a) Determine the air pressure above the water in the tank to make the process happen. Neglect the losses due to the valve and elbow, but consider the energy losses due to the pipe.

step 1: Set up Bernoulli's Eqn and begin cancelling things out

$$\to \frac{P_1}{\gamma} + \frac{V_1^2}{2\cdot g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2\cdot g} + z_2 + h_L$$

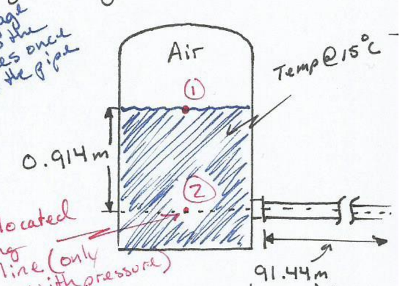
solve for ($P_1$)

velocity of water level @ surface is negligible ($V_1 = 0$)

we're starting from pt. 1 ∴ the height diff. = 0 ($z_1 = 0$)

$P_2 = 0$ because pt. 2 is in line with the faucet opening

we know it's 0 gage because of how the water behaves once it leaves the pipe




pt. 2 can be located anywhere along the dashed line (only the height matters with pressure)

→ Flow rate $Q = 0.0047\,m^3/s$

→ 0.0409 m Schedule 40 pipe

$$\to \boxed{\text{Reduced Eqn: } P_1 = \left(\frac{V_2^2}{2\cdot g} + z_2 + h_L\right)\cdot\gamma}$$

step 2:

→ Solve for $V_2$

→ $Q = A\cdot V \;\therefore\; V = Q/A$

→ $V = (0.0047\,m^3/s) \Big/ \left(\frac{\pi(0.0409\,m)^2}{4}\right)$

→ $\boxed{V = 3.58\,m/s}$

step 3:

→ Solve for $h_L$ (energy losses due to pipe)

→ $h_L = f \times \frac{L}{D} \times \frac{V^2}{2\cdot g}$

→ find friction factor $f$ by solving for Reynold's Number to determine if flow is laminar or turbulent

$$\to R_e = \frac{\rho\cdot V\cdot D}{\eta} = \frac{\left(1000\frac{\cancel{kg}}{\cancel{m^3}}\right)\left(3.58\,\cancel{m/s}\right)\left(0.0409\,\cancel{m}\right)}{1.15\times10^{-3}\,Pa\cdot s}$$

dynamic viscosity Table A.1 → $\eta$

$= 1.15\times10^{-3}\,\frac{N\cdot s}{m^2}$

$= \left(1.15\times10^{-3}\,\frac{\cancel{\frac{kg\cdot m}{s^2}}\cdot\cancel{s}}{\cancel{m^2}}\right)$

$\boxed{R_e = 127{,}229.2}$

*If $R_e > 2100$, then flow is turbulent*

∴ FLOW IS TURBULENT : use friction factor $f$ eqn for turbulent flow

$$\to f = \frac{0.25}{\left[\log\left(\left(\frac{1}{3.7\left(\frac{D}{\varepsilon}\right)}\right) + \left(\frac{5.74}{R_e^{\,0.9}}\right)\right)\right]^2} = 0.01725$$

Table 8.1 (→ $\frac{D}{\varepsilon}$)

Excel Spreadsheet calculator

$$\to h_L = 0.01725 \times \frac{91.44\,m}{0.0409\,m} \times \frac{(3.58\,m/s)^2}{2(9.81\,m/s^2)} = \boxed{25.16\,m = h_L}$$

step 4:

$$\rightarrow P_1 = \left(\frac{V_2^2}{2 \cdot g} + z_2 + h_L\right)\gamma$$

pressure 2 depth in SI units

Table A.1

$$P_1 = \left(\frac{(3.58 m/s)^2}{2(9.81 m/s^2)} + 0.914 m + 25.16 m\right)\left(9.81 \frac{KN}{m^3}\right) = 9,810 \frac{\frac{kg \cdot m}{s^2}}{m^3}$$

$$P_1 = 262.194 \frac{KN}{m^2}$$

$$P_1 = 262.19 \text{ KPa}$$

This value will be used throughout the rest of the problems

b) What is the water depth (y) in the open channel? The angle of the lateral walls is 60°. The width at the top of the water (T) is T = 2.309y (see Table 14.3 in the book). The channel slope is 0.1 percent and is made of <u>unfinished concrete</u>.

