P-21

Data + Variables

$V_{aircraft} = 900 km/h$

$P_1 = 40 kPa$

$T_1 = -35°C = 238.15 K$

$D_{engine\ inlet} = 1.6 m$

$T_{turbine\ inlet} = 950°C = 1223.15 K$

$C_p = 0.240\ Btu/lbm\cdot°F$

$\eta_c = 80\%$

$\eta_T = 90\%$

$W_T = 500\ kJ/kg$

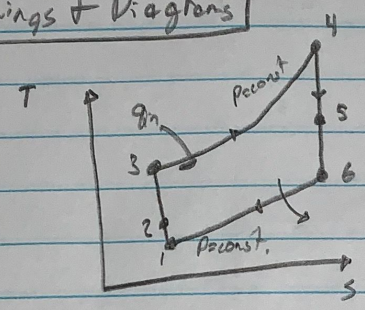
$k = 1.4$

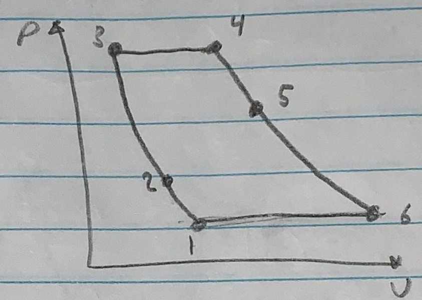
P-2|

Purpose|

The purpose of this problem is to determine the following:
- The pressure of combustion gases at the turbine exit
- The velocity of the gases at the nozzle exit
- The thrust for this engine if the diffuser inlet diameter is 1.6 m.

Drawings + Diagrams|





T-S diagram of turbojet cycle

Sources|

Cengel Yungus A., et al. Thermodynamics: an Engineering Approach. McGraw-Hill Education 2019

Design Considerations|

- Air behaves as an ideal gas R = 0.287
- C_p + C_v are variable
- Isentropic compression + Expansion processes
- Constant pressure Combustion

P-11

Summary

Having solved for the requirements of the Problem, a best option can be chosen.

Materials

Air, Brayton cycle engine w/ and w/o a regenerator.

Analysis

Based on the cases that I did, case C was the most thermally efficient with a value of 0.532. The second engineer was correct about increasing the compression ratio to 8.58. Also, raising the max temperature to 895.48°C helped improve thermal efficiency
The effectiveness of the heat exchanger was highest in case a, having a value of $\epsilon = 1.339$. This could very easily be an error.

$W_{net} = W_{out} - W_{in}$

$\boxed{W_{net} = 350.835\ kJ/kg}$ net work for case b.

heat addition + Rejection

$Q_{add} = h_3 - h_2$

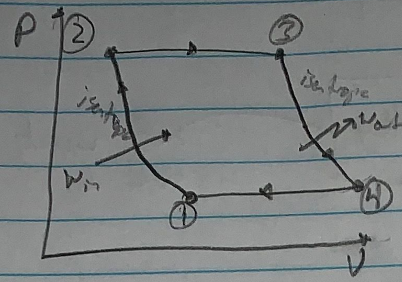
$\boxed{= 545.64\ kJ/kg}$

$Q_{rej} = h_4 - h_1$

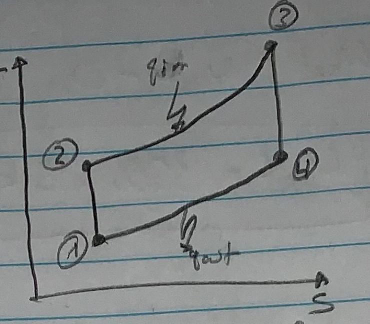
$\boxed{= 244.805}$

heat addition + rejection

1-b



P-V diagram for a brayton cycle



T-s diagram for a brayton cycle

1	2	3	4
$T_1 = 303.15K$	$P_2 = r_p P_1$	$T_3 = 1073.15K$	$P_1 = P_4$
$P_1 = 100\ kPa$	$P_2 = 1000kPa$	$\Rightarrow P_{r3} = 113.04$	$P_4 = 100kPa$
	$T_2 = 532K$	$P_4 = 1000kPa$	$T_4 = 543K$

