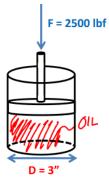
# **Pressure Definition – Example Problem**

Compute the pressure produced in the oil in a closed cylinder by a piston exerting a force of 2500 lb on the enclosed oil. The piston has a diameter of 3.00 in.

### Purpose:

Compute the pressure produced by a piston in a closed cylinder full of oil.

## **Drawing:**



### **Procedure:**

The pressure is obtained using its definition:

$$p = \frac{F}{A}$$

The force is given and the area (perpendicular to the force direction) can be computed from the corresponding geometric relation:

$$A = \frac{\pi}{4} * D^2$$

## **Calculations:**

$$A = \frac{\pi}{4} * D^2 = \frac{\pi}{4} * (3 \text{ in})^2 = 7.069 \text{ in}^2$$

Now.

$$p = \frac{F}{A} = \frac{2500 \text{ lbf}}{7.069 \text{ in}^2} = 353.68 \text{ lbf/in}^2$$

#### Sources

Mott, R., and Untener, J. Applied Fluid Mechanics. 7th Edition. 2015.

## **Design Considerations:**

The following must be assumed:

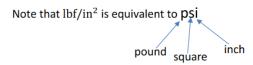
- 1) Incompressible fluid
- 2) Isothermal process

## **Data and Variables:**

F = 2500 lbf

D = 3 inches

F = 2500 lbf



Therefore,

$$p = 353.68 \text{ psi}$$

### **Summary:**

The pressure exerted by the piston is 353.68 psi

## **Materials:**

Oil

## Analysis:

- a) If piston diameter increases, pressure reduces.
- b) If applied force increases, pressure increases.

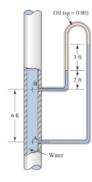
# Pressure measurement - Example Problem

For the compound differential manometer, calculate ( $P_A - P_B$ ).

## Purpose:

Compute the pressure differential of the manometer.

#### Drawing:



## Sources:

Mott R. and Untener J. Applied Fluid Mechanics. 7<sup>th</sup> Edition. Pearson. 2014.

### **Design Considerations:**

The following must be assumed:

- 1) Incompressible fluid
- 2) Isothermal process

## **Data and Variables:**

sg(oil) = 0.90 All distances in the drawings

## **Procedure:**

This problem is solved using the simple equation of increment of pressure:

$$\triangle p = \gamma * h$$

We will move along the manometer starting from a point of my interest and ending in another point of my interest.

While doing so, remember that when moving downward along the manometer we use a positive increment of pressure, but when moving upward we use a negative value.

Also remember that if moving horizontally within the same fluid, there is no pressure increment.

Oil (sg = 0.90)

A

Water

PLEASE NOTE THAT YOU CANNOT GO FROM POINT A AND B THROUGH THE PIPE.

$$P_B - \gamma_{water} * 2 \text{ ft}$$

$$P_B - \gamma_{water} * 2 \text{ ft} - \gamma_{oil} * 3 \text{ ft}$$

$$P_B - \gamma_{water} * 2 \text{ ft} - \gamma_{oil} * 3 \text{ ft} + \gamma_{water} * (6 \text{ ft} + 2 \text{ ft} + 3 \text{ ft}) = P_A$$

$$P_A - P_B = \gamma_{water} * 9 \text{ ft} - \gamma_{oil} * 3 \text{ ft}$$

At this point, we need the specific weight of water and oil:

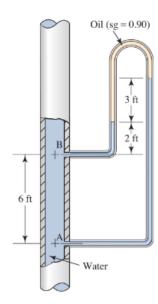
 $\gamma_{water} = 62.4 \ lb/ft^3$  From table in the back of the book

$$\gamma_{oil} {=} \mathsf{sg(oil)} * \gamma_{water} = 0.90*62.4 \ \mathrm{lb/ft^3} = 56.16 \ \mathrm{lb/ft^3}$$

Finally, plugging in values:

$$P_A - P_B = 62.4 \frac{\text{lb}}{\text{ft}^3} * 9 \text{ ft} - 56.16 \frac{\text{lb}}{\text{ft}^3} * 3 \text{ ft}$$

$$P_A - PB = 393.12 \frac{\text{lb}}{\text{ft}^2}$$



## **Summary:**

The differential pressure in the compound manometer is 393.12 lb/ft² (or 2.73 psi).

## Materials:

Water and Oil.

## **Analysis:**

The pressure at any point in the fluid only depends directly on the fluid column height and on the specific weight of the fluid.

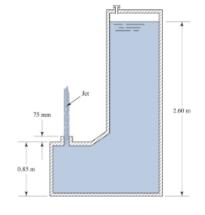
# Bernoulli's Equation - Example Problem

To what height will the jet of fluid rise for the conditions shown in the figure?

## **Purpose**

Compute the maximum height the fluid jet will have.

## **Drawing**



## Sources

See note below.

## **Design Considerations**

The following must be assumed:

- Incompressible fluid
- 2) Isothermal process
- 3) Steady State
- 4) No energy losses

### **Data and Variables**

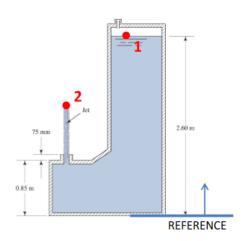
Various length of the vessel provided in the figure. p=0.0 psig above the water surface.

 $\gamma_{water} = 9.81 \text{ KN/m}^3 \text{ (from back of the book)}$ 

#### **Procedure**

We have to pick a reference point (always do this first!). In this case we can choose any point. I will pick the one shown.

Be careful with all the units. Change what needs to be changed for consistency during unit cancelation when plugging in values in the equations.



We will be using Bernoulli's Equation since the fluid is moving

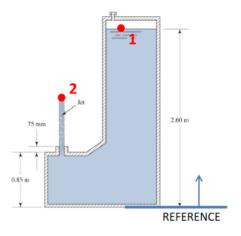
$$\frac{p_1}{\gamma} + \frac{{V_1}^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{{V_2}^2}{2g} + Z_2$$

As always, we need to decide the 2 points we will apply the equation to. Remember to pick the points wherever you know the most or wherever you would like to know something about.

For this problem, we know a lot more about the water surface where the pressure is atmospheric pressure (0 psig). We should use that as one of the points for Bernoulli's equation. As for the 2<sup>nd</sup> point, it should be at the top of the jet because we are being asked about that height.

Solving for the height of point 2 from Bernoulli's equation:

$$Z_2 = \frac{p_1 - p_2}{\gamma} + \frac{{V_1}^2 - {V_2}^2}{2g} + Z_1$$



The velocity at 1 is assumed to be negligible if the tank is large. If it is not large, then we cannot have steady state. We must assume is large.

The velocity at B should be ZERO because at the jet highest point the jet comes to rest. Similar to when you are throwing a ball vertically. At its highest point the velocity is zero.

$$Z_2 = \frac{p_1 - p_2}{\gamma} + \frac{{V_1}^{20} - {V_2}^{20}}{2g} + Z_1$$

So,

$$Z_2 = \frac{p_1 - p_2}{\gamma} + Z_1$$

The pressure at 1 and 2 is atmospheric pressure

$$Z_2 = \frac{p_1^0 - p_2^0}{\gamma} + Z_1$$

Finally,

$$Z_2 = Z_1$$

## **Summary**

The highest point the jet will reach is 2.6 m.

### Materials

Water.

## **Analysis**

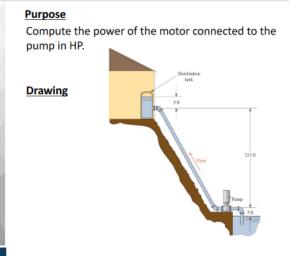
If we consider energy losses then the height decreases. The water at point 2 has less energy to reach a higher point.

Also note that if there is any pressure above the water surface (higher than atmospheric pressure), the jet will reach a higher point. Examine the expression we derive to note that:

$$Z_2 = \frac{p_1 - p_2}{\gamma} + Z_1$$

# Bernoulli's Equation-Example Problem

Professor Crocker is building a cabin on a hillside and has proposed the water system shown in the figure. The distribution tank in the cabin maintains a pressure of 30.0 psig above the water. There is an energy loss of 15.5 ft in the piping. When the pump is delivering 40 gal/min of water, compute the horsepower delivered by the motor to the pump. The pump has an efficiency of 72 percent.



### Sources

See note for details.

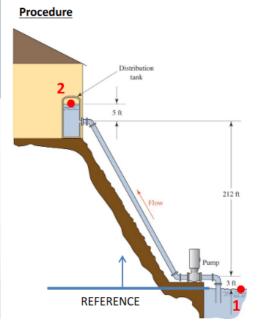
### **Design Considerations**

The following must be assumed:

- 1) Incompressible fluid
- 2) Isothermal process
- Steady state

## **Data and Variables**

$$Q = 40 \text{ gal/min} = 0.089 \text{ ft}^3/\text{s}$$
  
 $h_L = 15.5 \text{ ft}$   
 $p = 30 \text{ psig}$   
Efficiency pump= 0.72  
Dimensions given in the drawing



Pick a reference.

We will be using Bernoulli's Equation with energy losses and pump head

$$h_A + \frac{p_1}{\gamma} + \frac{{V_1}^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{{V_2}^2}{2g} + Z_2 + h_{L12}$$

We will also need the equation for power of the motor of a pump

$$P_m = \frac{P_A}{\eta_A} = \frac{\gamma Q h_A}{\eta_A}$$

Let us now pick the points for the Bernoulli's equation.

The best locations should be on top of both liquid surfaces. We know everything there! Also, the points are one before the pump and the other after the pump, which allows  $h_A$  to appear in the Bernoulli's equation.

Solving for  $h_A$  using points 1 and 2

$$h_{A} = \frac{p_{2} - p_{1}^{0}}{\gamma} + \frac{{\sqrt{2}^{2} - {\sqrt{1}^{2}}^{2}}}{2g} + Z_{2} - {Z_{1}^{0}} + h_{L\,12}$$

Simplifying with the known values from points 1 and 2:

$$h_A = \frac{p_2}{\gamma} + Z_2 + h_{L12}$$

Substituting values:

$$h_A = \frac{30\frac{\text{lb}}{\text{in}^2} * \frac{144 \text{ in}^2}{1 \text{ ft}^2}}{624 \frac{\text{lb}}{\text{ft}^3}} + 220 \text{ ft} + 15.5 \text{ ft}$$

$$h_A = 304.73 \text{ ft}$$

To get hydraulic required power,  $P_A = \gamma Q h_A$ :

$$P_A = 624 \frac{\text{lb}}{\text{ft}^3} * 0.089 \frac{\text{ft}^3}{\text{s}} * 304.73 \text{ ft}$$

$$P_A = 1692.35 \frac{\text{lb} * \text{ft}}{\text{s}}$$

From table in appendix: 1 HP = 550 lb\*ft/s, so:

$$P_A = 3.077 \text{ HP}$$

Finally, the electrical motor power (using pump efficiency):

$$P_m = \frac{3.077 \text{ HP}}{0.72}$$

$$P_m = 4.274 \text{ HP}$$

## Summary

The power of the motor of the pump is 4.274 HP.