STEP 1: Use the Normal Discharge Equation for open channels (make sure it is the equation for SI units!)

→ $Q = \left(\frac{1.00}{n}\right) \cdot A \cdot R^{2/3} \cdot S^{1/2}$

$Q$ = flow rate = given in prompt = $0.0047 \frac{m^3}{s}$

$n$ = manning's number for surface type: select from Table 14.1 = 0.017

$A$ = Area of the open channel for a specific shape: select from Table 14.3 $= 1.73y^2$

$R$ = Hydraulic Radius: select from Table 14.3 = $y/2$

$S$ = Slope of open channel = 0.001

STEP 2: solve for y = depth of channel

→ $$0.0047 = \left(\frac{1.00}{0.017}\right)\left(1.73y^2\right)\left(y/2\right)^{2/3}\left(0.001\right)^{1/2}$$

$$= \left(101.765y^2\right)\left(0.5y\right)^{2/3}\left(0.031623\right)$$

$$0.0047 = \left(3.21808y^2\right)\left(0.629961\,y^{2/3}\right)$$

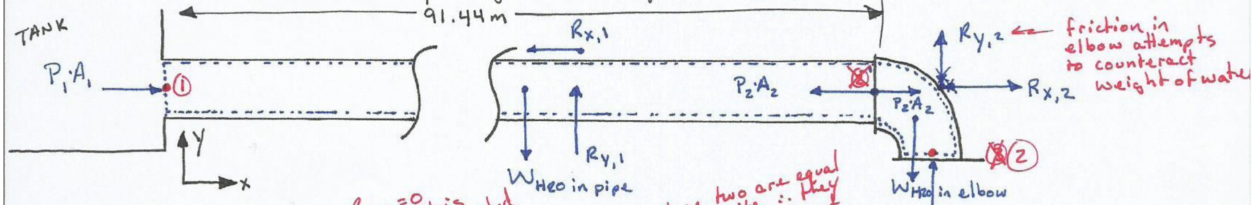
$$0.0047 = \left(3.21808y^{\frac{6}{3}}\right)\left(0.629961\,y^{2/3}\right)$$

$$\frac{0.0047}{2.027726} = y^{\frac{8}{3}}$$

$$\sqrt[8/3]{y^{8/3}} = \sqrt[8/3]{0.002318} = 0.002318^{3/8} = 0.102789 \cong 0.1\,m = y$$

c) The pipe needs to be supported. Your civil engineer colleague requires to know the relevant forces for the support design. Calculate the total horizontal and vertical forces in the whole system pipe-elbow.

STEP 1: Draw a free body diagram of the pipe with the elbow. Indicate a control volume.




STEP 2: Consolodate

$R_{x,1} = 0$ because it is a pipe connected to an elbow.

These two are equal and opposite ∴ they cancel out

→ Forces in the x-direction:

fluid isn't moving in x-direction @ point two

$$\rightarrow \Sigma F_x = \cancel{P_1 A_1}^{0} - \cancel{R_{x,1}}^{0} + R_{x,2} - \cancel{P_2 A_2}^{0} + \cancel{P_2 A_2}^{0} = \underbrace{\rho \cdot Q \left(\cancel{V_{2,x}}^{0} - V_{1,x}\right)}$$

use this because fluid is dynamic (if it were static, right-hand side would = 0)

→ Forces in the y-direction:

$$\rightarrow \Sigma F_y = -\cancel{W_{pipe}}^{0} + \cancel{R_{y,1}}^{0} + R_{y,2} - W_{elbow} + \cancel{P_3 A_3}^{0} = \rho \cdot Q \left(V_{2,y} - \cancel{V_{1,y}}^{0}\right)$$

