HW 2.2 Ch.5 #3, 22 and 29, 30

5.3) Given: $L = .1m$, $x=0$, $x = L$, $h_\infty = 200\ W/m^2 \cdot C$, $k = 10\ W/m \cdot C$; $g = 10^6\ W/m^3$
$T = 150°C$

Find: $T(0)$, $T(L)$

Solution:

At $x=0$ $T(0) = \frac{gL^2}{2k} + \frac{gL}{h_\infty} + T_\infty$

$= \frac{(10^6 W/m^3)(.1m)^2}{2(10 W/m \cdot C)} + \frac{(10^6 W/m^3)(.1m)}{200 W/m^2 C} + 150°C$

$T(0) = 1150°C$

At $x = L$ $T(L) = \frac{gL}{h_\infty} + T_\infty$

$= \frac{(10^6 W/m^3)(.1m)}{200 W/m^2 C}$

$T(L) = 650°C$

5.22) Given: $L = 10cm$ $T_L = 200°C$ $k = 20 W/m \cdot C$ $g = 2000 W/m^3$
$x = 0$ $x = L$

Find: $T(0) = ?$

Solution:

At $x=0$ $T(0) = \frac{gL^2}{2k}\left[1 - \left(\frac{x}{L}\right)^2\right] + T_\infty$

$= \frac{(2000 W/m^3)(.1m)^2}{2(20 W/m \cdot C)}\left[1 - \left(\frac{x}{.1m}\right)^2\right] + 200°C$

$= .5(1 - 100x^2) + 200$

$= .5(1 - 100(0)^2) + 200$

$= .5(1-0) + 200$

$T(0) = 200.5°C$

5.2a) Given: $\frac{d^2T(x)}{dx^2} + \frac{g}{k} = 0$ in $0<x<L$

$\frac{dT(x)}{dx} = 0$ at $x=0$

$-k\frac{dT(x)}{dx} = h_\infty[T(x)-T_\infty]$ at $x=L$

Find: a) finite-difference formulation of this heat conduction problem by dividing the region $0<x<L$ into 5 equal parts

Subdivision: $\Delta X = \frac{L}{m} = \frac{.05}{5} = .01m$

internal nodes, $(m=2-5)$ 1 2 3 4 5 6 $T_{m-1} - 2T_m + T_{m+1} + \frac{(.01)^2(10^6)}{10} = 0$

$= T_{m-1} - 2T_m + T_{m+1} = -10$ for $m=2,3,4,5$

at $x=0$, $2T_2 - 2T_1 + \frac{(.01)^2(10^6)}{10} = 0$

$= 2T_2 - 2T_1 = -10$ for $m=1$

at $x=L^-$, $2T_5 - (2 + \frac{2\cdot .01\cdot 200}{10})T_6 + \frac{(.01)^2\cdot 10^6}{10} + \frac{2\cdot .01\cdot 200}{10}\cdot 100 = 0$

$= 2T_5 - 2.4T_6 = -50$ for $m=6$

matrix:

$$\begin{bmatrix} -2 & 2 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 2 & -2.4 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} -10 \\ -10 \\ -10 \\ -10 \\ -10 \\ -10 \end{bmatrix}$$

b,c)

$$= \begin{matrix} 1.000 & -2.000 & 2.000 & -10.00 \\ 1.000 & -2.000 & 1.000 & -10.00 \\ 1.000 & -2.000 & 1.000 & -10.00 \\ 1.000 & -2.000 & 1.000 & -10.00 \\ 1.000 & -2.000 & 1.000 & -10.00 \\ 2.00 & -2.400 & .000 & -50.00 \end{matrix}$$

Node	Temperature finite difference	exact
1	475.00	475.00
2	470.00	470.00
3	455.00	455.00
4	430.00	430.00
5	395.00	395.00
6	350.00	350.00

5.30) Given: $\frac{d^2T(x)}{dx^2} + \frac{g}{k} = 0$ in $0 < x < L$

$T(x) = T_1$ at $x = 0$

$-k \frac{dT(x)}{d(x)} = h_\infty[T(x) - T_\infty]$ at $x = L$

Find: a) four equal parts

b) 8 equal parts

$\Delta x = \frac{L}{M} = \frac{.1}{4} = .025$

1 2 3 4

for $m = 2$ to 4, $T_{m-1} 2T_n + T_{n+1} + \frac{(.025)^2 (2\times10^6)}{20}$

for $m = 1$, $2T_2 - 2T_1 + \frac{(.025^2)(2\times10^6)}{20}$