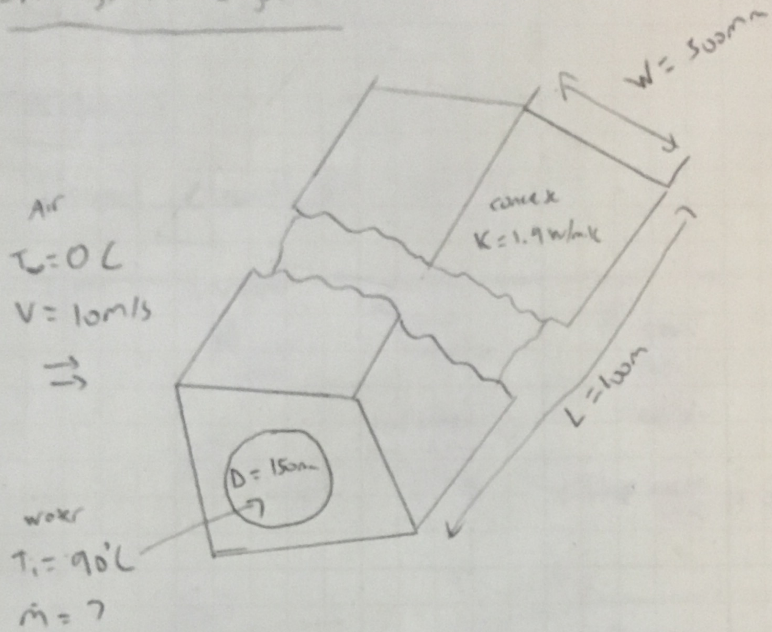
(1) Purpose

Calculate the heat loss and the mass flow rate of water, if the difference between the inlet and outlet temperatures of water flowing through pipe is 5C

Drawings and Diagrams



Sources

Bayazitoglu, Y., Ozisik, N., "A textbook Of Heat transfer Fundamentals", Begell House, 20

Design considerations

- Steady state
- Constant properties
- Steel pipe has negligible resistance
- 2 D heat transfer.

Data and variables

- See Drawing
- $T_i - T_o = 5°C$

$90°C - T_o = 5°C$

$T_o = 85°C$

Table B-1
Properties of air at 0°C

$T \approx 280K$ (Interpolate)

$K = 25.34 \times 10^{-3}$ W/mK

$V = 14.522 \times 10^{-6}$ m²/sec

Pr = .710

Table B-2
Properties of water at 90°C

T = 90°C (Interpolate)

K = .674 W/m·K

$V = .329 \times 10^{-6}$ m²/s

Pr = 1.98

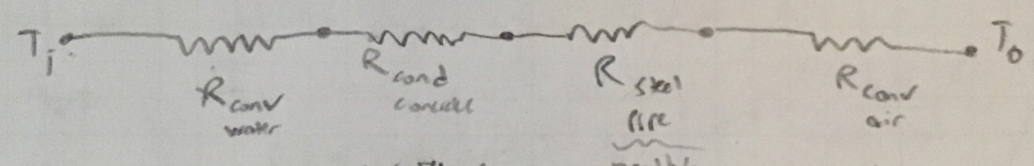
CP = 4207 J/kg·K

Procedure

Use thermal circuit to obtain equation for heat transfer. Use Nusselt correlation to get convective coefficients. If more fluid properties required for Nusselt correlation then iteration is needed.. Then solve for Q after calculating thermal resistances and then solve for mass flow rate.

Calculations

Thermal circuit for figure is:



$$\frac{1}{\pi D L h_{water}} \qquad \frac{\ln(.54w/R)}{2\pi k L} \qquad \frac{1}{4wLh_{air}}$$

(Shape factor case 11 table 4.2 (p.127))

and:

$$Q = \frac{\Delta T}{\Sigma R} = \frac{T_i - T_o}{R_{conv_{water}} + R_{cond_{con}} + R_{conv_{air}}}$$

$$Q = \frac{T_i - T_o}{\frac{1}{\pi D L h_{water}} + \frac{\ln(.54\,w/R)}{2\pi k L} + \frac{1}{4wLh_{air}}}$$

Plug values:

$$Q = \frac{90^\circ C - 85^\circ C}{\frac{1}{\pi(.15m)(100m)(h_{water})} + \frac{\ln\left(.54\left(\frac{.3}{.075}\right)\right)}{2\pi(1.4\ w/m{\cdot}K)(100m)} + \frac{1}{4(.3m)(100m)(h_{air})}}$$

Find: h_{air} and h_{water}

$$h_{air_\infty} = \frac{Nu \cdot k}{L}$$, use Nusselt correlations and fluid properties

$$R_{conv_{air}} = \frac{1}{4wL\,h_{\infty_{air}}} \quad \text{at Concrete Surface } L \longrightarrow h_{\infty} = \frac{Nu_0\,K}{L}$$