### **Materials**

Water.

## **Analysis**

The power delivered by the motor to the pump will always be greater than the required hydraulic power, this is because of some internal energy losses of the pump.

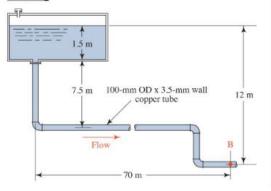
# Major and Minor losses - Example Problem

Water at 10 C at the rate of 900 L/min from the reservoir and through the pipe shown in the figure. Compute the pressure at point B.

### **Purpose**

Compute the pressure at point B.

#### Drawing



### Sources

See note for details.

### **Design Considerations**

The following must be assumed:

- 1) Incompressible fluid
- 2) Isothermal process
- 3) Steady state

## **Data and Variables**

Dimensions provided in the drawing

D = 99.3 mm = 0.0993 m (from book's appendices tables)

 $Q = 900 L/min = 0.02 m^3/s$ 

Roughness = 0.0015 mm (from book's table)

$$\gamma = 9.81 \text{ KN/m}^3$$
;  $\rho = 1000 \frac{\text{kg}}{\text{m}^3}$ ;  $v = 1.30 \times 10^{-6} \frac{\text{m}^2}{\text{s}}$ ;

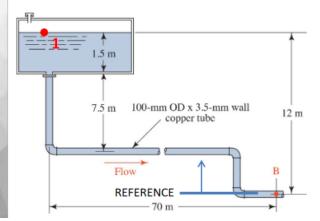
### **Procedure**

Pick a reference.

We will be using Bernoulli's Equation with the term of total energy losses between points 1 and 2:

$$\frac{p_1}{\gamma} + \frac{{V_1}^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{{V_2}^2}{2g} + Z_2 + h_{L12}$$

As always, we need to decide the 2 points we will apply the equation to. Remember to pick the points wherever you know the most or wherever you would like to know something about.



For this problem, we know everything about the water surface where the pressure is atmospheric pressure (0 psig), the velocity is negligible (if not zero), and the elevation is 12 m (using our reference).

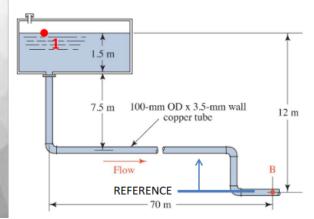
As for the 2<sup>nd</sup> point, it should be at the point B described in the problem because that is where we want to know the pressure.

## **Procedure**

$$\frac{p_1}{\gamma} + \frac{{V_1}^2}{2g} + Z_1 = \frac{p_B}{\gamma} + \frac{{V_B}^2}{2g} + Z_B + h_{L1B}$$

How many sources of energy losses do you recognize? (Pause the audio/video to think....)

We have the losses due to pipe friction and the losses due to the fittings: one entrance in the tank, and 3 elbows. I cannot neglect the minor losses this time. At least a priori because I am not sure the system is long enough to neglect all those minor losses (there is a lot of energy losses in elbows).



For the major energy losses due to friction, we use Darcy-Weisbach equation:

$$h_L = f \frac{L}{D} \frac{V^2}{2g}$$

To find the friction factor (f) we must calculate Reynolds number and relative roughness

$$Re = \frac{\rho VD}{\mu} = \frac{VD}{v}$$
 Relative roughness  $= \frac{D}{\epsilon}$ 

As for the minor losses, we will use:  $h_L = K \frac{V^2}{2g}$ 

## Calculations

$$\frac{p_1^{0 \text{ psig}} + \sqrt[6]{\frac{1}{2}}}{\gamma} + \frac{V_1^{0}}{2g} + Z_1 = \frac{p_B}{\gamma} + \frac{V_B^{2}}{2g} + \frac{V_B^{0}}{2g} + h_{L1B}$$

From the Bernoulli's equation what we are interested in is on the pressure at B. Let us consider what we know and then solve for that pressure.

The velocity on the water surface is zero and the elevation of point 2 is zero due to our chosen reference.

The pressure at the water surface is zero gage.

Finally, solving for  $p_R$ :

$$\frac{p_B}{v} = Z_1 - \frac{{V_B}^2}{2q} - hL_{1B}$$

The energy losses include the pipe losses and the 3 elbows losses:

$$h_{L\,1B} = f \frac{L}{D} \frac{V^2}{2g} + 3 * K \frac{V^2}{2g}$$

Substituting them into Bernoulli's equation:

$$\frac{p_B}{\gamma} = Z_1 - \frac{{V_B}^2}{2g} - f\frac{L}{D}\frac{V^2}{2g} + 3 * K\frac{V^2}{2g}$$

Note that the velocity in the energy losses equation should be the same as the velocity at point B due to continuity (or conservation of mass), so:

$$\frac{p_B}{\gamma} = Z_1 - \frac{{V_B}^2}{2g} - f \frac{L}{D} \frac{{V_B}^2}{2g} + 3 * K \frac{{V_B}^2}{2g}$$

or, 
$$\frac{p_B}{\gamma} = Z_1 - \frac{{V_B}^2}{2g} \left( 1 + f \frac{L}{D} + 3K \right)$$

We can obtain the velocity at point B using the flow rate equation:

$$V_B = \frac{Q}{A} = \frac{0.02 \frac{\text{m}^3}{\text{s}}}{\frac{\pi}{4} * 0.0993^2 \text{ft}^2} = 2.59 \frac{\text{m}}{\text{s}}$$

We compute the Reynolds Number and relative roughness to then go to the Moody's chart to obtain the friction factor

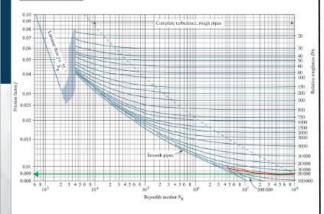
$$Re = \frac{2.59 \frac{\text{m}}{\text{s}} * 0.0993 \text{ m}}{1.30 \times 10^{-6} \frac{\text{m}^2}{\text{s}}} = 1.978 \times 10^5$$

$$\frac{\epsilon}{D} = \frac{1.5 \times 10^{-6} \text{m}}{0.0993 \text{ m}} = 1.51 \times 10^{-5}$$

So, from the Moody's chart

$$f \cong 0.0155$$

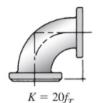
### Calculations



So, from the Moody's chart

$$f_T \cong 0.0086$$

We also need the resistance coefficient K for the elbow. We search for that in the book (assuming long radius elbow):



(b) 90° long radius elbow

We require  $f_T$ , which we can get from Moody's chart using the value of  $\frac{\epsilon}{R}$ ,

Please note that you could have also used the equation (which is better if using spreadsheets):

$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\varepsilon)} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

In this case, since  $f_T$  does not depend on Reynolds number, the equation reduces to:

$$f_T = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\varepsilon)}\right)\right]^2}$$

Thus, the resistance coefficient K for the elbow is:

$$K = 0.172$$

Plugging all the values into the Bernoulli's equation :

$$\frac{p_B}{\gamma} = Z_1 - \frac{{V_B}^2}{2g} \left( 1 + f \frac{L}{D} + 3K \right)$$

$$\frac{p_B}{\gamma} = 12.0 \text{ m}$$

$$-\frac{\left(2.59 \frac{\text{m}}{\text{s}}\right)^2}{2 * 9.81 \frac{\text{m}}{\text{s}^2}} \left(1 + 0.0155 \frac{80.5 \text{ m}}{0.0993 \text{ m}} + 3 * 0.172\right)$$

Computing and using the specific weight of water:

$$\gamma = 9.81 \text{ KN/m}^3$$

We get:

$$p_B = 70.49 \ kPa$$

### **Summary**

The pressure at point B is 70.49 kPa.

### **Materials**

Water.

## **Analysis**

There will always be energy losses due to friction in a pipe were flow flows, this can be reduced with different factors but never eliminated.

Note that we could have neglected the energy losses due to the elbows because they represent only the 3.66% of the total energy losses. Can you get that percentage?

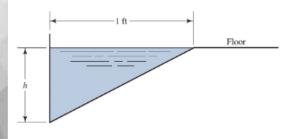
# Open channel flow- Example Problem

A square storage room is equipped with automatic sprinklers for fire protection the spray 1000 gal/min of water. The floor is designed to drain this flow evenly to troughs near each outside wall. The troughs are shaped as shown in the figure. Each trough carries 250 gal/min, is laid on a 1-percent slop, and is formed of unfinished concrete. Determine the minimum depth h

#### Purpose

Determine the minimum depth h

### **Drawing**



#### Sources

See note for details.

### **Design Considerations**

The following must be assumed:

- Incompressible fluid
- Isothermal process
- Steady state

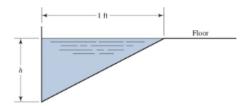
## **Data and Variables**

Q=250 gal/min = 0.557 ft3/s

n = 0.017 for unfinished concrete (from table in the book)

S = 0.01

### **Procedure**



Using the open channel flow equation:

$$Q = \frac{1.49}{n} A S^{\frac{1}{2}} R^{\frac{2}{3}}$$

Note that we use the US units version of the equation.

The definition of the hydraulic radius is:

$$R = \frac{A}{WP}$$

Since the transversal cut of the channel flow is a triangle, we can use Pythagoras theorem:

$$a^2 = b^2 + c^2$$

## Calculations

Let us stat by putting all the known values on one side of the open channel flow equation:

$$AR^{\frac{2}{3}} = \frac{nQ}{1.49S^{\frac{1}{2}}}$$

Substituting the known values:

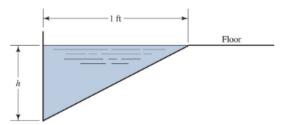
$$AR^{\frac{2}{3}} = \frac{0.017 * 0.557 \frac{\text{ft}^3}{\text{s}}}{1.49 * (0.01)^{\frac{1}{2}}} = 0.0635$$

Now, let us obtain the equations for area and hydraulic radius based on the data provided in the problem. For area:

$$A = (1.0 \text{ ft}) * h * \frac{1}{2} = \frac{h}{2}$$

For the wetted perimeter:

$$WP = h + L$$



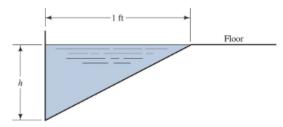
Using Pythagoras theorem, the L variable can be obtained as a function of the unknown h:

$$L = \sqrt{1 + h^2}$$

Thus, the hydraulic radius is:

$$R = \frac{A}{WP} = \frac{\frac{h}{2}}{h + \sqrt{1 + h^2}}$$

### **Calculations**



Note that I could also modify the equation this way:

$$h = \frac{2 \times 0.0635}{\left(\frac{1}{2} \frac{h}{h + \sqrt{1 + h^2}}\right)^{\frac{2}{3}}}$$

The advantage of this is that my guessed h value will provide a new h value that I can use in the next iteration. The second tab of the excel spreadsheet shows the results of this technique. Unfortunately this idea does not always work as there are equations that are mathematically unstable. So, be careful.

