The Euclidean Algorithm follows the principle:

GCD(a,b)=GCD(b,amod b)GCD(a, b) = GCD(b, a \mod b)GCD(a,b)=GCD(b,amodb)

1. 228÷44=5228 \div 44 = 5228÷44=5 remainder 228−44×5=228−220=8228 - 44 \times 5 = 228 - 220 = 8228−44×5=228−220=8
   → GCD(228,44)=GCD(44,8)GCD(228, 44) = GCD(44, 8)GCD(228,44)=GCD(44,8)
2. 44÷8=544 \div 8 = 544÷8=5 remainder 44−8×5=44−40=444 - 8 \times 5 = 44 - 40 = 444−8×5=44−40=4
   → GCD(44,8)=GCD(8,4)GCD(44, 8) = GCD(8, 4)GCD(44,8)=GCD(8,4)
3. 8÷4=28 \div 4 = 28÷4=2 remainder 8−4×2=08 - 4 \times 2 = 08−4×2=0
   → GCD(8,4)=4GCD(8, 4) = 4GCD(8,4)=4

Thus, **GCD(228, 44) = 4**.

i	ri	qi	si	ti
0	19	-	1	0
1	9	2	0	1
2	1	9	1	-2
3	0	-	-2	5

Since the last nonzero remainder is **1**, we take t2=−2t_2 = -2t2=−2 and adjust it into the modulo domain:
−2mod 19=17-2 \mod 19 = 17−2mod19=17.

Thus, **the multiplicative inverse of 9 mod 19 is 17**.

Question 3: Euler's Totient Function φ(3200)φ(3200)φ(3200)

Euler's totient function is given by:

φ(n)=n∏p∣n(1−1p)φ(n) = n \prod_{p \mid n} \left(1 - \frac{1}{p}\right)φ(n)=np∣n∏(1−p1)

Factorizing 3200:

3200=27×523200 = 2^7 \times 5^23200=27×52 φ(3200)=3200×(1−12)×(1−15)φ(3200) = 3200 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{5}\right)φ(3200)=3200×(1−21)×(1−51) =3200×12×45= 3200 \times \frac{1}{2} \times \frac{4}{5}=3200×21×54 =3200×410=3200×0.4=1280= 3200 \times \frac{4}{10} = 3200 \times 0.4 = 1280=3200×104=3200×0.4=1280

Thus, **φ(3200) = 1280**.

Fermat's theorem states:

ap−1≡1mod pa^{p-1} \equiv 1 \mod pap−1≡1modp

For an inverse:

ap−2≡a−1mod pa^{p-2} \equiv a^{-1} \mod pap−2≡a−1modp

Thus, we compute:

919−2=917mod 199^{19-2} = 9^{17} \mod 19919−2=917mod19

Breaking it using modular exponentiation:

92=81≡5mod 199^2 = 81 \equiv 5 \mod 1992=81≡5mod19 94=(92)2=52=25≡6mod 199^4 = (9^2)^2 = 5^2 = 25 \equiv 6 \mod 1994=(92)2=52=25≡6mod19 98=(94)2=62=36≡17mod 199^8 = (9^4)^2 = 6^2 = 36 \equiv 17 \mod 1998=(94)2=62=36≡17mod19 916=(98)2=172=289≡4mod 199^{16} = (9^8)^2 = 17^2 = 289 \equiv 4 \mod 19916=(98)2=172=289≡4mod19 917=916×9=4×9=36≡17mod 199^{17} = 9^{16} \times 9 = 4 \times 9 = 36 \equiv 17 \mod 19917=916×9=4×9=36≡17mod19

Thus, **the multiplicative inverse of 9 mod 19 is 17**.

Question 5: Compute 5300mod 315^{300} \mod 315300mod31 using Euler's Theorem

Euler's theorem states:

aφ(n)≡1mod na^{φ(n)} \equiv 1 \mod naφ(n)≡1modn

For prime 31,

φ(31)=31−1=30φ(31) = 31 - 1 = 30φ(31)=31−1=30

Thus,

530≡1mod 315^{30} \equiv 1 \mod 31530≡1mod31

Since 300=30×10300 = 30 \times 10300=30×10,

5300=(530)10≡110≡1mod 315^{300} = (5^{30})^{10} \equiv 1^{10} \equiv 1 \mod 315300=(530)10≡110≡1mod31

Thus, **5300≡1mod 315^{300} \equiv 1 \mod 315300≡1mod31**.