Test 2 - Joel Adriano

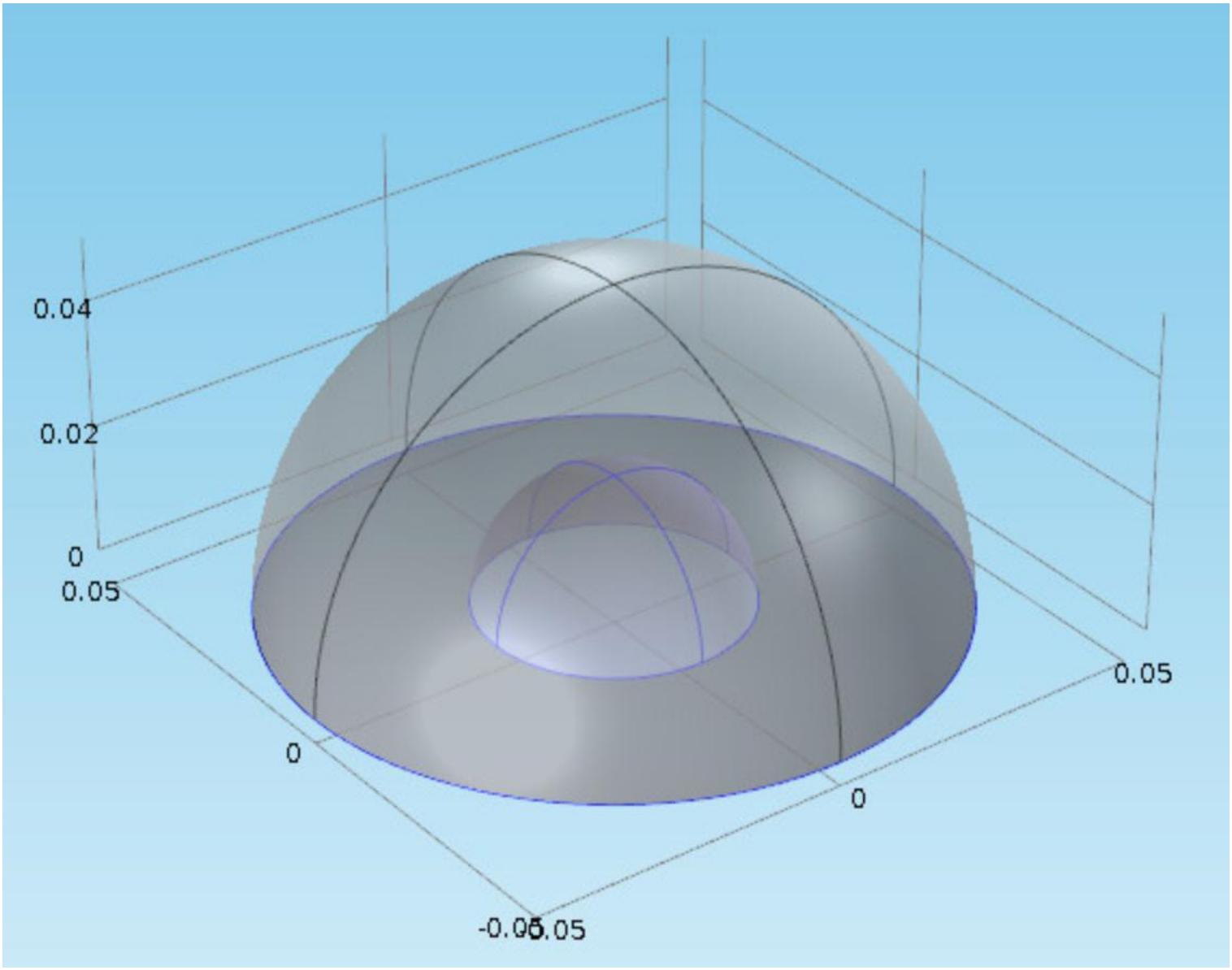
Question 1

Purpose

The purpose of the first question of this test is to observe steady heat generation. It can be assumed the element is at steady-state heat generation because the generating heat (go) (and thermal conductivity) is constant. It is required to find and develop a one-dimensional temperature profile equation to solve this question.