Let us substitute the area and the hydraulic radius:

$$AR^{\frac{2}{3}} = 0.0635$$
  $\frac{h}{2} \left( \frac{1}{2} \frac{h}{h + \sqrt{1 + h^2}} \right)^{\frac{2}{3}} = 0.0635$ 

It is not possible to solve for the unknown h directly. We most proceed to iterate to solve this (trial and error).

We will assume different values of h until the left hand side of the equation matches the right hand side of the equation. Let us use excel for this.

h (ft)	LHS	% diff
1	0.175025	175.63%
2	0.381966	501.52%
0.5	0.071985	13.36%
0.25	0.026497	-58.27%
0.4	0.052738	-16.95%
0.45	0.062238	-1.99%
0.46	0.064169	1.05%
0.455	0.063202	-0.47%
0.456	0.063396	-0.16%
0.457	0.063589	0.14%
0.4565	0.063492	-0.01%

### Summary

The height of the channel is 0.4565 ft.

### Materials

Water.

### Analysis

The flow rate that passes through an open channel depends directly of the size and the shape of such.

The problem we just solved is the typical problem of an open channel design. This type of problems is the longest because it involves an iteration process to solve for the equations.

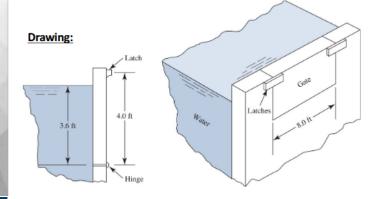
On this problem, one dimension was given. In a typical design this does not happen. When designing the open channel, you could fix one of the dimensions (we typically need two), or, BETTER, use another equation to have 2 equations with 2 unknowns. The second equation could come from the most efficient sections for open channels (see table in the book).

## Forces due to pressure - Example Problem

A rectangular gate is installed in a vertical wall of a reservoir, as shown in the figure. Compute the force on each of the two latches shown.

### Purpose:

Compute the force on each latch.



### Sources:

See note for details.

## **Design Considerations:**

The following must be assumed:

- 1) Incompressible fluid
- 2) Isothermal process

## **Data and Variables:**

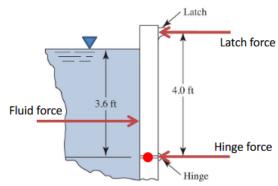
Dimensions of gate and water as fluid.

$$\gamma_{water} = 62.4 \, lb/ft^3$$

From table in the back of the book.

### **Procedure:**

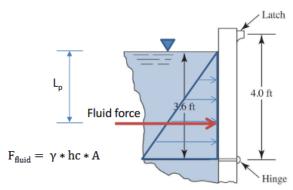
The latches are holding the vertical wall in position and that is the reason they excerpt a force on the wall. The hinges also help. To obtain the force in the latches, let us start with a free body diagram of the vertical wall:



Applying Newton's Law ( $\sum F=0$ ) will not help because we do not know the force by the hinges.

The best approach is to use an extended version of Newton's Law:  $\sum M = 0$ , and it is better to do it with respect to the hinge location.

We will need the fluid force, which is normal to the area and can be calculated using the equation we studied:



We also need the fluid force location, which is 2/3 of the 3.6 ft vertical distance, measured from the liquid surface  $(L_n)$ .

Let us start with the moment ( $\sum M = 0$ ) with respect to a point at the hinge:

$$F_{fluid} * (3.6 - Lp) ft = Flatc_{hes} * 4ft$$

We can calculate  $F_{fluid}$  and then solve for  $F_{latches}$ .

Let us proceed to get the fluid force with the discussed equation. So, first:

$$hc = \frac{3.6 ft}{2} = 1.8 ft$$
  $A = 3.6 ft * 8 ft = 28.8 ft^2$   $F_{fluid} = 62.4 \frac{lb}{ft^3} * 1.8 ft * 28.8 ft^2 = 3,234.816 lb$ 

As for the location of the fluid force from water surface:

$$Lp = \frac{2}{3} * 3.6 \text{ ft} = 2.4 \text{ ft}$$

Let us go back to the moment equation with  $F_{\text{latches}}$  solved for:

$$F_{latches} = \frac{F_{fluid} * (3.6 - Lp) \; ft}{4 \; ft} \;\; = \; \frac{3,234.816 \; lb \; * (3.6 \, - 2.4) \; ft}{4 \; ft}$$

$$F_{latches} = 970.44 \ lb$$



The resultant force due to pressure at the wall is 3,234.816 lb and the force excerpted by both latches is 970.44 lb (or 485.22 lb per latch).

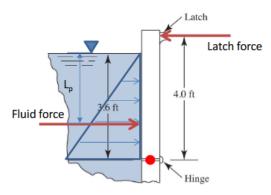
## Materials:

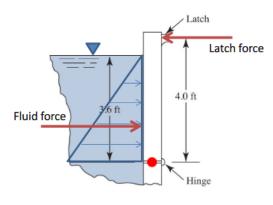
Just water

### **Analysis:**

The resultant force due to pressure, depends directly on the height of fluid on the surface and on the geometry of such surface. There are many problems where  $\sum M=0$  will come very handy.

We can also obtain the force by the hinges by using the equation:  $\sum F = 0$ . Forces by the latches increase with water depth and reduces with wall height. Of course, water depth cannot be larger than wall height.





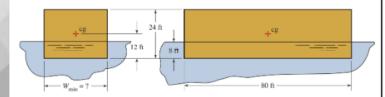
# **Buoyancy and Stability - Example Problem**

The figure shows a river scow used to carry bulk materials. Assume that the scow's center of gravity is at its centroid and that it floats with 8.00 ft submerged. Determine the minimum width that will ensure stability in seawater.

## **Purpose**

Calculate the width of the scow

### **Drawing**



### Sources

See note for details.

## **Design Considerations:**

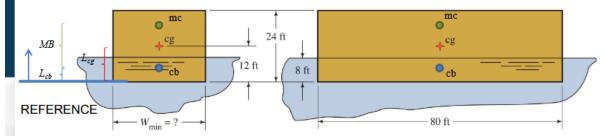
The following must be assumed:

- 1) Incompressible fluid
- 2) Isothermal process

## **Data and Variables:**

Dimensions of scow are given in the drawing.

### **Procedure**



First let us put a reference. For convenience, let us have it at the bottom of the scow.

We know that for a floating body to be stable, the metacenter MUST be above the center of gravity. The center of gravity location is given in this problem.

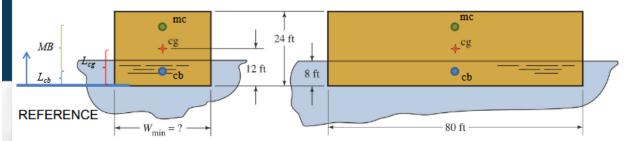
The center of buoyancy (cb) is located at the centroid of the submerged volume.

The metacenter (mc) is located above the center of buoyancy (cb) by a distance "MB".

For the floating body to be stable, the point "mc" should be above the point "cg", thus:

 $L_{cb} + MB > L_{cg}$ 

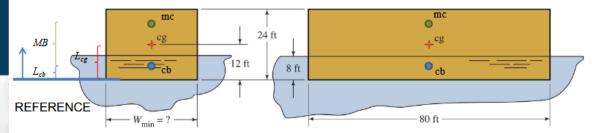
### Procedure



$$L_{cb} + MB > L_{cg}$$

The center of gravity distance to the reference is given. We can compute the distance to the center of buoyancy. And using the inequality, we can get the width from the MB distance.

### Calculations



The center of gravity distance  $(L_{\it cg})$  is at 12 ft,

$$L_{cg} = 12 \text{ ft}$$

For the center of buoyancy distance  $(L_{cb})$ :

$$L_{cb} = \frac{8 \text{ ft}}{2} = 4 \text{ ft}$$

From the inequality, the MB distance is:

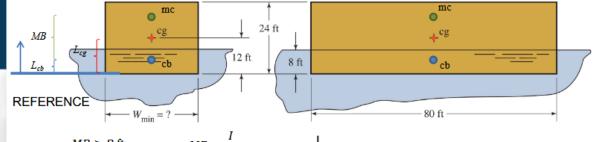
$$MB > L_{cg} - L_{cb}$$
  $MB > 8 \text{ ft}$ 

We know that:  $MB = \frac{I}{V_d}$ 

The moment of inertia of inertia of the imaginary plane formed by the interception of the scow and the liquid surface is:

$$I = \frac{80 \text{ ft} * W_{min}^3}{12}$$

Please note that the moment of inertia is with respect to the pivoting axis in the imaginary plane the scow could turn easily!



$$MB > 8 \text{ ft}$$
  $MB = \frac{I}{V_d}$ 

$$I = \frac{80 \text{ ft} * W_{min}^3}{12}$$

As for the displaced (submerged) volume:

$$V_d = W_{min} * 80 \text{ ft} * 8 \text{ ft}$$

Plugging in the values we know and the equations we obtained:

$$\frac{I}{V_d} > 8 \text{ ft}$$
  $\frac{80 \text{ ft} * W_{min}^3}{12}$   $\frac{12}{W_{min} * 80 \text{ ft} * 8 \text{ ft}} > 8$   $\frac{W_{min}^2}{96 \text{ ft}} > 8 \text{ ft}$   $W_{min} > 27.71 \text{ ft}$ 

## Summary

The minimum width of the scow so it can float stable is 27.71 ft.

## **Materials**

Water.

## **Analysis**

Note that the 80 ft length of the scow does not affect whether it is stable or not. However, it must be larger than  $W_{min}$ . Otherwise the instability could happen in the other pivoting direction.

We did not check whether the scow is really floating or not. We MUST do that first. In this problem, we did not do it because we do not have information about the scow's weight and we were told it is floating.

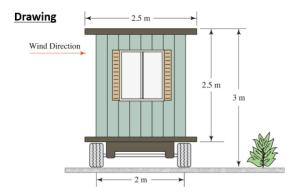
What we have to do is equate weight to buoyancy force. We typically obtain from this the volume of the object under the liquid.

# Drag and Lift - Example Problem

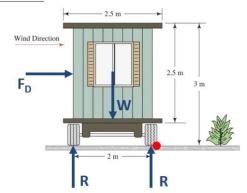
Determine the wind velocity required to overturn the mobile home sketched in the figure if it is 10 m long and weighs 50 kN. Consider it to be a square cylinder. The width of each tire is 300 mm. The air is at 0 C

## **Purpose**

Determine the wind velocity that will overturn the mobile home.



## **Procedure**



The key on this problem is to realize that the mobile home overturn is mathematically a moment. Thus, the mobile home is on the verge of turning when the algebraic sum of moments is equal to zero

$$\sum_{i} M_i = 0$$

#### Sources

See note for details.

