16.6) Given

$V = 22.0 \; ft/s$

mass flow RATE

$$\dot{m} = \rho A v = (62.4)(2.95 \times \tfrac{1}{144})(22)$$

$$m \approx 28.12 \quad lb \cdot m/s$$

initial horizontal velocity $= v_1$

final horizontal velocity $= -v_2 \cos(180 - \theta) = v_2 \cos\theta$

acting on VANE

$$F_H = m\,(-v_2 \cos(\theta) + v_1)$$

$$= m \times v \; (-\cos\theta + 1)$$

$$= (28.12)(22)\;(\cos(130) - 1)$$

$$\boxed{F_H = 1016.29 \;\; lb \cdot m \cdot ft/s}$$

vertical force

$$F_V = v_2 \sin(130)(m) = (22)(\sin(130))(28.12)$$

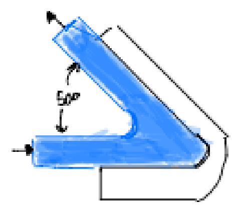
$$\boxed{F_V = 473.91 \;\; lb \cdot m \cdot ft/s}$$

Graham Charboneau | MET 330 | Homework 1.7 |

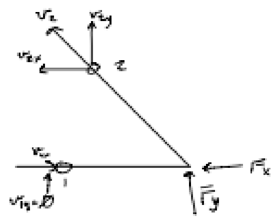
Chapter 16:

Problem 16-6) Figure 16.13 shows a free stream of water at 180°F being deflected by a stationary vane through a 130° angle. The entering stream is has a velocity of 22.0 ft/s. The cross-sectional area of the stream is constant at 2.95 in^2 throughout the system. Compute the forces in the horizontal and vertical directions exert3d on the water by the vane.

Figure 16.13



FBD



Given: $V = 22.0$ ft/s
$A = 2.95\ in^2 = 0.0205\ ft^2$
$T_w = 180°F$
$\theta = 180° - 130° = 50°$

- $\rho_{w@180°F} = 1.88\ \frac{slugs}{ft^3}$
- $Q = Av = (0.0205\ ft^2)(22.0\ \frac{ft}{s})$
  $Q = 0.45\ \frac{ft^3}{s}$

$$\boxed{\begin{aligned} \therefore\ F_x &= 30.57\ lb \\ F_y &= 14.25\ lb \end{aligned}}$$

cos of complete horizontal

- $V_{1x} = -V \times 1$
  $V_{1x} = -22\ \frac{ft}{s}$

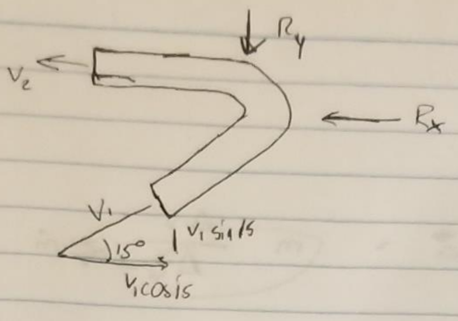
- $V_{2x} = V \cdot \cos(50°)$
  $= 22\ \frac{ft}{s} \cos(50°)$
  $V_{2x} = 14.14\ \frac{ft}{s}$

- $V_{2y} = V \cdot \sin(50°)$
  $= 22\ \frac{ft}{s} \sin(50°)$
  $V_{2y} = 16.85\ \frac{ft}{s}$

- $F_x = \rho Q (V_{2x} - V_{1x})$
  $= (1.88)(0.45)(14.14 - (-22))\ \frac{slug}{ft^3} \cdot \frac{ft^3}{s} \cdot \frac{ft}{s} = slug \cdot \frac{ft}{s^2}$
  $= (1.88)(0.45)(36.14)$ lb
  $\underline{F_x = 30.57\ lb}$

- $F_y = \rho Q (V_{2y} - V_{1y})$
  $= (1.88)(0.45)(16.85 - 0)\ \frac{slug}{ft^3} \cdot \frac{ft^3}{s} \cdot \frac{ft}{s} = 1\ lb$
  $\underline{F_y = 14.25\ lb}$

20)



$V_1 = 30\,m/s$

$N_{dia} = 200\,mm$

$d = .2m$

density of water = $1000\,kg/m^3$

$$Q = A \cdot V = \frac{\pi}{4} d^2 \times V$$
$$= \frac{\pi}{4} \cdot .2m^2 \cdot 30\,m/s$$
$$= .943\,m/s$$

$$R_x = \rho Q\left[(V_2) - (-V_1 \cos\theta)\right]$$
$$= \rho Q V (1 + \cos\theta)$$

$$= 1{,}000 \times .943 \times 30\,(1.966) = \boxed{55618.14N \text{ in x direction}}$$

$$R_y = \rho Q V (0 + \sin\theta)$$

$$= 1000 \times .943 \times 30\,(.25) = \boxed{7072.5\ N \text{ in Y direction}}$$

$V_x = V \cdot \cos 15 = 30 \cos 15° = \boxed{28.98\,m/s = V_x}$ stationary

$V_y = V \cdot \sin 15 = 30 \sin 15° = \underline{7.76\,m/s}$ stationary

$V_{2x} = V_x - V_{car} = 28.98\,m/s - 12\,m/s = \underline{16.98\,m/s = V_{2x}}$ moving car

$$V_{resultant} = \sqrt{V_{(2x)}^2 + (V_y)^2} = \sqrt{(16.98m/s)^2 \cdot (7.76m/s)^2} = 18.67m/s = V_R$$

$$M_e = \rho A V_r = \rho \times \frac{\pi}{4} d^2 \times V_r = 1000 \times \left(\frac{\pi}{4} \times .2^2\right) \times 18.67 = 586.6\,kg/s$$