These two cancel each other since the fluid isn't moving in the y-direction inside the pipe.

open to atmosphere ∴ $P_3 = 0$ gage

fluid isn't moving in y-direction @ pt one

→ $\rho$ = density of $H_2O$ @ 15°C = 1000 kg/m³

Q = volume flow rate = 0.0047 m³/s

$V_{1,x} = Q/A_1 = (0.0047\,m^3/s) / (\pi (0.0409\,m)^2/4) = 3.58\,m/s$

$V_{2,y} = Q/A_2 = V_{1,x} = 3.58\,m/s$

*Note to self: if this doesn't work, try adding $R_{x,1}$ & $R_{x,2}$ together and having them equal to each other*

$$\Sigma F_x = R_{x,2} = \rho \cdot Q(-3.58\,m/s) = (1000\,kg/\cancel{m^3})(0.0047\,\cancel{m^3}/s)(-3.58\,m/s)$$

$$\boxed{R_{x,2} = -16.83\,N}$$

The negative sign indicates that I drew the arrow in the wrong direction in the FBD and that the force is acting in the negative x-direction

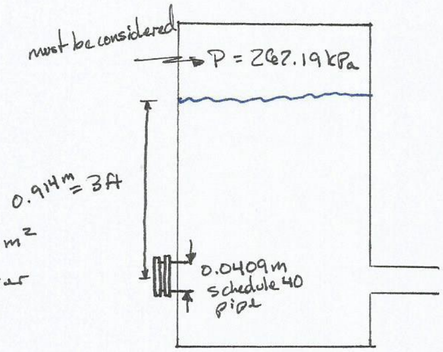
ANSWERS

$$\Sigma F_y = R_{y,2} - W_{elbow} = \rho \cdot Q(V_{2,y})$$

$$= R_{y,2} = (1000\,kg/m^3)(0.0047\,m^3/s)(3.58\,m/s) + W_{elbow}$$

$$\boxed{R_{y,2} = 16.83\,N + \text{Weight of fluid in the elbow}}$$

d) Compute the force acting upon the blind flange at the left-hand-side of the tank




STEP 1: Eqn for the resultant force on a submerged plane area

This would be the only force to calculate if the tank was open to the atmosphere BUT we have a pressurized tank ∴ we have to add a secondary force

→ $F_R = \gamma \cdot h_c \cdot A + F_2$

→ $\gamma$ = specific weight of water = 9.81 kN/m³

→ $h_c$ = distance from surface of fluid area to centroid plane of flange = 0.914355m

→ $A$ = area of flange cross-section = 0.001314 m²

→ $F_2$ = secondary force due to pressurized air over water surface = $P \times A = \gamma h_2 \times A$

→ $h_2 = \dfrac{P}{\gamma}$ ← This concept is a piezometric head (pg. 73 of text)

→ $F_R = \gamma \cdot h_c \cdot A + P \times A$

$= (9.81\,kN/\cancel{m^3})(0.914\cancel{m})(0.001314\cancel{m^2}) + (262.19\,\frac{kN}{\cancel{m^2}})(0.001314\cancel{m^2})$

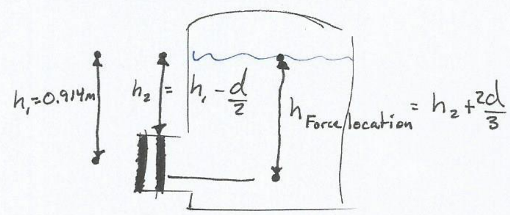
$F_R = 11.78N + 344.52N$

$F_R = 356.3N$

STEP 2: The resultant force is located at the center of pressure

→ center of pressure (and resultant force) are located at 1/3 the height of the area of the flange: h = diameter ∴ $\frac{d}{3}$