## **Design Considerations**

The following must be assumed:

- 1) Incompressible fluid
- 2) Isothermal process
- 3) Steady state

## **Data and Variables**

- Dimensions of the mobile home in the figure
- Weight = 50 kN

• Air @ 0 C -> 
$$ho=$$
 1.292  $rac{\mathrm{kg}}{\mathrm{m}^3}$ ,  $v=$  1.33  $\ \sqcup \hspace{-0.8em}\downarrow \hspace{-0.8em} 0^{-5} rac{\mathrm{m}^2}{\mathrm{s}}$ 

We must select a point to apply the moment balance equation. I prefer to use the pivoting point if overturning.

The forces that the mobile home experience are: the weight, the drag, and the reacting forces of the ground towards the tires.

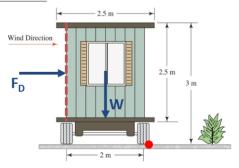
At the moment of the overturn, the reaction on the left tire is zero. The reaction on the right tire is not zero but it does not contribute to the moment with respect to the point we picked.

Thus, the moment balance equation becomes

$$F_D \times d_D - W \times d_W = 0$$

where  $d_D$  and  $d_W$  are the moment arms with respect to the pivoting point.

The wind velocity will show up in the drag force term and we will be able to solve for it.



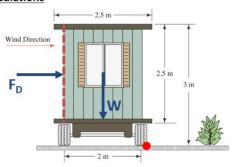
$$F_D \times d_D - W \times d_W = 0$$

The drag force equation is.

$$F_D = C_D \left(\frac{\rho V^2}{2}\right) A$$

The area here is the area perpendicular to the drag force. In this case, it is the side wall of the mobile home.

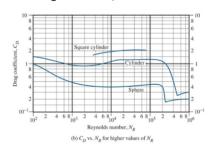
### Calculations

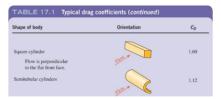


I will start using the one from the table  $C_D=1.60$ . If I see that the Reynolds number is larger than  $10^4$ , I will change it to  $C_D=2.0$  following the figure. If on the contrary, the Reynolds number is less than  $3x10^4$ , I will leave  $C_D=1.60$ . Looking at the figure it looks like the drag coefficient reduces its value with lower Reynolds but we do not know its exact value, thus we leave  $C_D=1.60$  and warn our client about the results.

The drag force located is in the middle of that area, and the weight location is in the centroid of the mobile home. This helps to determine the moment arms  $d_D$  and  $d_W$ .

As far as the drag coefficient, we have 2 sources:





Substituting what we know in the moment balance

$$C_D\left(\frac{\rho V^2}{2}\right)A\times d_D - W\times d_W = 0$$

$$\begin{split} &1.60 \times \left(\frac{1.292 \frac{\text{kg}}{\text{m}^3} \textit{V}^2}{2}\right) (2.5 \text{m} \times 10 \text{m}) \times \left(\frac{2.5 \text{m}}{2} + (3.0 \text{m} - 2.5 \text{m})\right) \\ &-50000 \text{ N} \times \left(\frac{2.0 \text{m}}{2} + \frac{300 \times 10^{-3} \text{m}}{2}\right) = 0 \end{split}$$

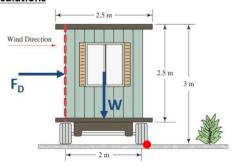
So,

$$45.22 \times V^2 \text{ kg} - 57500.00 \text{ N. m} = 0$$

Solving for velocity,

$$V = \sqrt{\frac{57500.00 \text{ kg} \frac{\text{m}}{\text{s}^2}.\text{m}}{45.22 \text{ kg}}}$$

$$V = 35.659 \frac{\text{m}}{\text{s}}$$



Let us now compute the Reynolds number to verify that  $C_D=1.60$  was correctly chosen:

$$Re = \frac{VL}{v} = \frac{35.659 \frac{\text{m}}{\text{s}} \times 2.5 \text{ m}}{1.33 \times 10^{-5} \frac{\text{m}^2}{\text{s}}}$$

Note that the dimension used (2.5 m) is the length of the mobile home parallel to the flow.

## Calculating

$$Re = 6.7 \times 10^6$$

This is larger than 10<sup>4</sup>, so I will not adjust  $C_D$ .

### Summary

The velocity of the wind that will overturn the home mobile is  $35.659 \frac{m}{s}$ .

## **Materials**

Air.

## **Analysis**

In this type of problems, you will have to iterate when solving for velocity because the drag coefficient depends on Reynolds number. If it weakly depends on it, then you will not need to iterate.

Other type of problem where you will need to iterate is if you are asked for the dimension that is used for Reynolds.

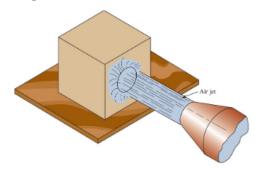
# Impulse Theorem- Example Problem

The figure shows a device for cleaning debris using a 1 ½ in diameter jet of air issuing from a blower nozzle. As shown, the jet is striking a rectangular box-shaped object sitting on a floor. If the air velocity is 25 ft/s and the entire jet is deflected by the box, what is the heaviest object that could be moved? Assume that the box slides rather than tumbling over and that the coefficient of friction is 0.60. The air has a density of 2.40 x 10<sup>-3</sup> slugs/ft<sup>3</sup>.

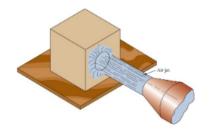
### **Purpose**

Determine the heaviest object that can be moved

### **Drawing**



### **Procedure**



The box can only move because of the applied force due to the air impacting one side of it. Thus, Newton's law is in order. Let us start with a free body diagram of the forces the box "feels".

The forces involved are the weight of the box (which is the unknown in this problem), the vertical reaction of the table against the box, the force due to the air blowing into the box, and the friction force of the sliding box on the table.

#### Sources

See note for details.

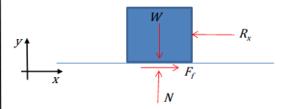
### **Design Considerations**

The following must be assumed:

- 1) Incompressible fluid
- 2) Isothermal process
- 3) Steady state

### **Data and Variables**

- 1 ½ inches air jet diameter
- V=25 ft/s
- Friction coefficient = 0.60
- ρ= 2.40 x 10<sup>-3</sup> slugs/ft<sup>3</sup>



Since the box is not moving vertically, the forces in the vertical direction should be in balance, thus

$$N = W$$

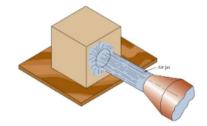
The box will start moving in the horizontal direction when the air force is larger than the friction force:

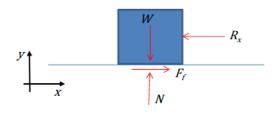
$$R_x > F_f$$

Let us get the equations for  $F_f$  and  $R_\chi$ . The friction force depends on the normal force and the friction coefficient

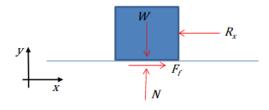
$$F_f = \mu N$$

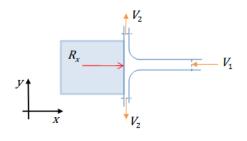
### Procedure





## **Procedure**

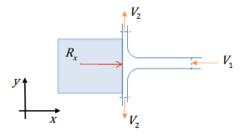




And since the normal force is equal to the weight:

$$F_f = \mu W$$

As far as the reaction force, let us have a control volume of the air only



The reaction force  $R_{\chi}$  acts in the opposite direction against the flow.

Applying impulse theorem in the horizontal direction:

$$\sum F_x = \rho Q(V_{2x} - V_{1x})$$

$$\sum F_x = \rho Q(V_{2x} - V_{1x})$$

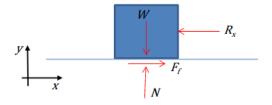
The only force in the horizontal direction in the water jet control volume is  $R_x$ . The horizontal component of the velocity at point 2 is zero. And the horizontal component of the velocity at point one is -V. Note that the velocity is in the negative direction of x. So,

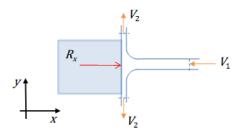
$$R_x = \rho Q(0 - (-V))$$

Knowing that Q is equal to  $V \times A$ , we get

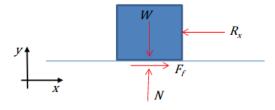
$$R_x = \rho \frac{\pi}{4} D^2 V^2$$

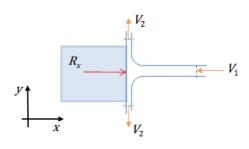
We will use all the above equations to solve for weight.





## Calculations





Knowing that for the box to start moving in the horizontal direction the air force is larger than the friction force:

$$R_x > F_f$$

And substituting the derived equations:

$$\rho \frac{\pi}{4} D^2 V^2 > \mu W$$

Solving for weight:

$$W < \frac{1}{\mu} \rho \frac{\pi}{4} D^2 V^2$$

Substituting values:

$$W < \frac{1}{0.60} 2.40 \times 10-3 \frac{\text{slugs}}{\text{ft}^3} \frac{\pi}{4} \left( 1.5 \text{ in } \frac{1 \text{ ft}}{12 \text{ in}} \right)^2 \left( 25 \frac{\text{ft}}{\text{s}} \right)^2$$

We get:  $W < 0.0307 \frac{\text{slugs x ft}}{\text{s}^2}$ 

Or:

## **Summary**

The box should be lighter than 0.0307 lb to slide over the table with than air jet.

## **Materials**

Air.

## **Analysis**

The maximum weight the box can have until it starts sliding is 0.0307 lb. The larger the velocity or jet diameter of air the heavier objects could slide. This is an intuitive statement that aligns with the equation we derived:

$$W<\frac{1}{\mu}\rho\frac{\pi}{4}D^2V^2$$

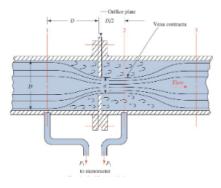
# Instrumentation - Example Problem

An orifice meter is to be installed in a 12-in ductile iron pipe carrying water at 60 F. A mercury manometer is to be used to measure the pressure difference across the orifice when the expected range of the flow rate is from 1500 gal/min to 4000 gal/min. The manometer scale ranges from 0 to 12 in of mercury. Determine the appropriate diameter of the orifice

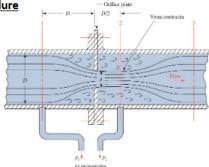
## **Purpose**

Calculate the diameter of the orifice

## **Drawing**



## **Procedure**



Let us use the equation for an orifice plate flow meter

$$V_1 = C \times \frac{\left[2gh\left(\frac{\gamma_m}{\gamma_w} - 1\right)\right]}{\left(\frac{A_1}{A_2}\right)^2 - 1}$$

Or:

$$Q = A_1 \times C \times \sqrt{\frac{2gh\left(\frac{\gamma_m}{\gamma_w} - 1\right)}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

### Sources

See note for details.