$P_{r1} = 1.0601 \longrightarrow$

$$\frac{P_2}{P_1} = \frac{P_{r2}}{P_{r1}}$$

$$\Rightarrow P_{r2} = \left(\frac{P_2}{P_1}\right) P_{r1}$$

$$P_{r2} = 10.601$$

$$\Rightarrow T_2 = 532K$$

$\longrightarrow$

$$P_{r4} = \left(\frac{P_4}{P_3}\right) P_{r3}$$

$$P_{r4} = 11.304$$

$$\Rightarrow T_4 = 543K$$

$$\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{W_{out_{3-4}} - W_{in_{1-2}}}{q_{in_{2-3}}}$$

$w_{in} = h_2 - h_1 = 233.37 kJ/kg$

$w_{out} = h_3 - h_4 = 584.2 kJ/kg$

$q_{in} = h_3 - h_2 = 595.64 kJ/kg$

$h_1 = 90.605\ kJ/kg$
$h_2 = 323.97\ kJ/kg$
$h_3 = 919.61\ kJ/kg$
$h_4 = 335.41\ KJ/kg$

$\boxed{\eta_{th} = 0.589}$ — thermal efficiency for case b

1-d Now that the states are calculated, Net work and heat exchanger effectiveness can be calculated.

Net work.

$$W_{net} = W_{turb.} - W_{comp.} = (h_4 - h_5) - (h_2 - h_1)$$

$T_4 = 1168.63K \Rightarrow h_4 = 1030.9\ kJ/kg$
$T_5 = 605.29K \Rightarrow h_4 = 400.66\ kJ/kg$
$T_2 = 585.29K \Rightarrow h_2 = 379.61\ kJ/kg$
$T_1 = 303.15K \Rightarrow h_1 = 90.605\ kJ/kg$
from table A-17

$$\boxed{\Rightarrow W_{net} = 341.24\ kJ/kg}$$ net work for case d

calculating heat exchanger effectiveness for the cycle

$$\epsilon = \frac{q_{regen,act}}{q_{regen,max}} = \frac{T_3 - T_2}{T_5 - T_2}$$

$T_3 = 595.29K$
$T_5 = 605.29K$
$T_2 = 585.29K$

$$\boxed{\epsilon = 0.5}$$ heat exchanger effectiveness for case d

$T_{max} = 895.48°C = 1168.63K = T_4$

1

$P_1 = 100 kPa$

$T_1 = 303.15K$

$v_1 = \frac{RT_1}{P_1} = 0.87 \frac{m^3}{kg}$

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$

$\Rightarrow T_2 = 585.29K$

$v_2 = \frac{RT_2}{P_2}$

$= 0.168 \frac{m^3}{kg}$

2

$P_2 = r_p P_1$

$P_2 = 1000kPa$

$T_2 = 585.29K$

$v_2 = 0.168 \frac{m^3}{kg}$

3

$T_3 = T_5 - 10$

$P_3 = P_2 = 1000kPa$

$T_3 = 595.29K$

4

$T_4 = 1168.63K$

$P = 10P_5 = 1000kPa$

$v_4 = \frac{RT_4}{P_4} = 0.335 \frac{m^3}{kg}$

$\frac{T_5}{T_4} = \left(\frac{P_5}{P_4}\right)^{\frac{k-1}{k}}$

$\Rightarrow T_5 = 605.29K$

$v_5 = \frac{RT_5}{P_5}$

$\Rightarrow v_5 = 1.737 \frac{m^3}{kg}$

5

$P_5 = P_6 = 100kPa$

$T_5 = 605.29K$

$v_5 = 1.737 \frac{m^3}{kg}$

6

$P_6 = P_1 = 100kPa$

$T_6 = 595.29$

$v_6 = 1.708 \frac{m^3}{kg}$

$h_5 - h_6 = h_3 - h_2$

$\cancel{c_p}(T_5 - T_6) = \cancel{c_p}(T_3 - T_2)$

$\Rightarrow T_6 = T_5 - (T_3 - T_2)$

$T_6 = 595.29K$

$v_6 = \frac{RT_6}{P_6}$

$\Rightarrow v_6 = 1.708 \frac{m^3}{kg}$

$\eta_{th} = 1 - \frac{T_6 - T_1}{T_4 - T_3}$

$\boxed{\eta_{th} = .490}$

P-1

1-c

the P-V and Ts diagrams from 1-a are the same. the only variable changed is $r_p = 8.58$.

calculating net work.