→ $\dfrac{d}{3} = \dfrac{0.0409m}{3} = 0.013633m$




$h_{\text{Force location}} = h_1 - \dfrac{d}{2} + \dfrac{2d}{3} = 0.914m - \dfrac{0.0409m}{2} + \dfrac{2(0.0409m)}{3}$

$h_F = 0.921m$ from the top of the water

e) What is the largest hickory wood log the open channel can carry? The log has a square cross section. The density of hickory wood is 830 kg/m$^3$. Is it stable?

very important

→ Think this through! The depth of the water in the open channel will be factor here. The log has a square cross-section, so the base will equal the height. Even though wood is buoyant, it will sink some. It needs to be above the channel bottom in order to move.

Step 1: start with the buoyant force equation

→ $F_{Buoyant} = W_{object}$

back of the book → $(\gamma_{fluid})(V_{fluid\,displaced}) = (\gamma_{object})(V_{object})$

$b \times h \times L$

b = length of cross-sectional area. Since the cross-sectional area is square, b = b selected for $V_{object}$.

h = water level on the cross-sectional area. If this value exceeds the height of the log, then the log is stuck and can't float

L = length of the log. Assume 1 ft in length.

$b = h \therefore b^2$

$b \times h \times L = b^2 \times L$

b = base length of cross sectional area. Since the cross section is square, b = h.

h = height dimension of cross-sectional area. Keep changing this value till the h-value from $V_{fluid\,displaced}$ is slightly smaller than the value of water depth in the trough.

L = length of log. assume 1 foot in length.

$\gamma_{object} = (S.G._{log})(\gamma_{H_2O})$

$S.G._{log} = \rho_{log} / \rho_{H_2O\,@\,15°C}$

$\therefore \gamma_{object} = \left(\frac{830\ kg/m^3}{1000\ kg/m^3}\right)(9.81\ kN/m^3)$

$\gamma_{object} = 8.14\ kN/m^3$

Step 2: solve this problem by using excel

→ New equation:

→ $(\gamma_{H_2O})(b_{log} \times h_{waterline} \times L_{log})$

$= (\gamma_{object})(b_{log}^2 \times L_{log})$

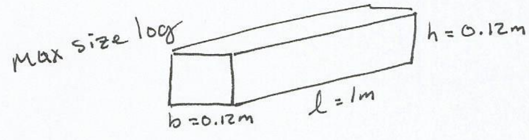
→ $h_{waterline}$ is what we're solving for. It cannot exceed the value of the water depth. Once this is solved for, take the square root of $b_{log}^2$ to find the dimension for the cross-sectional area.

→ $h_{water\,line} = \dfrac{(\gamma_{object})(b_{log}^2 \times L_{log})}{(\gamma_{H_2O})(b_{log})(L_{log})}$

if the water line height = 0.099 m (just under the "y-value" found in pt b)), then b = 0.12 m

→ ~~Assuming log equal to the mass or volume of a hickory log~~

∴ the biggest area a log can have is = 0.0144 m$^2$



step 3: determine if the log is stable.

→ locate the following: center of buoyancy, center of gravity, metacenter

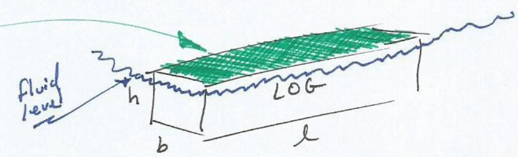
→ center of buoyancy is located at half the depth of the submerged portion of the log: $\frac{h_{water\ line}}{2} = \frac{0.099m}{2} = 0.0495m$ below water level

→ center of gravity is located in the center of the cross-sectional area: $\frac{b}{2} = \frac{0.12m}{2} = 0.06m$ from the top of the log

→ metacenter location <u>FROM</u> the center of buoyancy (MB):