## **Design Considerations**

The following must be assumed:

- 1) Incompressible fluid
- 2) Isothermal process
- 3) Steady state

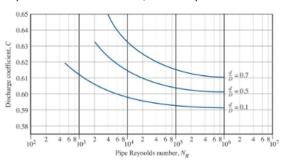
## **Data and Variables**

- Range of flow rate: 1500 gal/min 4000 gal/min = 3.34 ft<sup>3</sup>/s – 8.90 ft<sup>3</sup>/s
- Range of manometer: 0 in − 12 in = 0 ft − 1ft
- A<sub>1</sub>=0.8536 ft<sup>2</sup> (pipe area from table in back of the book)

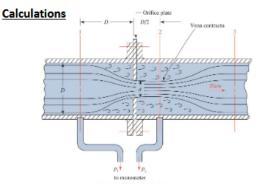
On paper, what we need to do is to solve for  $A_2$ , which depends on d, our unknown:

$$A_2 = \frac{A_1}{\sqrt[2]{\frac{2gh\left(\frac{Y_m}{Y_w} - 1\right)}{\left(\frac{Q}{A_1 \times C}\right)^2} + 1}}$$

The problem is the value of *C*, which depends on *d*:

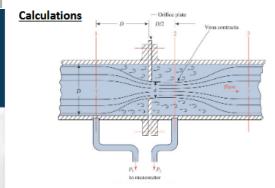


Thus, we will have to iterate.



Let us plugin in numbers we know.

For the h and Q values, since we were given a range, we will select the worst case scenario. This is the largest flow rate (8.90 ft<sup>3</sup>/s) with its corresponding largest manometer deflection (1 ft):



Now, we can get the value of C from the figure. We need  $d\!/\!D$  and Reynolds Number. From  $A_2$  solving for diameter, we get:

$$d = 0.7529 \, \text{ft}$$

Thus,

$$\frac{d}{D} = 0.7219$$

$$A_{2} = \frac{0.8536 \, \text{ft}^{2}}{2 \times 32.2 \, \frac{\text{ft}}{\text{s}^{2}} \times 1 \, \text{ft} \left( \frac{844.9 \, \frac{\text{lb}}{\text{ft}^{3}}}{62.4 \, \frac{\text{lb}}{\text{ft}^{3}}} - 1 \right)}{\left( \frac{8.90 \, \frac{\text{ft}^{3}}{\text{s}}}{0.8536 \, \text{ft}^{2} \times C} \right)^{2}} + 1$$

So,

$$A_2 = \frac{0.8536 \text{ ft}^2}{\sqrt[2]{\frac{807.5801 \frac{\text{ft}^2}{\text{s}^2}}{108.7104 \frac{\text{ft}^2}{\text{s}^2} \left(\frac{1}{C}\right)^2} + 1}} = \frac{0.8536 \text{ ft}^2}{\sqrt[2]{7.4287 \times C^2 + 1}}$$

We do not know C yet because it depends on d (and therefore  $A_2$ ). So, let us assume it and later check if the assumption was correct. Taking C = 0.60

$$A_2 = 0.4453 \, \text{ft}^2$$

To calculate the Reynolds number, we will use the traditional equations:

$$V = \frac{Q}{A} \qquad \qquad Re = \frac{\rho VD}{\mu} = \frac{VD}{v}$$

We get,

$$Re=8.99\times 10^5$$

Reading from figure, the  $\it C$  value is about 0.618. Trying again, we get

$$A_2 = 0.4357 \text{ ft}^2$$
  
 $d = 0.7448 \text{ ft}$   
 $\frac{d}{D} = 0.7142$ 

Using same Reynolds number, from figure, the  $\it C$  value is about 0.617, which is close to the previous one, so we are fine.

## **Summary**

The diameter of the orifice is 0.7448 ft or 8.938 in.

## Materials

Water.

## **Analysis**

This is a typical problem of flow meter design.

We could have chosen to use in the equations a maximum Mercury deflection of less that 12 in, to be on the safe side.

With the designed orifice flow meter, we can compute the mercury deflection for the lowest flow rate. Using the equation we used, solve for h:

$$h = \frac{\left(\frac{Q}{A_1 \times C}\right)^2 \times \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}{2g\left(\frac{\gamma_m}{\gamma_w} - 1\right)}$$

Substituting values:

$$h = \frac{\left(\frac{3.34 \frac{\text{ft}^3}{s}}{0.8536 \text{ ft}^2 \times 0.617}\right)^2 \times \left[\left(\frac{0.8536 \text{ ft}^2}{0.4357 \text{ ft}^2}\right)^2 - 1\right]}{2 \times 32.2 \frac{\text{ft}}{s^2} \left(\frac{844.9 \frac{\text{lb}}{\text{ft}^3}}{62.4 \frac{\text{lb}}{\text{ft}^3}} - 1\right)}$$

$$h = 0.0478 \text{ ft} = 0.5732 \text{ in}$$

Which is a readable value.

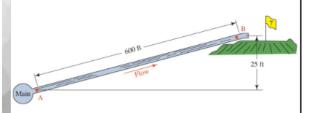
# Series pipeline - Example Problem

The figure shows a pipe delivering water to the putting green on a golf course. The pressure in the main is at 80 psig and it is necessary to maintain a minimum of 60 psig at point B to adequately supply a sprinkler system. Specify the required size of Schedule 40 steel pipe to supply 0.50 ft<sup>3</sup>/s of water at 60 F.

### **Purpose**

Calculate the diameter of the pipeline to satisfy the required flow rate for the given pressure drop.

## **Drawing**



#### Sources

See note for details.

### **Design Considerations**

The following must be assumed:

- 1) Incompressible fluid
- 2) Isothermal process
- Steady state

#### **Data and Variables**

Pressure at main = 80 psig

Pressure at base of sprinkler (point B) = 60 psig

Flow rate = 0.5 ft<sup>3</sup>/s

Wall roughness = 0.000150 ft (table in the book)

$$\gamma = 62.4 \frac{1b}{ft^3}$$
;  $v = 1.21 \times 10^{-5} \frac{ft^2}{s}$  (back of the book)

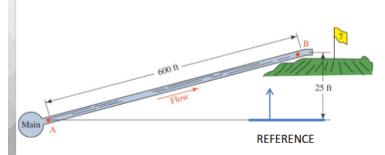
### **Procedure**

Pick a reference.

We will be using Bernoulli's Equation:

$$\frac{p_1}{\gamma} + \frac{{V_1}^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{{V_2}^2}{2g} + Z_2 + h_{L12}$$

As always, we need to decide the 2 points we will apply the equation to. For this type of problems, since the unknown is not clearly explicit in the Bernoulli's equation (it will show up but it is not clearly seen right now), we MUST put the points wherever we know everything (or at least the most).



For this problem, we know the info at points A and B. We do not know velocity up front, however we can make use of the equation  $Q=V^*A$ . We do not know the area, but it depends on what we are looking for.

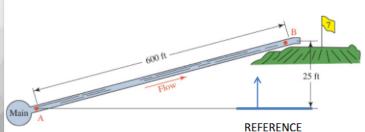
At the end, Bernoulli's equation will become the equation that will allow us to solve for diameter.

### **Procedure**

$$\frac{p_A}{\gamma} + \frac{{V_A}^2}{2g} + Z_A = \frac{p_B}{\gamma} + \frac{{V_B}^2}{2g} + Z_B + h_{LAB}$$

How many sources of energy losses do you recognize between A and B?

The ONLY one is the energy losses in the actual 600 ft long pipe (major losses). In real life, this is very atypical. However, since the pipe is long, the minor losses should not be relevant.



For the major energy losses due to friction, we use Darcy-Weisbach equation:

$$h_L = f \frac{L}{D} \frac{V^2}{2a}$$

To find the friction factor (f) we must calculate Reynolds number and relative roughness

$$Re = \frac{\rho VD}{\mu} = \frac{VD}{v}$$
 Relative roughness  $= \frac{D}{\epsilon}$ 

### Calculations

$$\frac{p_A}{\gamma} + \frac{{V_A}^2}{2g} + Z_A^0 = \frac{p_B}{\gamma} + \frac{{V_B}^2}{2g} + Z_B + h_{LAB}$$

The fluid velocity at A and B are the same, so the corresponding terms cancel each other. Following the reference, the elevation at point A is zero.

Note that if the velocities at A and B were different, there is no problem. For such case, we keep the terms and later use  $Q=V^*A$  in them.

I will manipulate the equation to have all I know at the left hand side (LHS) of the equation.

$$\frac{p_A - p_B}{\gamma} - Z_B = h_{LAB}$$

Substituting the equation for energy losses:

$$\frac{p_A - p_B}{\gamma} - Z_B = f \frac{L}{D} \frac{V^2}{2g}$$

The velocity V is the velocity in the pipe, which we do not know because we do not know the diameter. At this point, let us substitute the equation  $Q=V^*A$  for V:

$$V = \frac{4Q}{\pi D^2}$$

Substituting,

$$\frac{p_A - p_B}{v} - Z_B = f \frac{L}{D} \frac{1}{2a} \frac{16Q^2}{\pi^2 D^4}$$

Or:

$$\frac{p_A - p_B}{\gamma} - Z_B = f \frac{8LQ^2}{a\pi^2} \frac{1}{D^S}$$

$$\frac{p_A - p_B}{\gamma} - Z_B = f \frac{8LQ^2}{g\pi^2} \frac{1}{D^S}$$

At this stage, it seems like all we have to do is to solve for diameter. Let us do it and see what the new equation implies. So,

$$D = \int_{\gamma}^{5} \frac{f \frac{8LQ^2}{g\pi^2}}{\frac{p_A - p_B}{\gamma} - Z_B}$$

Analyzing each term in the equation we notice that from all of them we DO know the length (L), the flow rate (Q), the pressures (p), the elevation (Z), and of course the constants.

However, what we do not know is the friction factor (f).

### Calculations

- Guess friction factor (f).
- 2) Calculate diameter with equation we derived.
- Compute Reynolds number and relative roughness.
- With the values from 3), read friction factor (f) from Moody chart or compute it using equation.
- 5) Compare newly calculated friction factor (f) to the one we use before. If they are the "same", stop the iteration. If not, use new friction factor (f) and go to step 2).

$$D = \int_{-\infty}^{5} \frac{f \frac{8LQ^2}{g\pi^2}}{\frac{p_A - p_B}{\gamma} - Z_B} \qquad f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\varepsilon)} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

$$Re = \frac{VD}{v}$$

 $\text{Relative roughness} = \frac{D}{\epsilon}$ 

The friction factor (f) depends on Reynolds number and relative roughness:

$$Re = \frac{VD}{v}$$
 Relative roughness  $= \frac{D}{\epsilon}$ 

Both of them depend on diameter, which is what we are solving for.

