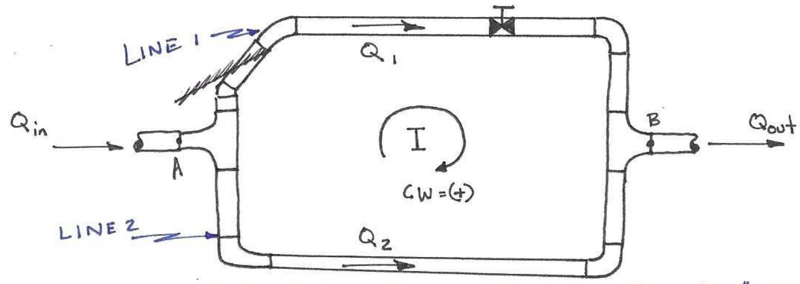
PROBLEM #1 PART A

PURPOSE: The purpose of this problem is two-fold: through use of the Hardy-Cross method, find the flow rate in each branch, and the pressure at point A. Through use of the Hardy Cross method, multiple iterations will ensure that the correct answer is found.

DIAGRAM:



* Value for Q_2 will be negative since it works "against" the positive nomenclature *

SOURCES: - Mott & Utener. Applied Fluid Mechanics. 7th edition. Pearson 2015.
- Class lecture notes.

DESIGN CONSIDERATIONS:

1) Incompressible fluids
2) Assume steady state
3) Energy ~~Minor~~ losses are due to the following:
   a) general losses in pipes
   b) minor losses due to fittings

MATERIALS:

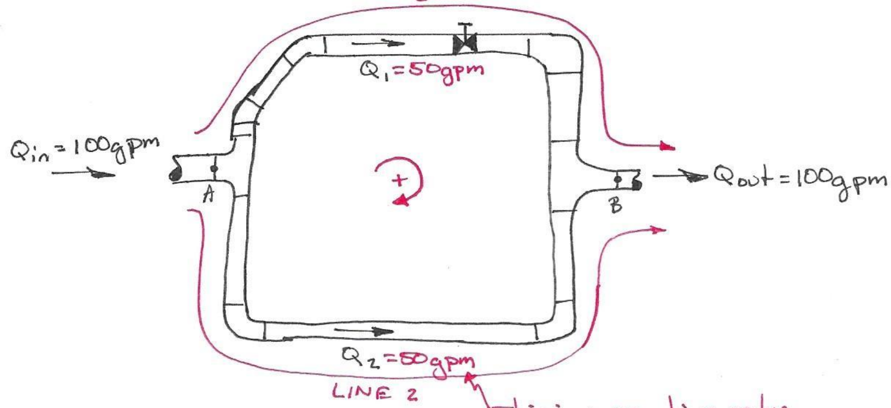
→ schedule 40 steel pipe
→ turpentine flowing through the pipe

DATA & VARIABLES:

→ All pipes are schedule 40 commercial steel
→ Pipes in line 1 are 2 inch nominal pipe
→ Pipes in line 2 are 1.5 inch nominal pipe
→ The pressure at B is 10 psig
→ Q_{in} & Q_{out} are the same and equal to 100 gallons per minute
→ γ_{H_2O} = specific weight of water = 62.4 lb/ft^3
→ $\gamma_{turpentine}$ = 54.20 lb/ft^3 (table B.2)
→ ε = pipe roughness = 1.5×10^{-4} ft (table B.2)
→ ν = kinematic viscosity of turpentine = 1.70×10^{-5} ft^2/s (table B.2)
→ Length of pipe in Line 1 = 150 ft
→ Length of pipe in Line 2 = 125 ft

PROCEDURE & CALCULATIONS:

I'm starting the Hardy-Cross method as the following:



This is a negative value while in Excel, because according to the positive nomenclature it's working "against" this flow rate

→ The equations I've used in my Excel file are the following:

→ Reynold's Number (Re) $= \dfrac{4Q}{\pi \cdot D \cdot \nu} = \dfrac{4(\text{flow rate})}{\text{pi} \cdot \text{diameter} \cdot \text{Kinematic viscosity}}$

→ f = Friction factor (turbulent) $= \dfrac{0.25}{\left[\log\left(\dfrac{1}{3.7(D/\varepsilon)} + \dfrac{5.74}{Re^{0.9}}\right)\right]^2}$

→ $k = \dfrac{8(L + \Sigma L_e)f}{\pi^2 \cdot g \cdot D^5}$

→ $h_L = kQ^2$

→

→ Analysis of minor losses in Line 1

→ tee_1, $45°\,elbow_1$, $45°\,elbow_2$, $90°\,elbow_1$, tee_2, Globe valve

→ $K = 60f_T$, $K = 16f_T$, $K = 16f_T$, $K = 30f_T$, $K = 60f_T$, $K = 340f_T$

→ $K = (L_e/D)f_T$. If I need L_e, which I do, I can do the following for each of the above values:

→ $L_e = (60)(D_1)$, $L_e = (16)(D_1)$, $L_e = (16)(D_1)$, $L_e = (30)(D_1)$, $L_e = (60)(D_1)$, $L_e = (340)(D_1)$

→ D_1 (Diameter for Line 1) = Inner diameter = 2.067 in

→ $L_e = (60)(2.067) + (16)(2.067) + (16)(2.067) + (30)(2.067) + (60)(2.067) + (340)(2.067)$

→ Put the whole line above in Excel calculator in cell K6. It will stay the same through the rest of this problem

→ I realized I made a mistake. The values in the L_e eqn above should be the inner diameter in <u>feet</u> not <u>inches</u>. I've changed this accordingly in cell K6 of problem #1 sheet

→ $L_e = 89.9406$

→ Analysis of minor losses in Line 2

→ tee, 90° elbow$_1$, 90° elbow$_2$, tee$_2$

→ $K = 60f_T$, $K = 30f_T$, $K = 30f_T$, $K = 60f_T$

→ $K = (Le/D)f_T$; so I can now do the following again to find the total value for L_e:

→ $Le = (60)(D_2) + (30)(D_2) + (30)(D_2) + (60)(D_2)$

$Le = (60)(0.1342)(2) + (30)(0.1342)(2) = 24.156$

→ Put the whole equation above in Excel cell K7. It will stay the same through the rest of this problem.

→ It has taken three iterations to get the percentage error to drop to zero, which is a suprising low count. Another suprising thing is that the final flow rate through line 1 is greater than the final flow rate through line 2. I thought it would've been the other way around.

→ The pressure at point A is calculated to be essentially the same value as the pressure at point B, which is 10 psig. The equation used to calculate this was the following:

→ $\dfrac{\Delta P}{\gamma_{turpentine}} = h_L$

→ h_L = energy losses = kQ^2

→ $P_A = (h_L)(\gamma_{turpentine}) + P_B$

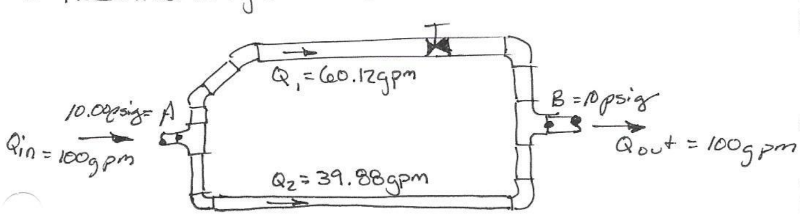
→ The flow rate in Line 1 is 60.12 gpm; the flow rate in Line 2 is 39.88 gpm

→ The final pressure at point A is 10.00 psig

→ In all honesty I couldn't find anything in the textbook about critical velocities, so I don't know if the velocities calculated in this problem are anywhere near critical or not.