Drawings & Diagrams

The shape in which we are trying to find the rate of heat dissipation is a hollow hemisphere.



Sources

Heat Transfer Fundamentals 1st Edition, Begell House, Inc. by Yildiz Bayazitoglu & Necati Ozisik

Design Considerations

Since this substance is specifically a hollow hemisphere, prior considerations may be followed.

Data & Variables

$$g = g_0 \frac{b^2}{r^2} \ W/m^3$$

$$k = k_0 \frac{b^2}{r^2} \ W/mK$$

$$h = T_\infty$$

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 k \frac{dT}{dr} \right) + g = 0$$

$$r = a, b$$

$$\frac{dT}{dr} = 0$$

$$-k \frac{dT}{dr} = h(T - T_\infty)$$

Procedure

To gather second constant, the derivative of the first constant needs to be solved for using differential equations.

Calculations

Obtaining the temperature profile equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 k \frac{dT}{dr}\right) + g = 0 \Rightarrow \frac{1}{r^2}\frac{d}{dr}\left(r^2 k_0 \frac{b^2}{r^2}\frac{dT}{dr}\right) + g_0\frac{b^2}{r^2} = 0$$

$$\Rightarrow \left[\frac{k_0 b^2}{r^3}\frac{d^2T}{dr^2} + g_0\frac{b^2}{r^2} = 0\right]\frac{r^3}{k_0 b^2}$$

$$\frac{d^2T}{dr^2} + \frac{g_0 r}{k_0} = 0$$

$$\left[\frac{d}{dr}\left(\frac{dT}{dr}\right) + \frac{g_0 r}{k_0} = 0\right]dr$$

$$\int d\left(\frac{dT}{dr}\right) + \int \frac{g_0 r}{k_0}dr = \int 0\,dr$$

$$\left[\frac{dT}{dr} + \frac{g_0 r^2}{k_0} = C_1\right]dr$$

$$\int dT + \int \frac{g_0 r^2}{k_0}dr = \int C_1\,dr + \int 0\,dr$$

$$T + \frac{g_0}{k_0}\frac{r^3}{3} = C_1 r + C_2$$

$$\boxed{T = -\frac{g_0 r^3}{3k_0} + C_1 r + C_2}$$

$$\frac{dT}{dr} + \frac{g_0 r^2}{k_0} = C_1$$

$$0 + \frac{g_0 a^2}{k_0} = C_1$$

$$\boxed{C_1 = \frac{g_0 a^2}{k_0}}$$

$$-k_0\frac{b^2}{b^2}\left[C_1 - \frac{g_0 r^2}{k_0}\right] = h\left(\frac{g_0}{k_0}\frac{r^3}{3} + C_1 r + C_2 - T_\infty\right)$$

$$-k_0\frac{b^2}{b^2}\left[\frac{g_0 a^2}{k_0} - \frac{g_0 b^2}{k}\right] = h\left(\frac{g_0}{k_0}\frac{b^3}{3} + \frac{g_0}{k_0}ab + C_2 - T_\infty\right)$$

$$-k_0\frac{g_0}{k_0}\left[a - b\right] = h\left[\frac{g_0}{k_0}b\left(a - \frac{b^2}{3}\right) + C_2 - T_\infty\right]$$

$$-\frac{g_0}{h}(a-b) = \frac{g_0}{k_0}b\left(a - \frac{b^2}{3}\right) + C_2 - T_\infty$$

$$C_2 = -\frac{g_o}{h}(a-b) - \frac{g_o}{k_o} b\left(a - \frac{b^2}{3}\right) + T_\infty$$

$$T = -\frac{g_o}{k_o}\frac{r^3}{3} + C_1 r + C_2$$

$$T = -\frac{g_o}{k_o}\frac{r^3}{3} + \frac{g_o}{k_o} a^2 r - \frac{g_o}{h}(a-b) - \frac{g_o}{k_o} b\left(a - \frac{b^2}{3}\right) + T_\infty$$

If heat generation (go) is zero, the only thing that is left is Tinf (100°F) because the other terms considered are zeroed out.

Summary

It can be concluded from the temperature profile equation that if there is no heat generation (go), then the only temperature affecting the semi-sphere element is the atmosphere.