$W_{net} = W_{turbine} - W_{comp.} = (h_4 - h_5) - (h_2 - h_1)$

$T_4 = 1073.15\,K \Rightarrow h_4 = 919.61\,kJ/kg$
$T_5 = 580.70\,K \Rightarrow h_5 = 374.8\,kJ/kg$
$T_2 = 560.23\,K \Rightarrow h_2 = 353.37\,kJ/kg$
$T_1 = 303.15\,K \Rightarrow h_1 = 90.605\,kJ/kg$

$\boxed{W_{net} = 282.05\,kJ/kg}$ net work for case c

calculating heat exchanger effectiveness for the cycle

$\epsilon = \frac{q_{regen,act}}{q_{regen,max}} = \frac{T_3 - T_2}{T_5 - T_2}$

$T_3 = 570.70\,K$
$T_5 = 580.70\,K$
$T_2 = 560.23\,K$

$\boxed{\epsilon = 0.511}$ heat exchanger effectiveness for case c

$\boxed{r_p = 8.58}$

1

$P_1 = 100\,kPa$

$T_1 = 303.15\,K$

$v_1 = \frac{RT_1}{P_1}$

$v_1 = 0.870\,\frac{m^3}{kg}$

2

$P_2 = r_p P_1$

$P_2 = 858\,kPa$

$T_2 = 560.23\,K$

$v_2 = 0.187\,\frac{m^3}{kg}$

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$

$\Rightarrow T_2 = 560.23\,K$

$v_2 = \frac{RT_2}{P_2}$

$\Rightarrow v_2 = 0.187$

3

$T_3 = T_5 - 10$

$P_3 = P_2 = 858\,kPa$

$T_3 = 570.70\,K$

4

$T_4 = 1073.15\,K$

$P_4 = 8.58 P_5$

$P_4 = 858\,kPa$

$v_4 = \frac{RT_4}{P_4}$

$\Rightarrow v_4 = 0.359\,\frac{m^3}{kg}$

$\frac{T_5}{T_4} = \left(\frac{P_5}{P_4}\right)^{\frac{k-1}{k}}$

$\Rightarrow T_5 = 580.70\,K$

$v_5 = \frac{RT_5}{P_5}$

$\Rightarrow v_5 = 1.667\,\frac{m^3}{kg}$

5

$P_5 = P_6 = 100\,kPa$

$T_5 = 580.70\,K$

$v_5 = 1.667\,\frac{m^3}{kg}$

6

$P_6 = P_1 = 100\,kPa$

$T_6 = 570.23\,K$

$h_5 - h_6 = h_3 - h_2$

$c_p(T_5 - T_6) = c_p(T_3 - T_2)$

$\Rightarrow T_6 = T_5 - (T_3 - T_2)$

$T_6 = 570.23\,K$

$\eta_{th} = 1 - \frac{(T_6 - T_1)}{(T_4 - T_3)}$

$\boxed{\eta_{th} = 0.532}$

1-C

After calculating all of the states, the Net work of the cycle can be calculated

$$w_{net} = w_{turbine} - w_{comp} = (h_4 - h_5) - (h_2 - h_1)$$

$T_4 = 1073.15\,K \Rightarrow h_4 = 919.67\ kJ/kg$
$T_5 = 555.84\,K \Rightarrow h_5 = 348.79\ kJ/kg$
$T_2 = 585.29\,K \Rightarrow h_2 = 379.61\ kJ/kg$
$T_1 = 303.15\,K \Rightarrow h_1 = 90.605\ kJ/kg$
from table A-17

$\boxed{w_{net} = 281.815\ kJ/kg}$ Net work for case ~~4~~

Calculating heat exchanger effectiveness of the cycle.