SUMMARY:

→ The final diagram is as follows:



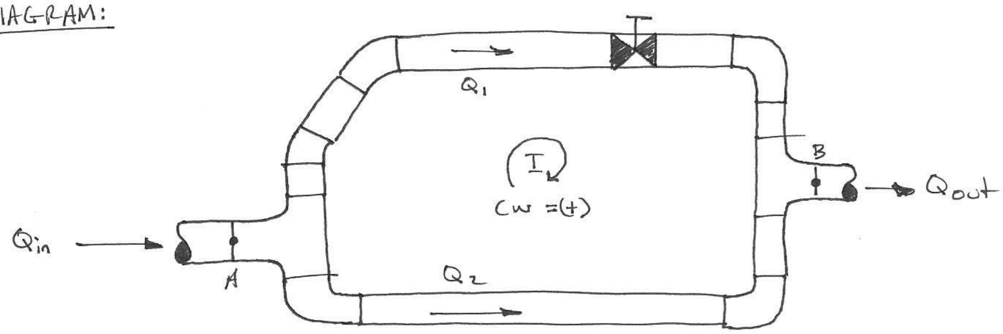
ANALYSIS:

→ I would have thought that the flow rate values would have been switched. Current flow takes the path of least resistance and so I would have expected the greater value to be in Line 2. It may be that I miscalculated in my Excel file.

PROBLEM #1 PART B

PURPOSE: The people in charge of the system want it to perform faster by increasing the entrance flow rate from 100gpm to 150 gpm. This overhaul is years after the system was initially set up, and the pressures at points A & B should remain the same, therefore the pipe diameters will need to increase in size to accomodate the increased flow rate.

DIAGRAM:



→ This diagram is the same as the one in part A, but with increased pipe sizes

SOURCES: - Mott & Utener. Applied Fluid Mechanics. 7th edition. Pearson 2015.
- Class lecture notes

DESIGN CONSIDERATIONS:

1) Incompressible fluids
2) Assume steady state
3) Energy losses are due to the following:
   a) general losses in pipes
   b) minor losses due to fittings

MATERIALS:

→ schedule 40 steel pipe
→ turpentine flowing through the pipes

DATA & VARIABLES:

→ All data and variables are the same as part A of this problem, with the exception of there there are unknown diameters of the pipes.

→ $Q_{in} = Q_{out} = 150$ gpm

PROCEDURE & CALCULATIONS:

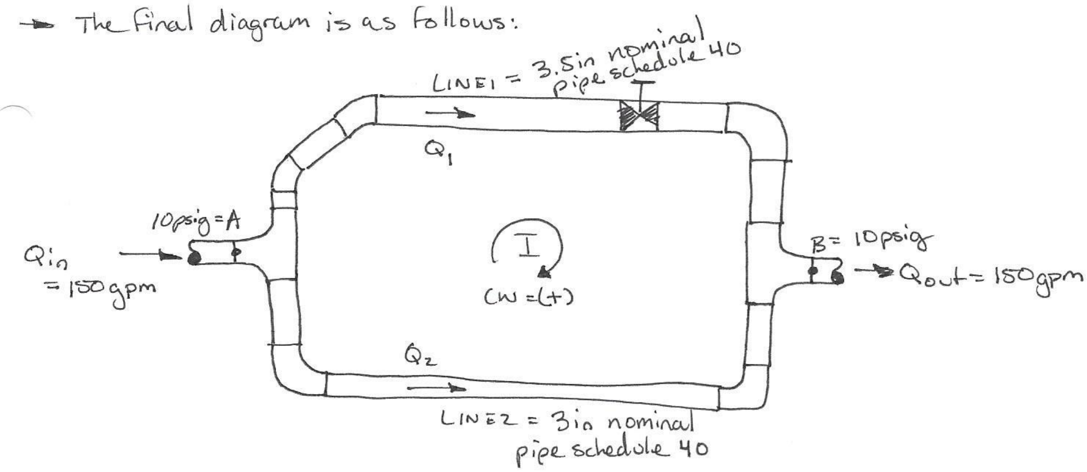
- I took the Excel file for part A and began to manipulate a copy of it, labeled Problem #1B. I changed the value from 100gpm = Q_{in} (50 gpm in each line) to 150gpm = Q_{in} (75 gpm in each line)
- I then saw that my final calculation at the bottom of the file for the pressure value at point A had changed (it increased). So then I began to increase the diameter values by one value increments given in Table F.1 (table for Schedule 40 diameters) until the pressure value at A had lowered back down to the correct value.
- Therefore the corrected values for the larger diameters are:

  Line 1: ~~diame~~ Inner diameter is 0.2957ft, therefore is 3.5in nominal pipe schedule 40

  Line 2: Inner diameter is 0.2557ft, therefore is 3in nominal pipe schedule 40

SUMMARY:

- The final diagram is as follows:



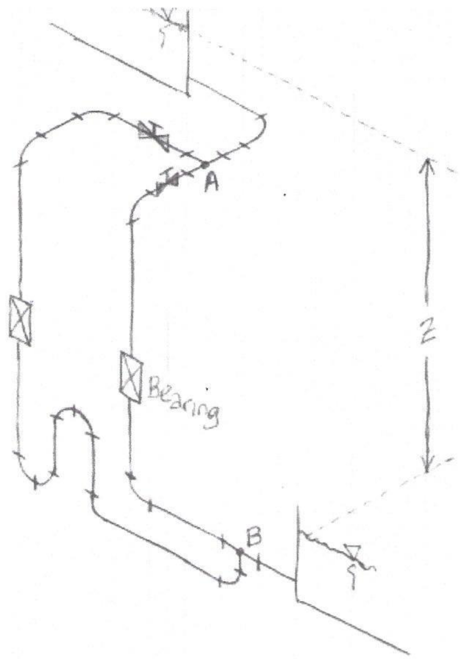
ANALYSIS:

- I did not have to play with the resistance coefficient of the globe value to ensure a flow rate of 150 gpm.

PROBLEM #2

PURPOSE: The purpose is to determine the flow rate of oil to the bearing in each of the given branche

DIAGRAM:



SOURCES: - Mott & Utener. Applied Fluid Mechanics. 7^{th} edition. Pearson 2015.
- Class lecture notes.

DESIGN CONSIDERATIONS:

1) Incompressible fluids
2) Assume steady state
3) Energy losses are due to the following:
   a) general losses in pipes
   b) minor losses due to fittings

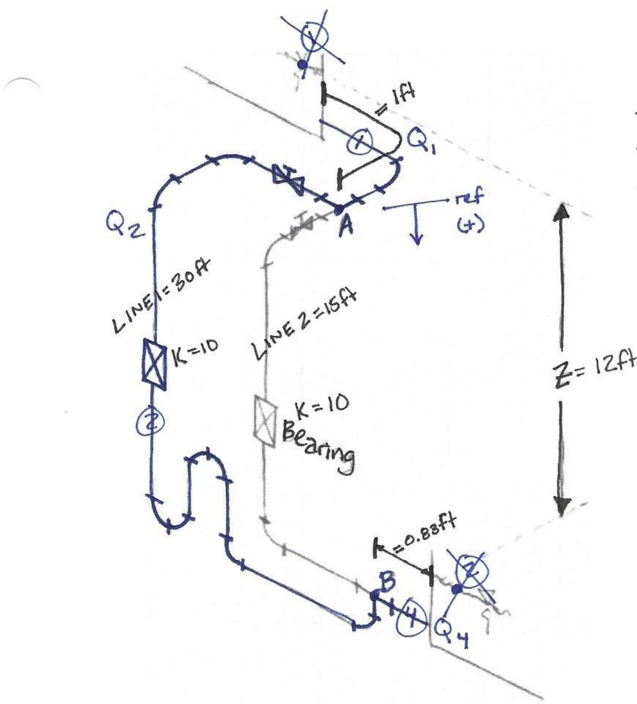
MATERIALS:

→ 1/2 standard type K copper tubing
→ oil flowing through the pipes

DATA & VARIABLES:

→ All pipes are ~~sched~~ 1/2 standard K copper tubing
→ Pipes in lines 1 and 2 have an inner diameter of 0.0439 ft
→ Each line has a gate valve and a bearing with a resistance coefficient of 10
→ The oil's specific gravity is 0.888
→ The oil's kinematic viscosity is $9 \times 10^{-4} m^2/s = 0.00968 ft^2/s$
- Length of pipe in Line 1 is 30ft
→ Length of pipe in Line 2 is 15ft
→ elevation between the tanks is 12ft
→ The length of pipe from the top open tank to point A is 1ft
→ The length of pipe from point B to the bottom open tank is 10 inches

PROCEDURE & CALCULATIONS:



FROM TOP TANK TO BOTTOM TANK
LINE 1 CONTAINS:

- 9 90° Elbows
- 1 Gate valve
- 1 Bearing
- 1 Tee "flow through branch"
- 1 Tee "flow through branch" in reverse
- 30ft of 1/2 standard type K copper tubing

$$\underset{\text{pipes}}{h_L} = f\frac{L}{D}\frac{V^2}{2g} = \frac{fL}{2g\,D}\frac{1}{A^2}Q^2$$

$$\underset{\text{minor}}{h_L} = K\frac{V^2}{2g} = \frac{K}{2gA^2}Q^2$$

$$Q_1 = Q_2 + Q_3 = Q_4$$

→ Write Bernoulli's Eqn and start canceling and reduce the equation.

$$\frac{P_A}{\gamma} + \cancel{\frac{V_A^2}{2g}}^{0} + \cancel{Z_A}^{0} = \frac{P_B}{\gamma} + \cancel{\frac{V_B^2}{2g}}^{0} + Z_B + h_L \qquad Q = V \cdot A$$

$$\frac{P_A - P_B}{\gamma} - Z_B = h_L \quad \text{(Reduced Bernoulli's Eqn)}$$

These two tees are different because they use the entrance flow rates in the calculations, which are not equal.

$$h_L = \underset{\text{LINE 1}}{\text{general losses}} + \underset{\text{LINE 1}}{\text{minor losses}} = \underset{\text{ELBOWS}}{(9)(30f_T)} + \underset{\text{GATE VALVE}}{8f_T} + \underset{\text{BEARING}}{10} + \underset{\text{TEE1}}{60f_T} + \underset{\text{TEE2}}{60f_T} + f\frac{L}{D}$$

→ Put the h_L eqn above in terms of flow rates

$$\frac{P_A - P_B}{\gamma} - Z_B = (9)\left(\frac{30f_T}{2gA^2}Q^2\right) + \left(\frac{8f_T}{2gA^2}Q_2^2\right) + \left(\frac{10}{2gA_2}Q_2^2\right) + \left(\frac{60f_T}{2gA^2}Q_1^2\right) + \left(\frac{60f_T}{2gA^2}Q_2^2\right) + \left(\frac{fL}{2gD}\frac{1}{A^2}Q_1^2\right)$$

this portion needs to be split up. 1 of these elbows is with the Q_1 and the other 8 are Q_2.

this portion also needs to be redone as some of the general pipe losses are Q_1 & some are Q_2

$$\frac{P_{\Delta AB}}{\gamma} - Z_B = \left(\frac{30f_T}{2gA^2}\right)Q_1^2 + \left(\frac{30f_T}{2gA^2}\right)(Q_2^2)(8) + \left(\frac{8f_T}{2gA^2}\right)(Q_2^2) + \left(\frac{10}{2gA_2}\right)(Q_2^2) + \left(\frac{60f_T}{2gA^2}\right)(Q_1^2)$$
$$+ \left(\frac{60f_T}{2gA^2}\right)(Q_2^2) + \left(\frac{fL}{2gD}\frac{1}{A^2}\right)(Q_1^2) + \left(\frac{fL}{2gD}\frac{1}{A^2}\right)(Q_2^2)$$

→ f_T can be found by calculating both Reynold's number and the relative roughness of the copper tube and using Moody's Diagram or the Equation for f_T.