Materials

Semi-sphere element

Analysis

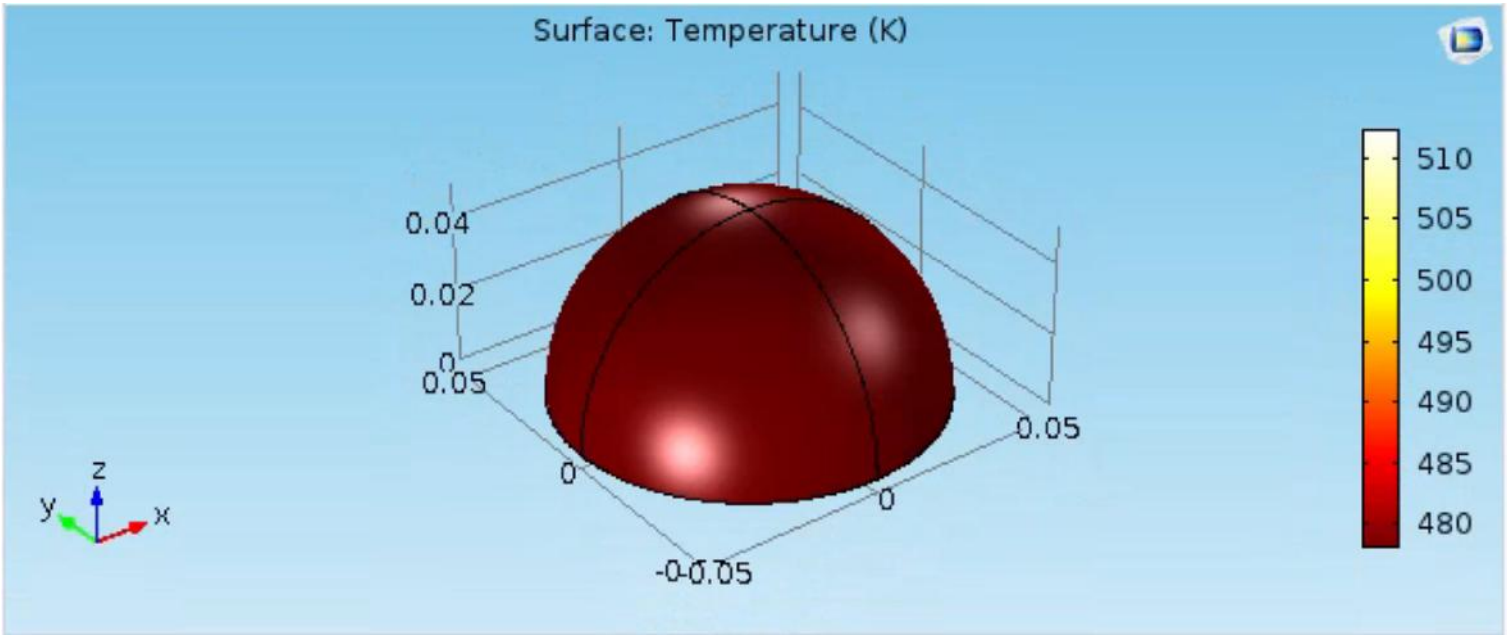
This test requires the knowledge of steady heat generation within a substance and the application of differential equations.