$$\epsilon = \frac{q_{regen,act}}{q_{regen,max}}$$

$T_3 = 545.84\,K$
$T_5 = 555.84\,K$
$T_2 = 585.29\,K$

$$\epsilon = \frac{T_3 - T_2}{T_5 - T_2}$$

$\boxed{\epsilon = 1.339}$ Effectiveness of heat exchanger

1

$P_1 = 100\,kPa$

$T_1 = 303.15K$

$v_1 = \frac{RT_1}{P_1} = 0.87\frac{m^3}{kg}$

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$

$\Rightarrow T_2 = 585.29K$

$v_2 = \frac{RT_2}{P_2}$

$= 0.168\frac{m^3}{kg}$

2

$P_2 = r_p P_1$

$P_2 = 1000\,kPa$

$T_2 = 585.29K$

$v_2 = 0.168\frac{m^3}{kg}$

3

$T_3 = T_5 - 10$

$P_3 = P_2$

$P_3 = 1000\,kPa$

$T_3 = 545.84K$

$v_3 = \frac{RT_3}{P_3}$

$\Rightarrow v_3 = 0.157\frac{m^3}{kg}$

4

$T_4 = 1073.15K$

$P_4 = 10P_5 = 1000\,kPa$

$v_4 = \frac{RT_4}{P_4} = 0.308\frac{m^3}{kg}$

$\frac{T_5}{T_4} = \left(\frac{P_5}{P_4}\right)^{\frac{k-1}{k}}$

$\Rightarrow T_5 = 555.84K$

$v_5 = \frac{RT_5}{P_5}$

$\Rightarrow 1.595\frac{m^3}{kg}$

5

$P_5 = P_6 = 100\,kPa$

$T_5 = 555.84K$

$v_5 = 1.595\frac{m^3}{kg}$

6

$P_6 = P_1 = 100\,kPa$

$T_6 = 595.29K$

$v_6 = 1.708\frac{m^3}{kg}$

$q_{hot\,stm} = q_{cold\,stm}$

$h_5 - h_6 = h_3 - h_2$

$c_p(T_5 - T_6) = c_p(T_3 - T_2)$

$\Rightarrow T_6 = T_5 - (T_3 - T_2)$

$T_6 = 595.29K$

$v_6 = \frac{RT_6}{P_6}$

$\Rightarrow 1.708\frac{m^3}{kg}$

$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{c_p(T_6 - T_1)}{c_p(T_4 - T_3)}$

$= 1 - \frac{T_6 - T_1}{T_4 - T_3}$

$\boxed{\eta_{th} = 0.416}$

Thermal Efficiency for case a

1-a

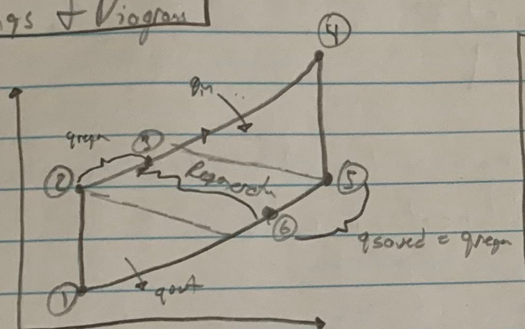
MET 350 Foster Grubbs

Test 11

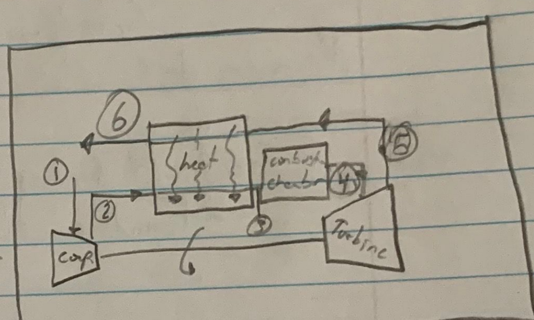
Purpose the purpose of this problem is to determine the network, the heat addition and rejection, the thermal efficiency, heat exchanger effectiveness for the following cases:

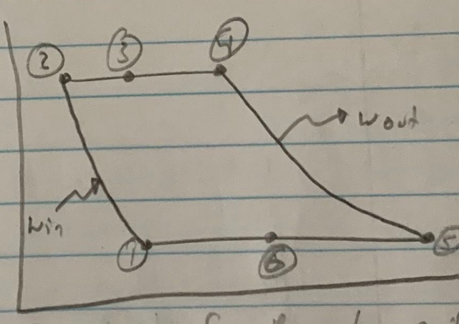
a) the original design
b) " " " w/o regeneration
c) " " " w/ a pressure ratio of 8.58
d) " " " w/ a max pressure of 895.48°C
e) " " " removing the compressor w/ a two stage compressor w/ inter cooling

Drawings & Diagram



T-s diagram for Brayton Cycle w/ regeneration





P-V diagram for Brayton Cycle w/ regeneration

Sources

Cengel Yunus A., et al. Thermodynamics: an Engineering Approach. McGraw-Hill Education 2019

Design Considerations

Assume the following:

- Air behaves as an ideal gas $R = 0.287 \frac{kJ}{kg \cdot K}$

Data + Variables

$T_1 = 30°C = 303.15\,K$ $T_{max} = 800°C = 1073.15$

$P_1 = 100\,kPa$ $\Delta T_{hot-cold} = 10°C$ @ end of regeneration $= 10K$

$r_p = 10$

$C_{min}/C_{max} = 1.00$

Procedure + Calculations

Start with finding each of the states for case a)

- Found on the next page.