→ Reynold's number will have to calculated based on a guess of Q_1

I forgot to multiply this value by 8

→ $$\frac{P_{\Delta AB}}{\gamma} - z_B = \left(\frac{30f_T + 60f_T}{2gA^2}\right)(Q_1^2) + \left(\frac{f_1 L_1}{2gD}\frac{1}{A^2}\right)(Q_1^2) + \left(\frac{\underbrace{30f_T}_{240f_T} + 8f_T + 10 + 60f_T}{2gA^2} + \frac{f_2 L_2}{2gD}\frac{1}{A^2}\right)(Q_2^2)$$

→ Calculate Reynold's number using Excel Calculator

→ Eqn to use: $Re = \frac{4Q_1}{\pi D \nu} = 14{,}969.35$ when $Q_1 = 5 ft^3/s$

→ Calculate Relative Roughness using Excel Calculator

→ $D/\varepsilon = 8{,}333.33$

→ $$f_T = \frac{0.25}{\left[\left(\log\left(\frac{\frac{1}{3.7}}{\left(\frac{D}{\varepsilon}\right)}\right)\right)^2\right]} = 0.0124$$

→ $A = \frac{\pi d^2}{4} = 0.0014 ft^2$

→ g = gravity = $32.2 ft/s^2$

→ Calculate f_1 (due to general pipe losses) by using the calculated relative roughness and Reynold's number

→ $$\frac{0.25}{\left[\left(\log\left(\left(\frac{1}{3.7\left(\frac{D}{\varepsilon}\right)}\right) + \left(\frac{5.74}{Re^{0.9}}\right)\right)\right)^2\right]} = 0.0281$$

→ $$\cancel{\frac{P_{\Delta AB}}{\gamma}} - z_B =$$

→ let's stop and analyze for a moment. We're not given any pressure values at A or B, so finding $\frac{P_{\Delta AB}}{\gamma}$ is going to be tricky. But I am given the height difference between two open tanks. So I am going to redesignate my points A & B to point 1 @ the top tank and point 2 @ the botom tank. This way the pressure difference is zero because both tanks are open to the atmosphere.

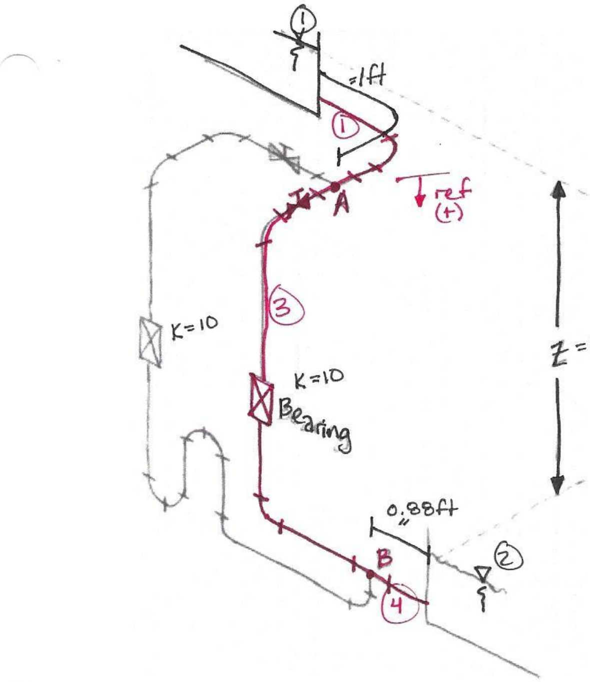
→ My new equation is as follows (In reduced form):