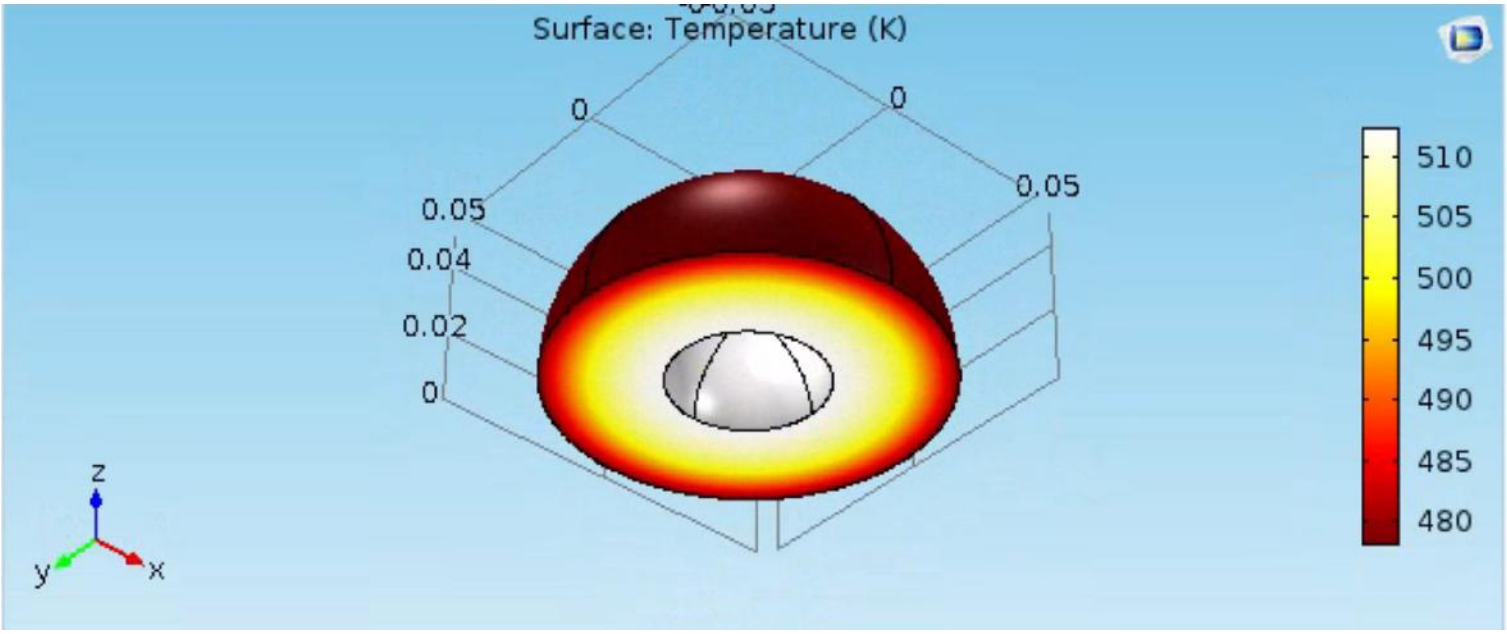
Question 2

Purpose

This question will need to be solved with COMSOL. This provides a physical solution of how steady-state heat generation works for a hollow hemisphere through a simulation based application. COMSOL can organize analytical and numerical temperature profiles into charts to provide comparison between the two.

Drawings & Diagrams





Sources

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Design Considerations

The substance's shape is a hollow hemisphere.

Data & Variables

$k_o = 10\ W/mK$
$h = 200\ W/m^2K$
$a = 2cm$
$b = 5cm$
$T_\infty = 100°C$
$g_o = 10^6\ W/m^3$

Procedure

The first step to solve for this question is to actually try to model a hollow hemisphere which involves join primitive shapes and using Boolean functions to cut away any bits to get the right model. The heat sources are then added to the substance (inner surface and bottom are subjected to dissipation of heat by convection with the value of "h" and the ambient temperature affects the surface of the hemisphere. The final step is to gather the heat transfer at the surface of the element and gather the plot for analytical and numerical plot graphs to get the efficiency error between them.

Calculations

$$T = -\frac{g_o}{k_o}\frac{r^3}{3} - \frac{g_o}{k_o}a^2 r - \frac{g_o}{h}(a-b) - \frac{g_o}{k_o}b\left(a - \frac{b^2}{3}\right) + T_\infty$$

$$T = -\frac{0}{10}\frac{3^3}{3} - \frac{0}{10}(2^2)(3) - \frac{0}{200}(2-5) - \frac{0}{10}(5)\left(2 - \frac{5^2}{3}\right) + 100$$

$$T = 0 - 0 - 0 - 0 + 100 \Rightarrow \boxed{T = 100°C}$$

Summary

From the first question, if there is no heat generation (go), there will only be the atmospheric temperature affecting it (Tinf).

Materials

COMSOL

Analysis

This question of the test requires the knowledge of using the COMSOL application and knowledge of steady heat generation.