$a = \tan^{-1} 7.76/16.98 = 24.56°$

$\beta = a - \theta = 24.56° - 15° = 9.56°$

$V_{ex}(par) = V_{\cancel{R}} \cos\beta = 18.67 m/s \cos 9.56°$

$= 18.40 m/s$

$(R_x)_r = M_e (\Delta V_{\cancel{xR}})$

$586.6 (18.40 - (16.98))$

$= \underline{832.97 N}$

$\boxed{\text{Effective force @ 12 m/s = 832.97 N in x direction}}$

$(R_y)_c = M_c \Delta V_{YR}$

$= 586.6 (0 - (-7.76)) = 4552 N$

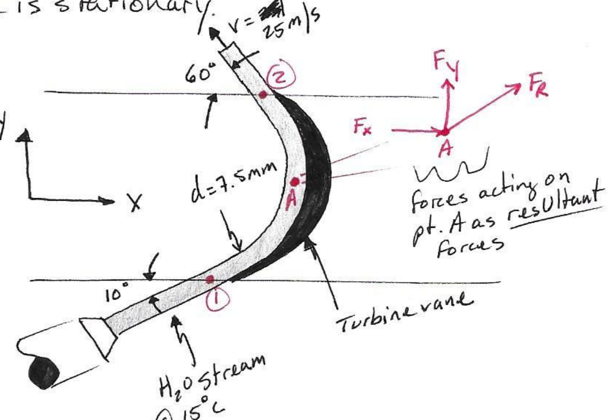
$\boxed{\text{Effective force @ 12 m/s = 4552N in Y direction}}$

16.29 | The incoming stream of water @ 15°C has a diameter of 7.50mm and is moving with a velocity of 25m/s. compute the force on one blade of the turbine if the stream is deflected through the angle shown and the blade is stationary.

→ Blade is motionless

→ Q = volume flow rate

→ $\rho_{H_2O @ 15°C} = 1000\ kg/m^3$



→ The force acting on the turbine blade is a resultant force from the water force acting in the x & y directions.

we know it's zero because the blade is stationary

→ $\Sigma F_x = 0 = \rho \cdot Q \cdot (\Delta V_x) \pm R_x$

→ $\Delta V_x = V_{x_2} - V_{x_1} = (-25)(\cos 60°) + (25)(\cos 10°) = 12.1202\ m/s$

→ $\rho = 1000\ kg/m^3$

→ $Q = A \cdot V = \left(\frac{\pi (0.0075\ m)^2}{4}\right)(25\ m/s) = 0.001104\ m^3/s$

$\Sigma F_x = 0 = (1000\ kg/m^3)(0.001104\ m^3/s)(12.1202\ m/s) + R_x$

$R_x = 13.3807\ \frac{kg \cdot m}{s^2} = 13.4\ N$

$\Sigma F_y = 0 = \rho \cdot Q \cdot \Delta V_y - R_y$

$= (1000\ kg/m^3)(0.001104\ m^3/s)(17.3094\ m/s) - R_y$

→ $\Delta V_y = V_{y_2} - V_{y_1} = (25\ m/s)\sin 60° - (25\ m/s)\sin 10° = 17.3094\ m/s$

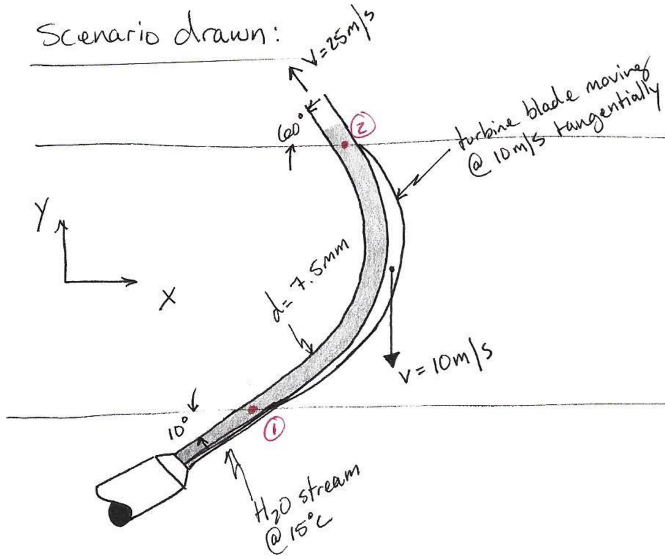
$R_y = 19.1096\ N$

$F_R = \sqrt{(13.4\ N)^2 + (19.1096\ N)^2}$

$F_R = 23.339\ N$

16.30 | Repeat problem 16.29 with the blade rotating as a part of the wheel at a radius of 200 mm and with a linear tangential velocity of 10 m/s. Also, compute the rotational speed of the wheel in rpm.

Scenario drawn:



$v = 10\ m/s$

$r = 0.2\ m$

$\omega = \frac{v}{r} = \frac{10\ \cancel{m}/s}{0.2\ \cancel{m}} = 50\ \frac{rad}{s}$

$rpm = \cancel{\frac{rev}{min} \times \frac{2\pi\ rad}{rev}}$

$rpm = \frac{2\pi\ rad}{min} \times \frac{1\ min}{60 s}$

$\frac{50\ \cancel{rad}}{\cancel{s}} \times \frac{60\ \cancel{s}}{1\ min} \times \frac{rev}{2\pi\ \cancel{rad}}$

$rpm = 4712.39\ \frac{rev}{min}$