→ $$-z_B = \left(\frac{90f_T}{2gA^2}\right)(Q_1^2) + \left(\frac{f_1 L_1}{2gDA^2}\right)(Q_1^2) + \left(\frac{\overset{308f_T}{\cancel{98f_T}} + 10}{2gA^2} + \frac{f_2 L_2}{2gDA^2}\right)(Q_2^2)$$

→ In terms of Q_1, Q_2 =

→ $$Q_2 = \sqrt{\frac{-z_B - \left(\frac{90f_T}{2gA^2} + \frac{f_1 L_1}{2gDA^2}\right)(Q_1^2)}{\left(\frac{\underset{308f_T}{\cancel{98f_T}} + 10}{2gA^2} + \frac{f_2 L_2}{2gDA^2}\right)}}$$

→ We need to find Q_1. We know that $Q_1 = Q_2 + Q_3$. We now have 3 variables and two equation so we must now find the third equation (Q_3) and solve for it in terms of Q_1. Then we can add our equations for Q_2 & Q_3 together.



FROM TOP TANK TO BOTTOM TANK

LINE 2 CONTAINS:

- → 3 90° Elbows
- → 1 Gate valve
- → 1 Bearing
- → 1 Tee "flow through run"
- → 1 Tee "flow through run"
- → 15 ft of 1/2 standard type K copper tubing

→ $h_{L\,\text{pipes}} = f\frac{L}{D}\frac{V^2}{2g} = \frac{fL}{2gD}\frac{1}{A^2}Q^2$

→ $h_{L\,\text{minor}} = K\frac{V^2}{2g} = \frac{K}{2gA^2}Q^2$

→ $Q_1 = Q_2 + Q_3 = Q_4$

Write Bernoulli's Eqn and start canceling and reduce the equation.

→ $\cancelto{0}{\frac{P_1}{\gamma}} + \cancelto{0}{\frac{V_1^2}{2g}} + \cancelto{0}{z_1} = \cancelto{0}{\frac{P_2}{\gamma}} + \cancelto{0}{\frac{V_2^2}{2g}} + z_2 + h_L$

as stated in the blue analysis on the previous page, both tanks are open and are ∴ 0psig.

→ $-z_2 = h_L$

→ h_L = general losses + minor losses = $\underset{\text{ELBOWS}}{(3)(30f_T)} + \underset{\text{TEE1}}{20f_T} + \underset{\text{TEE2}}{20f_T} + \underset{\text{GATE VALVE}}{8f_T} + \underset{\text{BEARING}}{10} + \underset{\text{PIPES 1\&4}}{f_1\frac{L_1}{D}} + \underset{\text{PIPE 3}}{f_3\frac{L_3}{D}}$

LINE 2   LINE 2

→ Put the h_L eqn above in terms of flow rates

$$-z_2 = (2)\left(\left(\frac{30f_T}{2gA^2}\right)Q_3^2\right) + \left(\frac{30f_T}{2gA^2}\right)(Q_1^2) + \left(\frac{20f_T}{2gA^2}\right)(Q_1^2) + \left(\frac{20f_T}{2gA^2}\right)(Q_3^2) + \left(\frac{8f_T}{2gA^2}\right)(Q_3^2) + \left(\frac{10}{2gA^2}\right)(Q_3^2)$$
$$+\left(\frac{f_1L_1}{2gDA^2}\right)(Q_1^2) + \left(\frac{f_3L_3}{2gDA^2}\right)(Q_3^2)$$

→ group like terms and reduce

→ $$\left(\frac{60f_T}{2gA^2} + \frac{f_1L_1}{2gDA^2}\right)(Q_1^2) + \left(\frac{88f_T+10}{2gA^2} + \frac{f_3L_3}{2gDA^2}\right)(Q_3^2) = -z_2$$

→ Rearrange the above equation, solving for Q_3:

$$\rightarrow \quad Q_3 = \sqrt{\frac{-z_2 - \left(\frac{60 f_T}{2gA^2} + \frac{f_1 L_1}{2gDA^2}\right)(Q_1^2)}{\left(\frac{88 f_T + 10}{2gA^2} + \frac{f_3 L_3}{2gDA^2}\right)}}$$

→ The friction factors f ; f_T due to general losses and minor losses respectively, will not change from Line 1 to Line 2.