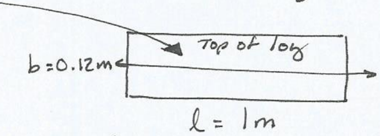
(pg. 103 of text) → $MB = I/Vd$

→ I = <u>least</u> moment of inertia of a horizontal section of the log taken at the surface of the liquid




→ $I = \frac{LB^3}{12} = \frac{(1m)(0.12m)^3}{12}$

$I = 0.000144\,m^4$




(view, from liquid surface, of log)

Vd = volume displaced

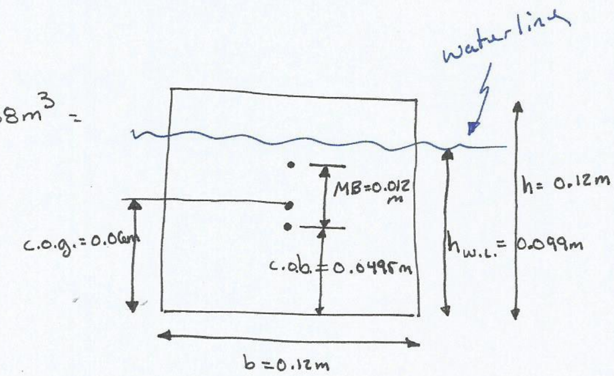
$= b \cdot h \cdot L$

$= (0.12m)(0.099m)(1m)$

$Vd = 0.01188\,m^3$

→ $MB = 0.000144m^4 / 0.01188m^3 =$

$MB = 0.012m$




→ metacenter location is c.o.b. + MB

$= 0.0495m + 0.012m$

$= 0.0615m$

→ c.o.g. location is 0.06m

metacenter is located above center of gravity (even if only a small bit), therefore the log is steady (but could easily be rolled)

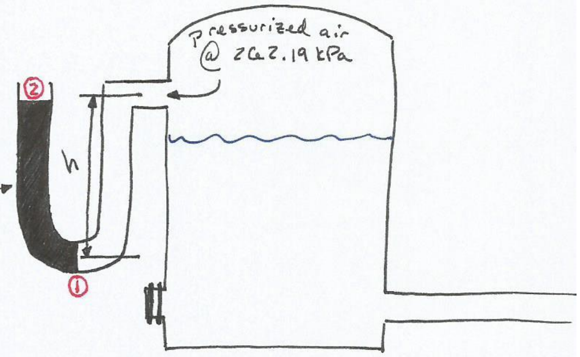
STABLE

f) To monitor the air pressure, your client proposes to use a U-tube manometer. If using mercury, what should the minimum dimension of the U-tube measured from the connection to the tank to the lowest point of the U-tube so it works properly?

→ This question is asking for the height of the u-tube manometer. In order to solve this, we need to consider some things:

→ The pressure @ 2 is open to the atmosphere, and is therefore 0 kPag.

→ The pressure @ 1 is equal to the pressurized air in the tank.




STEP 1: The manometer is setup to where the mercury column height is equal to the height of manometer.

→ $P_{@1} - P_{@2} = 0$

$P_{@1}$ = pressurized air = 262.19 kPa

$P_{@2} = \gamma_{Hg} \times h \quad = 132.8\,kN/m^3 \times h$

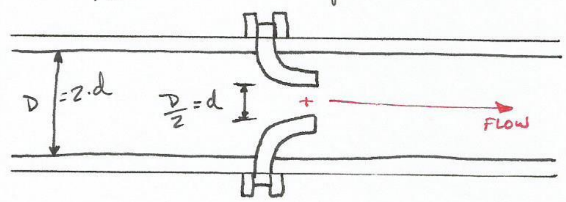
solve for    Table B.1

→ $262.19\,kPa - 132.8\,kN/m^3 \times h = 0$

→ $\dfrac{262.19\,\frac{kN}{m^2}}{132.8\,\frac{kN}{m^3}} = h = 1.97m$

g) Your client also proposes to use a flow nozzle to measure the flow. For a nozzle diameter to pipe diameter ratio of 0.5, what is the pressure drop across the nozzle?

