207.6)} \cdot 175^\circ\text{C}$$

$$T(t) - 25^\circ\text{C} = 145.14^\circ\text{C}$$

$$T(t) = 170.1^\circ\text{C}$$

6.10) Given: wood wall $\alpha = .82 \times 10^{-7} \text{ m}^2/\text{s}$ and $K = .15 \text{ W/m}\cdot\text{C}$ $T_i = 20^\circ\text{C}$

$$T_\infty = 500^\circ\text{C} \quad T(t) = 40^\circ\text{C} \quad h = 45 \text{ W/m}^2\cdot\text{C}$$

$$x = 0$$

Find: $t = ?$

Solution:

$$\frac{h\sqrt{\alpha t}}{K} = \frac{45 \text{ W/m}^2\cdot\text{C} \sqrt{.82 \times 10^{-7} \text{ m}^2/\text{s}} (t)}{.15 \text{ W/m}\cdot\text{C}}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = \text{erf} \frac{x}{2\sqrt{\alpha t}} - \exp\left(-\frac{hx}{K} + \frac{h^2 \alpha t}{K^2}\right) \text{erf}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{K}\right)$$

$$\frac{400 - 500}{20 - 500} = \text{erf}(0) - \exp(0) + \frac{45^2 (.82 \times 10^{-7}) (t)}{.15^2} \text{erf}(0) + \frac{(45 .82 \times 10^{-7}) (t)}{.15^2}$$

$$.208 = \exp\left(0 + \frac{.000166 t}{.0225}\right) \text{erf}\left(0 + \frac{45 \sqrt{.82 \times 10^{-7}} (t)}{.0225}\right)$$

$$t = 597.6 \text{ sec}$$

6.14) Given: marble plate: $\alpha = 1.3 \times 10^{-6} \text{ m}^2/\text{s}$ $K = 380 \text{ W/m}\cdot\text{C}$ $L = 3 \text{ cm}$
 $T_i = 130^\circ\text{C}$ $T_\infty = 30^\circ\text{C}$ $t = 2 \text{ min}$ $x = 0.015 \text{ m}$

Find: $T(x) = ?$

Solution: $\rightarrow$ center line

$$\frac{x}{L} = \frac{0.015 \text{ m}}{0.03 \text{ m}} = 0.5$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = 0.5$$

$$T = T_\infty + T_0 - T_\infty (0.5)$$

$$T = 30^\circ\text{C} + (130^\circ\text{C} - 30^\circ\text{C}) (0.5)$$

$$T = 80^\circ\text{C}$$