In this case we are forced to solve this by an iterative process where we guess the friction factor value until we get the right one. The procedure is as follows:

- 1) Guess friction factor (f).
- 2) Calculate diameter with equation we derived.
- 3) Compute Reynolds number and relative roughness.
- With the values from 3), read friction factor (f) from Moody chart or compute it using equation.
- 5) Compare newly calculated friction factor (f) to the one we use before. If they are the "same", stop the iteration. If not, use new friction factor (f) and go to step 2).

Regarding this procedure, the main question you might have at this point is how to determine that the friction factor is the "same" because we will never be able to match exactly the same number. For that, we will compute the percentage difference between the newly calculated friction factor and the one used before. Like this:

$$\%diff = \frac{f^{(old)} - f^{(new)}}{f^{(old)}} \times 100$$

The value could be negative or positive, it does not matter in this case. You will stop when the difference is less that certain threshold that you pick yourself.

In my experience there are 3 types of thresholds: 1%, 5%, and 10%. In engineering a difference of more than 10% is considered large already. So, anything below 10%, we could consider it as small. On the other hand, when doing calculations, 1% is a better threshold to minimize calculation errors. However, 5% could also be accepted if the results are taken as an approximation.

- Guess friction factor (f).
- 2) Calculate diameter with equation we derived.
- 3) Compute Reynolds number and relative roughness.
- With the values from 3), read friction factor (f) from Moody chart or compute it using equation.
- 5) Compare newly calculated friction factor (f) to the one we use before. If they are the "same", stop the iteration. If not, use new friction factor (f) and go to step 2).

$$D = \sqrt[5]{\frac{f\frac{8LQ^2}{g\pi^2}}{\frac{p_A - p_B}{\gamma} - Z_A}} \qquad f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\varepsilon)} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

$$Re = \frac{VD}{v}$$
 %diff =  $\frac{f^{\text{(old)}} - f^{\text{(new)}}}{f^{\text{(old)}}} \times 100$   
e roughness =  $\frac{D}{v}$ 

 $\text{Relative roughness} = \frac{D}{\epsilon}$ 

actual calculations is that the friction factor is a value that is always less than 0.1 (see Moody chart). So, a guess in step 1) larger than 0.1 is not recommended.

The other important note before we proceed for the

We can solve this problem using an excel spreadsheet. First, let us have the input data.

Input D	Input Data									
Specific Weight=	62.4	lb/ft^3								
Kinematic Viscosity=	1.21E-05	ft^2/s								
Pressure A=	80	psig	11520	lb/ft^2						
Pressure B=	60	psig	8640	lb/ft^2						
Z_B=	25	ft								
Tube Length=	600	ft								
Flow rate=	0.5	ft^3/s								
Wall Roughness=	0.00015	ft								
g=	32.2	ft/s^2								

Please note that I made the appropriate change of variables.

## Calculations

- Guess friction factor (f).
- 2) Calculate diameter with equation we derived.
- 3) Compute Reynolds number and relative roughness.
- 4) With the values from 3), read friction factor (f) from Moody chart or compute it using equation.
- 5) Compare newly calculated friction factor (f) to the one we use before. If they are the "same", stop the iteration. If not, use new friction factor (f) and go to step 2).

5	$f \frac{8LQ^2}{q\pi^2}$	0.25
$D = \sqrt{\frac{1}{2}}$	$\frac{p_A - p_B}{\gamma} - Z_A$	$\int \left[ \log \left( \frac{1}{3.7(D/\varepsilon)} + \frac{5.74}{Re^{0.9}} \right) \right]^2$

$$Re = \frac{VD}{v} \qquad \qquad \% diff = \frac{f^{(\text{old})} - f^{(\text{new})}}{f^{(\text{old})}} \times 100$$
 Relative roughness =  $\frac{D}{\epsilon}$ 

Now, let us implement the iterative procedure. The table for the iteration procedure looks like this:

						NEW	
Iteration	f	D (ft)	V (ft/s)	Re	D/e	f	%diff

Note that each column is aligned with each of the steps in the procedure. There is a column for velocity not included in the procedure. This is because Reynolds number equation needs velocity. We can get it using V=Q/A.

- Guess friction factor (f).
- 2) Calculate diameter with equation we derived.
- 3) Compute Reynolds number and relative roughness.
- With the values from 3), read friction factor (f) from Moody chart or compute it using equation.
- 5) Compare newly calculated friction factor (f) to the one we use before. If they are the "same", stop the iteration. If not, use new friction factor (f) and go to step 2).

$$D = \sqrt[5]{\frac{f\frac{8LQ^2}{g\pi^2}}{\frac{p_A - p_B}{\gamma} - Z_A}} \qquad f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\varepsilon)} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

$$Re = \frac{VD}{v}$$
 %diff =  $\frac{f^{\text{(old)}} - f^{\text{(new)}}}{f^{\text{(old)}}} \times 100$ 

Relative roughness =  $\frac{D}{\epsilon}$ 

The first iteration looks like this:

						NEW	
Iteration	f	D (ft)	V (ft/s)	Re	D/e	f	%diff
1	0.01000	0.282051	8.002499	1.87E+05	1880.338	0.01922	-92.177%

The percentage difference is very large. We need to continue iterating, which will give us the final answer:

						NEW	
Iteration	f	D (ft)	V (ft/s)	Re	D/e	f	%diff
1	0.01000	0.282051	8.002499	1.87E+05	1880.338	0.01922	-92.177%
2	0.01922	0.321416	6.162325	1.64E+05	2142.775	0.01913	0.459%
3	0.01913	0.321121	6.173667	1.64E+05	2140.806	0.01913	-0.002%
4	0.01913	0.321122	6.173619	1.64E+05	2140.814	0.01913	0.000%

<u>Summary</u> <u>Materials</u>

The minimum diameter of the pipe is 0.3211 ft. Water.

### **Analysis**

a) The actual diameter of the pipe will not be 0.3211 ft because there is no commercial pipe with such specific internal diameter. Using the table in the back of the book, the commercial pipe to use should be a 4 inches pipe Schedule 40. This one has a internal diameter of 0.3355 ft. We cannot use the smaller next to this one because its internal diameter is 0.2957 ft. If using this last one, the pressure at B will be lower than the required 60 psig.

b) If using the 4" Sch40 pipe, depending on how the sprinkler works, the pressure at its base will be larger than 60 psig and/or the flow rate will be larger. To know exactly what exact flow rate and pressure we will get, we will need information on the sprinkler.

We could still exactly match the requirement of pressure at B and flow rate without knowing information about the sprinkler. For such case, we will need the pipe we pre-selected (4" Sch40) and, very important, a valve. The valve has to be at a certain opening, which you must determine. I will show you how in a later problem.

c) If we had a fitting between point A and B, Bernoulli's equation becomes:  $\frac{p_A - p_B}{\gamma} - Z_B = f \frac{8LQ^2}{g\pi^2} \frac{1}{D^5} + K \frac{8Q^2}{g\pi^2} \frac{1}{D^4}$ 

For such case, we cannot solve for D directly. We have to solve numerically by an iteration process. I will show you how in the next problem.

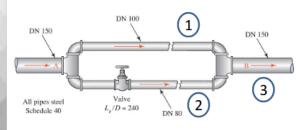
# Parallel pipeline - Example Problem

The figure shows branched system in which the pressure at A is 700 kPa and the pressure at B is 550 kPa. Each branch is 60 m long. Neglect losses at the junctions, but consider all elbows. If the system carries oil with specific weight of  $8.80 \text{ kN/m}^3$ , calculate the total volume flow rate. The oil has kinetic viscosity of  $4.8 \times 10^{-6} \text{ m}^2/\text{s}$ .

### **Purpose**

Compute the total volume flow rate.

### **Drawing**



### Sources

See note for details.

#### **Design Considerations**

The following must be assumed:

- 1) Incompressible fluid
- 2) Isothermal process
- 3) Steady state

#### **Data and Variables**

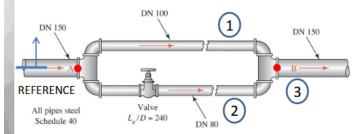
 $\begin{array}{lll} \text{L} = 60 \text{ m}; & \text{Le/D}_{\text{valve}} = 240; & \text{Le/D}_{\text{elbow}} = 30 \\ \text{Pressure at A: } 700 \text{ kPa} & \text{Pressure at B: } 550 \text{ kPa} \\ \text{D}_1 = 0.1023 \text{ m}; & \text{D}_2 = 0.0779 \text{ m}; & \text{D}_3 = 0.1541 \text{ m} \\ \text{Wall roughness} = 4.6 \times 10^{-5} \text{ m} \\ \gamma = 8.80 \text{ kN/m}^3 \; ; \; \upsilon = 4.8 \times 10^{-6} \frac{\text{m}^2}{\text{s}} \\ \end{array}$ 

## Procedure

Pick a reference.

Even on these problems, we will be using Bernoulli's equation. The difference is that we might need to apply it several times as several branches are involved. When doing so to different branches, the energy losses term has to be adjusted to the selected branch.

$$\frac{p_1}{\gamma} + \frac{{V_1}^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{{V_2}^2}{2g} + Z_2 + h_{L12}$$



The other equation to consider for this type of problems is the conservation of mass equation. In this case it is:

$$Q_3 = Q_1 + Q_2$$

As always, we need to decide the 2 points we will apply the equation to. Even for this type of problems, we have to put the points wherever we know the most, or wherever we want to know something about.

For this problem, we know the info at point A and B. We do not know velocity up front, however we could handle that by using the equation  $Q=V^*A$ .

The problem requires us to get  $Q_3$ . If we knew the flow rates 1 and 2 (in each branch), we could get it but we do not know them up front. Let us patiently continue and see how the equations evolve.

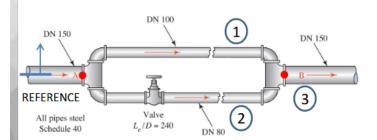
#### **Procedure**

$$\frac{p_A}{\gamma} + \frac{{V_A}^2}{2g} + Z_A = \frac{p_B}{\gamma} + \frac{{V_B}^2}{2g} + Z_B + h_{LAB}$$

Now, the traditional question: How many sources of energy losses do you recognize between A and B?

This time the answer is different because there are 2 ways to go from A to B. Thus, the answer depends on the path I choose. Both are valid and (most importantly!) both are needed.

For path 1, we have a pipe loss and 2 minor losses due to the elbows. For path 2, we have the pipe loss, a valve loss, and 2 elbow losses. Note that the problem stated that we can neglect the junction losses. I will later address this simplification.



For the pipe energy losses, we use Darcy-Weisbach equation:

$$h_L = f \frac{L}{D} \frac{V^2}{2g}$$

To find the friction factor (f) we must calculate Reynolds number and relative roughness

$$Re = \frac{\rho VD}{\mu} = \frac{VD}{v}$$
 Relative roughness  $= \frac{D}{\epsilon}$ 

### **Procedure**

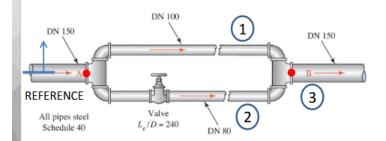
In contrast to how we handled the minor energy losses before, this time I will use the equivalent length (Le) technique. I am doing it just to teach you another technique. The procedure on how to solve this problem is the same regardless of the minor losses technique you choose.