Flow Nozzle Setup




STEP 1:

→ The equation(s) we're going to use are the following:

→ Discharge coefficient equation for a flow nozzle

→ $C = 0.9975 - 6.53\sqrt{\beta / N_R}$

→ $\beta$ = ratio of diameters = $\frac{\text{nozzle } \phi}{\text{pipe } \phi} = \frac{d}{D} = 0.5$ (given)

→ $N_R$ = Reynold's number

[redacted] = [redacted] 127229.2 (calculated in part(a))

→ $V_1 = C \cdot \sqrt{\frac{2g(P_1 - P_2)/\gamma}{(A_1/A_2)^2 - 1}}$

(we're looking for a pressure drop, so we need to solve for $P_2$, and then find the <u>difference</u> between $P_1$ & $P_2$)

→ we know that $V_1 = Q/A_1$

→ $Q = A_1 \cdot C \sqrt{\frac{2g(P_1 - P_2)/\gamma}{(A_1/A_2)^2 - 1}}$

given in prompt

known

we'll know this once we solve equation for it given above

→ $P_2 = P_1 - \frac{\left(\left(\frac{Q}{C \cdot A_1}\right)^2 \times \left((A_1/A_2)^2 - 1\right)\right)\gamma}{2 \cdot g}$

STEP 2: Start solving!

→ $C = 0.9975 - 6.53\sqrt{(0.5)/127229.2}$

$C = 0.985$

→ consider Figure 15.5 on page 401 of text

→ this indicates that the calculate value of C is in a steady region ✓

→ ~~QcA₁~~ $P_2 = P_1 - \frac{\left(\left(\frac{Q}{C \cdot A_1}\right)^2 \times \left((A_1/A_2)^2 - 1\right)\right)\gamma}{2 \cdot g}$

g.2) $P_2 = 262.19\,\frac{kN}{m^2} - \dfrac{\left(\dfrac{(0.0047\,m^3/s)}{(0.985)\left(\frac{\pi(0.0409m)^2}{4}\right)}\right)^2 \times \left(\left(\left(\frac{\pi(0.0409m)^2}{4}\right)\Big/\left(\frac{\pi\left(\frac{0.0409}{2}\right)^2}{4}\right)\right)^2 - 1\right)\gamma}{2 \times 9.81\,m/s^2}$

$= 262.19\,\frac{kN}{m^2} - \dfrac{\left(\dfrac{0.0047\,m^3/s}{(0.985)(0.001314\,m^2)}\right)^2 \times \left(\left(0.001314\,m^2/0.000328\,m^2\right)^2 - 1\right) \times \gamma}{19.62\,m/s^2}$

$9.81\,\frac{kN}{m^3}$

$= 262.19\,\frac{kN}{m^2} - \dfrac{\left(13.19\,m^2/s^2 \times 15.00 \times 9.81\,\frac{kN}{m^3}\right)}{19.62\,m/s^2}$

$= 262.19\,\frac{kN}{m^2} - 98.928\,\cancel{\frac{kN}{m\cdot s^2} \times \frac{s^2}{m}}$

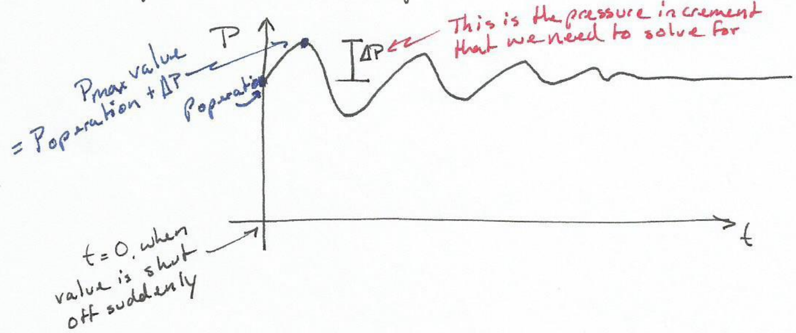
$P_2 = 163.26\,\frac{kN}{m^2} = 163.26\,kPa$

which is, incidentally, this value over here, so I could've just stopped here.