Properties of air at surface
Table B-1 interpolate
$T \approx 280K = 6°C$
$K = 25.34 \times 10^{-3}\ W/m{\cdot}K$
$\nu = 14.622 \times 10^{-6}\ m^2/sec$
$P_r = .710$

$$Re = \frac{vL}{\nu} = \frac{(10\ m/s)(100\ m)}{14.622 \times 10^{-6}\ m^2/sec}$$

$$Re = 67467278.4$$

$$Nu = .3 + \frac{.62\,Re_0^{\frac{1}{2}}\,Pr^{\frac{1}{3}}}{\left(1 + (.4/Pr)^{\frac{2}{3}}\right)^{\frac{1}{4}}} \left[1 + \left(\frac{Re_0}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$

$$Nu = .3 + \frac{.62\,(67467278.4)^{\frac{1}{2}}(.710)^{\frac{1}{3}}}{\left(1 + (.4/.710)^{\frac{2}{3}}\right)^{\frac{1}{4}}} \left[1 + \left(\frac{67467278.4}{282{,}000}\right)^{\frac{5}{8}}\right]^{4/5}$$

$$Nu = .3 + \frac{.62\,(8213.8)(.892)}{(1 + .682)^{\frac{1}{4}}} \left[1 + 30.67\right]^{\frac{4}{5}}$$

$$Nu = .3 + \frac{4542.6}{1.139}(15.87)$$

$$Nu = 63293.03612$$

$$h_{\infty} = \frac{Nu \cdot K}{L}$$

$$h_{\infty} = \frac{(63293)(25.34 \times 10^{-3}\ W/m{\cdot}K)}{100\ m}$$

$$h_{\infty} = 16.04\ W/m^2{\cdot}K$$

Now Find: h_{water}

Need to iterate to find h_{water}

1) Go to table p.25 and assume a h_{water}

2) Compute Q

3) Compute $\dot{m} = \frac{Q\,L}{C_v(\Delta T)}$

4) Go back and use $\dot{m}$ to find $h_{water} = \frac{Nu_D\,k}{D}$

4.1) $Re = \frac{4\dot{m}}{\pi D \mu}$

4.2) Pr from table B-2 for water at 90°C

4.3) Calculate Nu_D

4.4) Calculate h_{water}

5) Compare new h_{water} to old h_{water}.

from table p.25

we shall use $h_{water} = 39\ W/m^2{\cdot}C$ (forced convection case: L = .1m)

$$Q = \frac{90°C - 85°C}{\frac{1}{\pi(.15m)(100m)(39\,W/m^2 C)} + \frac{\ln\left(.54 - \left(\frac{.3}{.075}\right)\right)}{2\pi(1.9\,W/m K)(100m)} + \frac{1}{4(.3m)(100m)(16.04\,W/m^2.K}}$$

$$Q = \frac{5.°C}{.000544\,K/W + .000876\,K/W + .0005195\,K/W}$$

$$Q = \frac{5°C}{.00194}$$

$$Q = 2577.9\ W/m$$

$$\dot{m} = \frac{Q\,L}{C_p\,\Delta T} = \frac{(2577.9\,W/m)(100m)}{(4207\,J/kg{\cdot}K)(5°C)}$$

$$\dot{m} = 12.25\,kg/s$$

Go back to find h_{water}

$$Re = \frac{4(12.25\ kg/s)}{\pi(.15m)(.329\times10^{-6}m^2/s)}$$

$Re = 316129032.3$, $Pr = 1.98$ from (table B.2)

$$Nu_D = .3 + \frac{.62 Re^{1/2} Pr^{1/3}}{[1+(.4/Pr)^{2/3}]^{1/4}}\left[1+\left(\frac{Re}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$Nu_D = .3 + \frac{.62(316129032.3)^{1/2}(1.98)^{1/3}}{[1+(.4/1.98)^{2/3})^{1/4}}\left[1+\left[\frac{316129032.3}{282{,}000}\right]^{5/8}\right]^{4/5}$$

$$Nu_D = .3 + \frac{.62(681.2)(1.256)}{(1+.344)^{1/4}}\left[1+80.53\right]^{4/5}$$

$$Nu = .3 + \frac{530.5}{1.076}(33.8)$$

$$Nu_D = 16664.7$$

$$h_{water} = \frac{Nu\,k}{D} = \frac{(16664.7)(.674\ W/m\cdot K)}{.15m}$$

$$h_{water\ new} = 74880\ W/m^2\cdot K$$

Summary:

$Q = 2577.9\ w/m$
$\dot{m} = 12.25\ kg/s$ } from assumed $h_{water} = 39\ W/m^2\cdot C$ (p.25 table 1.2)

Our new $h_{water} = 74880\ w/m^2\cdot K$

The value for the h_{water} are very inaccurate and I would think again and pick a value from the table 1.2 p.25 but there aren't that many to fit this case.

the next closest value I would assume for h_{water} is

$h = 10,500\ W/m^2 C$ (forced convection: water at 1 kg/s inside 2.5 ID tube)

because that description / type of flow fits my case.

Materials

- Concrete
- Steel pipe
- water

Analysis

The iteration process for finding "h", heat transfer coefficient limits us with only Table 1.2 p.25 to assume a "h" value for and compute Q, heat loss and $\dot{m}$, mass flow rate. In reality we would want to use a better table for average h_{water}.

The h_{water} value is generally large looking at this table 1.2.