Now, let us work the Bernoulli's equation:

$$\frac{p_A}{\gamma} + \frac{{V_A}^2}{2g} + Z_A = \frac{p_B}{\gamma} + \frac{{V_B}^2}{2g} + Z_B + h_{LAB}$$

The velocities at points A and B are the same because at both points we DO have the same flow rate and we have the same pipe diameter.

The elevation at both points A and B is the same.



Thus, the equation becomes:

$$\frac{p_A - p_B}{\nu} = h_{LAB}$$

Or:

$$\frac{\Delta p}{v} = h_{LAB}$$

Note that the pressure drop is known.

### **Procedure**

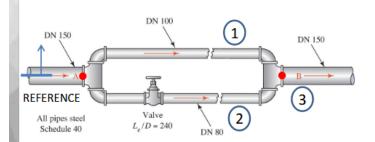
$$\frac{\Delta p}{\gamma} = h_{LAB}$$

Do not get desperate, be patient! Keep working on the equation(s).

Now, let us acknowledge that the energy losses equation depends on the path I choose to go from A to B. If so, the single equation I obtained using Bernoulli's become two equations, one for each branch:

$$\frac{\Delta p}{\gamma} = h_{LAB_1}$$

$$\frac{\Delta p}{\gamma} = h_{LAB_2}$$



Let us work now on the energy losses terms for each of those equations. For branch 1:

$$\frac{\Delta p}{\gamma} = f_1 \frac{(L_1 + 2 \times Le_{\text{elbow1}})}{D_1} \frac{V_1^2}{2g}$$

$$\frac{\Delta p}{\gamma} = f_1 \left( \frac{L_1}{D_1} + 2 \times \frac{Le}{D} \Big|_{\text{elbow1}} \right) \frac{V_1^2}{2g}$$

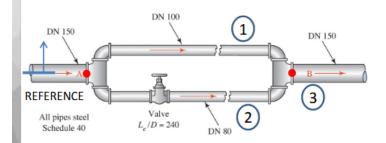
## Procedure

$$\frac{\Delta p}{\gamma} = f_1 \left( \frac{L_1}{D_1} + 2 \times \frac{Le}{D} \right|_{\text{elbow1}} \right) \frac{V_1^2}{2g}$$

let us substitute the equation  $Q=V^*A$  for V:

$$V = \frac{4Q}{\pi D^2}$$

$$\frac{\Delta p}{\gamma} = f_1 \left( \frac{L_1}{D_1} + 2 \times \frac{Le}{D} \right|_{elbow1} \right) \frac{1}{2g} \frac{16Q_1^2}{\pi^2 D_1^4}$$



Now, for branch 2, following similar steps, we get:

$$\frac{\Delta p}{\gamma}$$

$$= f_2 \left( \frac{L_2}{D_2} + 2 \times \frac{Le}{D} \right)_{\text{elbow2}}$$

$$+ \frac{Le}{D} \Big|_{\text{valve2}} \frac{1}{2g} \frac{16Q_2^2}{\pi^2 D_2^4}$$

$$\frac{\Delta p}{\gamma} = f_1 \left( \frac{L_1}{D_1} + 2 \times \frac{Le}{D} \right|_{\text{elbow1}} \right) \frac{1}{2g} \frac{16Q_1^2}{\pi^2 D_1^4}$$

$$\frac{\Delta p}{\gamma} = f_2 \left( \frac{L_2}{D_2} + 2 \times \frac{Le}{D} \right|_{\text{elbow2}} + \frac{Le}{D} \right|_{\text{valve2}} \frac{1}{2g} \frac{16Q_2^2}{\pi^2 D_2^4}$$

At this stage, it seems like all we have to do is to solve for flow rate in each branch. Then add them up to get the total flow rate. Thus:

$$Q_1 = \sqrt{\frac{\frac{\Delta p}{\gamma}}{f_1 \left(\frac{L_1}{D_1} + 2 \times \frac{Le}{D}\Big|_{\text{elbow1}}\right) \frac{8}{g\pi^2 D_1^4}}}$$

$$Q_{2} = \sqrt{\frac{\frac{\Delta p}{\gamma}}{f_{2} \left(\frac{L_{2}}{D_{2}} + 2 \times \frac{Le}{D}\Big|_{\text{elbow2}} + \frac{Le}{D}\Big|_{\text{valve2}}\right) \frac{8}{g\pi^{2}D_{2}^{4}}}}$$

### **Calculations**

$$Q_1 = \sqrt{\frac{\frac{\Delta p}{\gamma}}{f_1 \left(\frac{L_1}{D_1} + 2 \times \frac{Le}{D} \Big|_{\text{elbow1}}\right) \frac{8}{g\pi^2 D_1^4}}}$$

$$Q_2 = \sqrt{\frac{\frac{\Delta p}{\gamma}}{f_2 \left(\frac{L_2}{D_2} + 2 \times \frac{Le}{D}\Big|_{\text{elbow2}} + \frac{Le}{D}\Big|_{\text{valve2}}\right) \frac{8}{g\pi^2 D_2^4}}}$$

- 1) Guess friction factors ( $f_1$  and  $f_2$ ).
- 2) Calculate flow rates with equations we derived.
- 3) Compute each Reynolds number.
- With the values from 3), read friction factors (f<sub>1</sub> and f<sub>2</sub>) from Moody chart or compute it using equation.
- Compare newly calculated friction factors to the one we used before. If they are the "same", stop the iteration. If not, use new friction factors and go to step 2).

Analyzing each term in the equations we notice that from all of them we DO know the length (L), the pipe internal diameter (D), the pressures (p), the fitting equivalent lengths, and the constants.

However, what we do not know is the friction factor (f).

The friction factor (f) depends on Reynolds number and relative roughness. The later is known (as it was mentioned) but not the Reynolds number:

$$Re = \frac{VD}{v}$$

Reynolds number depends on velocity that depends on flow rate, which is what we are solving for.

We are forced to solve this by an iterative process where we guess the friction factor value until we get the right one. The procedure is shown next.

Regarding this procedure, the main question you might have at this point is how to determine that the friction factor is the "same" because we will never be able to match exactly the same number. For that, we will compute the percentage difference between the newly calculated friction factor and the one used before. Like this:

$$\% diff = \frac{f^{\text{(old)}} - f^{\text{(new)}}}{f^{\text{(old)}}} \times 100$$

The value could be negative or positive, it does not matter in this case. You will stop when the difference is less that certain threshold that you pick yourself.

In my experience there are 3 types of thresholds: 1%, 5%, and 10%. In engineering a difference of more than 10% is considered large already. So, anything below 10%, we could consider it as small. On the other hand, when doing calculations, 1% is a better threshold to minimize calculation errors. However, 5% could also be accepted if the results are taken as an approximation.

A quick note: we could have solved the problem by the other iterative technique I taught you before. We could solve it by guessing  $\mathcal Q$  values until the right-hand-side (RHS) of the equation becomes equal to the left-hand-side (LHS).

$$\frac{\Delta p}{\gamma} = f_1 \left( \frac{L_1}{D_1} + 2 \times \frac{Le}{D} \right|_{\text{elbow1}} \right) \frac{1}{2g} \frac{16Q_1^2}{\pi^2 D_1^4}$$

$$\text{LHS} \qquad \text{RHS}$$

$$\frac{\Delta p}{\gamma} = f_2 \left( \frac{L_2}{D_2} + 2 \times \frac{Le}{D} \right|_{\text{elbow2}} + \frac{Le}{D} \right|_{\text{valve2}} \right) \frac{1}{2g} \frac{16Q_2^2}{\pi^2 D_2^4}$$
LHS
RHS

The left-hand-side (LHS) of the equations is constant and can be computed ahead of time. The procedure is:

## **Calculations**

- 1) Guess friction factors ( $f_1$  and  $f_2$ ).
- 2) Calculate flow rates with equations we derived.
- 3) Compute each Reynolds number.
- With the values from 3), read friction factors (f<sub>1</sub> and f<sub>2</sub>) from Moody chart or compute it using equation.
- Compare newly calculated friction factors to the one we used before. If they are the "same", stop the iteration. If not, use new friction factors and go to step 2).

$$\begin{split} Q_1 &= \sqrt{\frac{\frac{\Delta p}{\gamma}}{f_1 \left(\frac{L_1}{D_1} + 2 \times \frac{Le}{D} \Big|_{\text{elbow1}}\right) \frac{8}{g\pi^2 D_1^4}}} \quad \% \text{diff} = \frac{f^{(\text{old})} - f^{(\text{new})}}{f^{(\text{old})}} \times 100 \end{split}$$

$$Q_2 &= \sqrt{\frac{\frac{\Delta p}{\gamma}}{f_2 \left(\frac{L_2}{D_2} + 2 \times \frac{Le}{D} \Big|_{\text{elbow2}} + \frac{Le}{D} \Big|_{\text{valve2}}\right) \frac{8}{g\pi^2 D_2^4}}$$

$$Re &= \frac{VD}{v} \qquad \qquad f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\varepsilon)} + \frac{5.74}{Re^{0.9}}\right)\right]^2} \end{split}$$

- 1) Guess flow rates  $(Q_1 \text{ and } Q_2)$ .
- 2) Compute Reynolds number in each branch.
- 3) With the values from 2), read friction factor ( $f_1$  and  $f_2$ ) from Moody chart or compute them using equation.
- Compute the right-hand-side (RHS) of both equations.
- Compare the RHS values to the corresponding LHS values for each equation. If they are the "same", stop the iteration. If not, go to step 1).

As before, to determine whether LHS=RHS, we will compute the percentage difference like this:

$$\%$$
diff =  $\frac{LHS - RHS}{LHS} \times 100$ 

The value could be negative or positive. Analyze the equation and you will notice that if it is negative is because the guessed flow rate is too large, and if it is positive is because the guessed flow rate is too small. You will stop when the difference is less that certain threshold as has been discussed before.

I will choose the 1<sup>st</sup> one. However, I must admit that this one works only in some cases when you are able to manipulate the equation and solve for the variable of interest, whereas the 2<sup>nd</sup> one works always.

We can solve this problem using an excel spreadsheet. First, let us have the input data.