$\Delta P = P_1 - P_2 = 262.19\,kPa - 163.26\,kPa = 98.9\,kPa = \Delta P$

h) If the valve ~~in~~ in the pipe closes suddenly, what is the pressure increment [just] after the sudden closing? The modulus of elasticity of steel is 200 GPa. Is there any change of cavitation in your system?

→ This problem presented has to do with the water-hammer phenomenon. As soon as a valve is shut off suddenly, the water hammer occurs. See the figure below for a depiction of how the pressure in the water behaves over time:




→ STEP 1: solve for $\Delta P$

→ $\Delta P = \rho \cdot c \cdot V_1$

$\rho$ = density of water @ 15°C

c = the ~~cost~~ return velocity of of the wave in the pipe during the water hammer effect

$V_1$ = velocity initially moving the water towards the valve before the valve abruptly closed.

$$c = \frac{\sqrt{E_0/\rho}}{\sqrt{1+\frac{E_0 \cdot D}{E \cdot \delta}}}$$

$E_0$ = bulk modulus of the fluid = 2,179 MPa (Table 1.3)

D = inner diameter of pipe = 0.0409m

E = elastic modulus of pipe material = 200 MPa (given)

$\delta$ = thickness of the pipe wall = (3.68mm) = 0.00368m (Table F.1)

$\rho$ = density of $H_2O$ @ 15°C = 1000 kg/m³

$$c = \frac{\sqrt{\left(2{,}179\ \frac{MN}{m^2}\right) / \left(1000\ kg/m^3\right)}}{\sqrt{1+\frac{\left(2{,}179\ \frac{MN}{m^2}\right)(0.0409m)}{\left(200\ \frac{MN}{m^2}\right)(0.00368m)}}}$$

$$= \left(\frac{2{,}179{,}000{,}000\ \frac{kg \cdot m}{s^2}}{m^2}\right) / \left(1000\ kg/m^3\right)$$

$$= 2{,}179{,}000\ \frac{kg \cdot m}{s^2} \cdot \frac{1}{m^2} \cdot \frac{m^3}{kg}$$

$$c = \frac{\sqrt{2{,}179{,}000\ \frac{m^2}{s^2}}}{\sqrt{1+121.09}}$$

$$\boxed{c = 133.6\ m/s = \text{return velocity}}$$

h.2) $V_1 = Q/A_1 = (0.0047\,m^3/s) \Big/ \left(\frac{\pi (0.0409\,m)^2}{4}\right)$

$V_1 = 3.6\,m/s$ ← This velocity is remarkably slower than the return velocity. The difference demonstrates just how dangerous a water hammer can be to a system and its surroundings!

→ Step 2: solve for pressure increment

→ $\Delta P = \rho \cdot C \cdot V_1$

$= \left(1000\,\frac{kg}{m^3}\right)(133.6\,m/s)(3.6\,m/s)$

$= 480960\,\frac{kg}{m^3}\cdot\frac{m^2}{s^2} = 480{,}960\,\frac{kg}{m \cdot s^2} = 480{,}960\,\frac{N}{m^2} = 481\,kPa = \Delta P$