→ We now have the following three variables and three equations

1. Q_1
2. Q_2
3. Q_3

1. $Q_1 = Q_2 + Q_3$
2. $$Q_2 = \sqrt{\frac{-z_B - \left(\left(\frac{90 f_T}{2gA^2} + \frac{f_1 L_1}{2gDA^2}\right)(Q_1^2)\right)}{\left(\frac{308 f_T + 10}{2gA^2} + \frac{f_2 L_2}{2gDA^2}\right)}}$$
3. $$Q_3 = \sqrt{\frac{-z_B - \left(\left(\frac{60 f_T}{2gA^2} + \frac{f_1 L_1}{2gDA^2}\right)(Q_1^2)\right)}{\left(\frac{88 f_T + 10}{2gA^2} + \frac{f_3 L_3}{2gDA^2}\right)}}$$

→ Our full equation is now

$$\rightarrow \quad 0 = -Q_1 + \sqrt{\frac{-z_B - \left(\left(\frac{90 f_T}{2gA^2} + \frac{f_1 L_1}{2gDA^2}\right)(Q_1^2)\right)}{\left(\frac{308 f_T + 10}{2gA^2} + \frac{f_2 L_2}{2gDA^2}\right)}} + \sqrt{\frac{-z_B - \left(\frac{60 f_T}{2gA^2} + \frac{f_1 L_1}{2gDA^2}\right)(Q_1^2)}{\left(\frac{88 f_T + 10}{2gA^2} + \frac{f_3 L_3}{2gDA^2}\right)}}$$

→ For the above equation, there are two unknowns: Q_1 ; friction factors f_2 ; f_3 (f_1 is known through solving Reynold's Number based on our first guess of Q_1). Since there are two unknowns, the plan will have two "layers" to solve the problem.

→ These are the steps I will take to solve this problem:

1) Guess f_2 & f_3 (I can't proceed in the problem unless I guess these values and compare the percentage errors later in the process)

2) Guess Q_1 (This guessed value will let me theoretically compute Reynold's Number, which will let me find friction factor (f_1))

3) Compute Reynold's Number.

4) Calculate friction factor (f_1) by using Reynold's Number and the value for Relative Roughness calculated earlier in this problem.

5) Compute the full RHS of the equation on the previous page.

6) Compare this RHS value to the LHS (which is = 0). Before guessing another value for Q_1, proceed to the next step of this process.

7) Since we now "know" Q_1, we can compute the individual values of Q_2 & Q_3.

8) Once we have the theoretical & individual values for Q_2 & Q_3, we can calculate new friction factor values for f_2 & f_3 (we'll call the new values f_2' & f_3').

9) Compare the values of f_2 & f_3 to f_2' & f_3' and calculate the percentage difference between the two sets.

10) Begin the whole process again until the percentage differences for f_2 & f_3 to f_2' & f_3' AND the LHS to the RHS are both zero percent.

→ Upon attempting to solve for Q_2 in terms of Q_1, I found that Excel could not calculate the value for it could not take the square root of ~~the~~ a negative number. ~~So there are a couple of ways in which I can solve this:~~ To solve this I had to make the Q_1 - Guessed flow rate much smaller, whi in turn made Reynold's number much smaller; Small enough to go to the laminar flow region. This meant that I needed to change my friction factor calculator from turbulent to laminar. No big dea

SUMMARY & ANALYSIS:

→ Upon completion of my calculations in Excel, I was unable to fully compute the flow rate to each bearing. I think my process was sound, but my calculations were flawed. The percentage error between the LHS & RHS was decreasing as it was supposed to, but then it began to increase drastically. The percentage difference between the guessed friction factors barely decreased the further I iterated.

→ I believe what I should have done was used a correction factor for both percentage errors to compute the next guess.