Input Data									
Specific Weight=	8.8	kN/m^3							
Kinematic Viscosity=	4.80E-06	m^2/s							
Pressure A=	700	kPa							
Pressrue B=	550	kPa							
Pipe length = L1 = L2=	60	m							
D1=	0.1023	m							
D2=	0.0779	m							
D3=	0.1541	m							
Wall Roughness=	4.60E-05	m							
Le/D valve=	240								
Le/D elbow=	30								

D1/e=	2223.91
D2/e=	1693.48
σ=	9.81 m/s^2

- l) Guess friction factors  $(f_1 \text{ and } f_2)$ .
- 2) Calculate flow rates with equations we derived.
- 3) Compute each Reynolds number.
- With the values from 3), read friction factors (f<sub>1</sub> and f<sub>2</sub>) from Moody chart or compute it using equation.
- Compare newly calculated friction factors to the one we used before. If they are the "same", stop the iteration. If not, use new friction factors and go to step 2).

$$Q_{1} = \sqrt{\frac{\frac{\Delta p}{\gamma}}{f_{1} \left(\frac{L_{1}}{D_{1}} + 2 \times \frac{Le}{D}\Big|_{elbow1}\right) \frac{8}{g\pi^{2} D_{1}^{4}}}}$$
%diff =  $\frac{f^{(old)} - f^{(new)}}{f^{(old)}} \times 100$ 

$$Q_{2} = \sqrt{\frac{\frac{\Delta p}{\gamma}}{f_{2} \left(\frac{L_{2}}{D_{2}} + 2 \times \frac{Le}{D}\Big|_{elbow2} + \frac{Le}{D}\Big|_{valve2}\right) \frac{8}{g\pi^{2} D_{2}^{4}}}}$$

$$Re = \frac{VD}{v} \qquad f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\epsilon)} + \frac{5.74}{Re^{0.9}}\right)\right]^{2}}$$

Now, let us implement the iterative procedure. The tables for the iteration procedure for each flow rate look like this:

FOR Q1							FOR Q2						
					NEW							NEW	
Iteration	f1	Q1 (m3/s)	V1 (m/s)	Re1	f1	%diff	Iteration	f2	Q2 (m3/s)	V2 (m/s)	Re2	f2	%diff
1							1						

Note that each column is aligned with each of the steps in the procedure. There is a column for velocity not included in the procedure. This is because Reynolds number equation needs velocity. We can get it using V=Q/A.

#### Calculations

- 1) Guess friction factors ( $f_1$  and  $f_2$ ).
- Calculate flow rates with equations we derived.
- 3) Compute each Reynolds number.
- 4) With the values from 3), read friction factors  $(f_1 \text{ and } f_2)$  from Moody chart or compute it using equation.
- Compare newly calculated friction factors to the one we used before. If they are the "same", stop the iteration. If not, use new friction factors and go to step 2).

$$Q_{1} = \sqrt{\frac{\frac{\Delta p}{f_{1}\left(\frac{L_{1}}{D_{1}} + 2 \times \frac{Le}{D}\Big|_{elbow1}\right) \frac{8}{g\pi^{2}D_{1}^{4}}}} \qquad \%diff = \frac{f^{(old)} - f^{(new)}}{f^{(old)}} \times 100$$

$$Q_{2} = \sqrt{\frac{\frac{\Delta p}{f_{2}\left(\frac{L_{2}}{D_{2}} + 2 \times \frac{Le}{D}\Big|_{elbow2} + \frac{Le}{D}\Big|_{valve2}\right) \frac{8}{g\pi^{2}D_{2}^{4}}}$$

$$Re = \frac{VD}{v} \qquad f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\varepsilon)} + \frac{5.74}{Re^{0.9}}\right)\right]^{2}}$$

The first iteration looks like this:

FOR Q1							FOR Q2						
					NEW							NEW	
Iteration	f1	Q1 (m3/s)	V1 (m/s)	Re1	f1	%diff	Iteration	f2	Q2 (m3/s)	V2 (m/s)	Re2	f2	%diff
1	0.01000	0.059116	7.19227	1.53E+05	0.01918	-91.767%	1	0.01000	0.026643	5.59007	9.07E+04	0.02105	-110.47%

The percentage difference is very large. We need to continue iterating, which will give us the final answer:

- 1) Guess friction factors ( $f_1$  and  $f_2$ ).
- 2) Calculate flow rates with equations we derived.
- 3) Compute each Reynolds number.
- With the values from 3), read friction factors (f<sub>1</sub> and f<sub>2</sub>) from Moody chart or compute it using equation.
- 5) Compare newly calculated friction factors to the one we used before. If they are the "same", stop the iteration. If not, use new friction factors and go to step 2).

$$Q_{1} = \sqrt{\frac{\frac{\Delta p}{\gamma}}{f_{1}(\frac{L_{1}}{D_{1}} + 2 \times \frac{Le}{D}|_{elbow1})\frac{8}{g\pi^{2}D_{1}^{4}}}} \qquad \%diff = \frac{f^{(old)} - f^{(new)}}{f^{(old)}} \times 10$$

$$Q_{2} = \sqrt{\frac{\frac{\Delta p}{\gamma}}{f_{2}(\frac{L_{2}}{D_{2}} + 2 \times \frac{Le}{D}|_{elbow2} + \frac{Le}{D}|_{valve2})\frac{8}{g\pi^{2}D_{2}^{4}}}}$$

$$Re = \frac{VD}{v} \qquad f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\varepsilon)} + \frac{5.74}{Re^{0.9}}\right)\right]^{2}}$$

FOR Q1							FOR Q2						
					NEW							NEW	
Iteration	f1	Q1 (m3/s)	V1 (m/s)	Re1	f1	%diff	Iteration	f2	Q2 (m3/s)	V2 (m/s)	Re2	f2	%diff
1	0.01000	0.059116	7.19227	1.53E+05	0.01918	-91.767%	1	0.01000	0.026643	5.59007	9.07E+04	0.02105	-110.47%
2	0.01918	0.042689	5.19373	1.11E+05	0.01993	-3.923%	2	0.02105	0.018365	3.85317	6.25E+04	0.02214	-5.214%
3	0.01993	0.041876	5.09477	1.09E+05	0.01998	-0.246%	3	0.02214	0.017904	3.75649	6.10E+04	0.02223	-0.378%
4	0.01998	0.041825	5.08851	108448.8	0.01998	-0.016%	4	0.02223	0.01787	3.74942	6.08E+04	0.02223	-0.028%
5	0.01998	0.041821	5.08811	1.08E+05	0.01998	-0.001%	5	0.02223	0.017868	3.74889	6.08E+04	0.02224	-0.002%
6	0.01998	0.041821	5.08808	1.08E+05	0.01998	0.000%	6	0.02224	0.017867	3.74885	6.08E+04	0.02224	0.000%

## **Summary**

The flow rate passing through the upper branch is  $0.0418 \text{ m}^3/\text{s}$  and the lower branch  $0.0178 \text{ m}^3/\text{s}$ . The total flow rate through the system is  $0.0597 \text{ m}^3/\text{s}$ .

## Materials

Water.

## **Analysis**

- 1) Note that to solve the problem I did not need the actual pressures at A and B, just the pressure difference.
- 2) The fluid velocity in each branch is larger than the critical velocity. I would suggest to increase the pipe diameter.
- 3) There are many other parallel pipeline type of problems that I will discuss in the next set of problems. Let me present a list before that, which is by no mean a comprehensive one.

## Analysis

There are a couple of other real industrial situations you might also face when working:

- 1. What if we want to consider the energy losses at the junctions? In real life you must.
- 2. On the problem we just discussed, we knew the pressure at the junctions. This is not going to happen in real life, we will not know them there. What to do? (Put the points where you know the most!!!)
- What if I want to control the flow rate with the valve? This is a more difficult task for a parallel system than for a series system but you might be asked in the industry to do this.
- 4. What if I know the total flow rate and the pipeline configuration? How do I get the pressure drop from junction to junction?
- 5. What is the procedure to design a parallel system that will handle a specified flow rate in each branch? We will begin with critical velocity criteria (as discussed in the analysis of an earlier problem). However after selecting the pipe size, we will not get the specific flow we want in the branches. We will need a valve. It is important to point out that the procedure of selecting a valve and determining its opening is not exactly the same as the one discussed in prior problems. Before, we were adjusting the flow rate for an existing system (there was a fixed pressure drop). Now we are designing the pipeline for specific flow rates, the pressure drop is not determined.
- We could increase the flow rate of a single pipeline system by adding a parallel branch to the single pipe.The increase can be controlled by the new branch pipe diameter and/or its length.

I will present all these other industrial situations by modifying this problem to discuss.

		Exceeds Standard	Meets Standard	Approaches Standard	Needs Attention
		100 points	70 points	40 points	0 points
1.	Purpose 5%	The purpose of the section to be answered is clearly identified and stated.	The purpose of the section to be answered is identified, but is stated in a somewhat unclear manner.	The purpose of the section to be answered is partially identified, and is stated in a somewhat unclear manner.	The purpose of the section to be answered is erroneous or irrelevant.
2.	Drawings & Diagrams	Clear and accurate diagrams are included and make the section easier to understand. Diagrams are labeled neatly and accurately.	Diagrams are included and are labeled neatly and accurately.	Diagrams are included and are labeled.	Needed diagrams are missing OR are missing important labels.
3.	Sources 5%	Several reputable background sources were used and cited correctly.	A few reputable background sources are used and cited correctly.	A few background sources are used and cited correctly, but some are not reputable sources.	Background sources are cited incorrectly.
4.	Design considerations (safety, cost, etc)	Design is carried out with applicable assumptions and full attention to safety and cost, etc.	Design is generally carried out with assumptions and attention to safety, cost, etc.	Design is carried out with some assumptions and some attention to safety, cost, etc.	Assumptions, safety and cost were ignored in the design.
5.	Data and variables	All data and variables are clearly described with all relevant details. The units of all values are shown.	All data and variables are clearly described with most relevant details.	Most data and variables are clearly described with most relevant details.	Data and variables are not described OR the majority lack sufficient detail.
6.	Procedure 25%	Procedure is described in clear steps. The step description is in a complete and easy to understand short paragraph.	Procedure is described in clear steps but the step description is not in a complete short paragraph.	Procedure is described in clear steps. The step description is in a complete short paragraph but it is difficult to understand.	Procedure is not described in clear steps at all.
7.	Calculations 20%	All calculations are shown and the results are correct and labeled appropriately. The units of all values are shown.	Some calculations are shown and the results are correct and labeled appropriately.	Some calculations are shown and the results labeled appropriately.	No calculations are shown OR results are inaccurate or mislabeled.
8.	Summary 5%	Summary describes the design, the relevant information and some future implications.	Summary describes the design and some relevant information.	Summary describes the design.	No summary is written.
9.	Materials 5%	All materials used in the design are clearly and accurately described.	Almost all materials used in the design are clearly and accurately described.	Most of the materials used in the design are clearly and accurately described.	Many materials are described inaccurately OR are not described at all.
10	0. Analysis 10%	The design is discussed and analyzed. Argumentative predictions are made about what might happen in case of change in the operation and how the design could be change.	The design is discussed and analyzed. Argumentative predictions are made about what might happen in case of change in the operation.	The design is discussed and analyzed. No argumentative predictions are made about what might happen in case of change in the operation and how the design could be change.	The design is not discussed and analyzed.