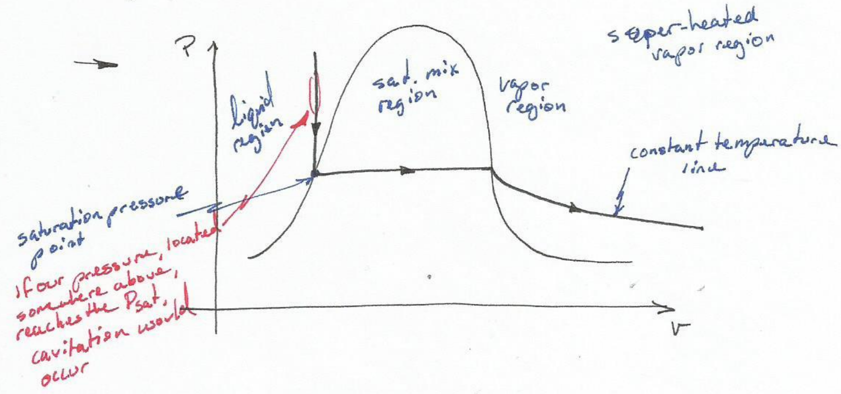
equivalent units Table K.1

pressure increment. It's huge! this system, if not prepared for this sudden increase in pressure, might very well just explode!

→ Step 3: consider cavitation

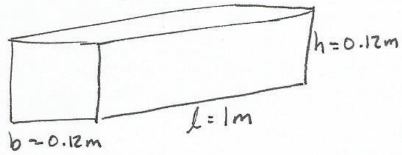
→ cavitation occurs if the velocity increases enough to where the pressure drops to the saturation pressure. "Cavitation-level" pressures can also be achieved by long lengths of pipe and changes in elevation or changes in cross-sectional area to increase velocity. In this scenario, none of these occur to the extreme of causing cavitation.

→

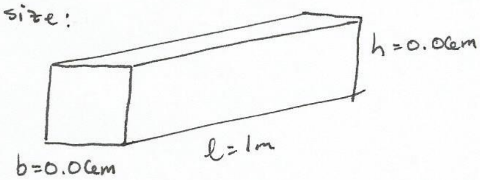


i) Assuming a log with half the size of the largest log that can be carried in the channel, what is the largest drag force it would experience if it gets stuck in the channel? Make any reasonable assumptions necessary.

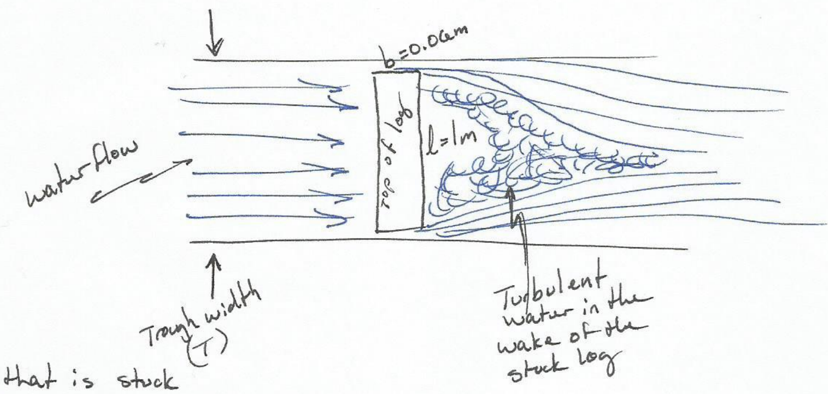
→ the maximum is:




→ a log half this size:




→ bird's eye view of the senario:




→ The log that is stuck is feeling a drag force as a result of the wake. [redacted]

Step 1: determine the equation to be used:

$F_D$ = drag = $C_D(\rho \cdot v^2/2)A$

$C_D$ = drag coefficient (located table 17.1) [redacted] = 1.60

$\rho$ = density of fluid = 1000 kg/m$^3$

V = velocity of flow acting on the stuck log = $V_1$ solved for in h.2) = 3.6 m/s

A = area being acted on perpendicularly by moving fluid = $l \times h$ = (1m)(0.06m) = 0.06m$^2$

→ [redacted]

i.2) $F_D = C_D(\rho \cdot v^2/2)A$

$$F_D = (1.60)\left(\frac{(1000\,kg/m^3)(3.6\,m/s)^2}{2}\right)(0.06\,m^2)$$

$F_D = 622.08 \frac{kg \cdot \cancel{m^2}}{\cancel{m^3} \cdot s^2} \cdot \cancel{m^2} = 622.08\,N$ = Force of drag on the ~~stag~~ stuck log