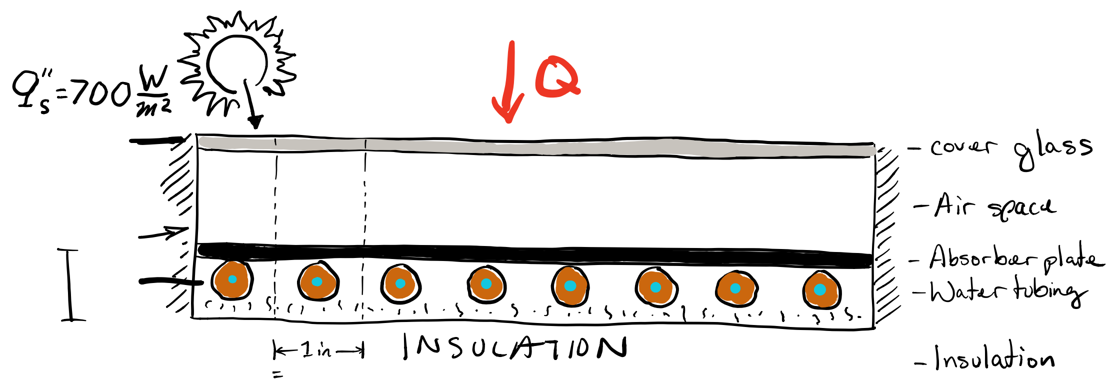
1. (55 points) A solar flux of 700 W/m2 is incident on a flat-plate solar collector used to heat water. 90% of the solar radiation passes through the cover glass and air, and is absorbed directly by the absorber plate. The remaining 10% is reflected away from the collector. Water flows through the tube passages on the backside of the absorber plate and is heated from an inlet temperature of 20oC. The convection coefficient of the water is 1000 W/m2.oK. The convection coefficient between the cover glass and the ambient air at 25oC is 50 W/m2.oK. The convection coefficient in the air space is 10 W/m2.oK. The type K copper tubes are 1⁄2 in. nominal size (5/8 in. outside diameter and 0.528 in. inside diameter) and they are separated by 1 in. center-to-center. The tubes are buried in a 1 in. thick slab made of silver. The back of this silver slab is well insulated. The cover glass thickness is 1/8 in. Assume the absorber plate to be very thin and highly conductive. The air space is 2 in. thick.
   A. What is the amount of heat collected by the water in one of the tubes?
   B. What is the air space temperature?
   C. If the outlet temperature of the water is 45oC, what is the flow rate? Assume the specific heat of the water to be 4179 J/kg.oK.
   D. The collector efficiency η is defined as the ratio of the useful heat collected to the rate at which solar energy is incident on the collector. What is the value of η?

↑ Doesn't offer much resistance to the circuit

PURPOSE:

Find out how much heat goes into the water flowing in the pipes

DRAWINGS & DIAGRAMS:



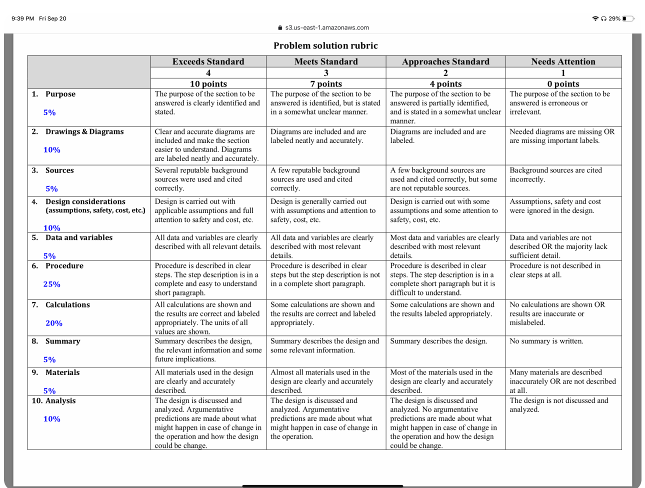
SOURCES:

→ Bayazitoglu, Y., Ozisik, N., "A textbook for heat transfer fundamentals", Begell House, Inc. (2012)

DESIGN CONSIDERATIONS:
(assumptions, safety, cost, etc.)

$A = (1\,in)(1\,m)$
$= (0.0254\,m)(1\,m)$
$= 0.0254\,m^2$

$h_{water} = 1000\,W/(m^2 \cdot °K)$
$h_{air} = 10\,W/(m^2 \cdot °K)$
$h_{glass-air} = 50\,W/(m^2 \cdot °K)$
$k_{silver} = 419\,W/(m \cdot °K)$
$k_{copper} = 386\,W/(m \cdot °K)$
$k_{glass} = 0.78\,W/(m \cdot °K)$



9:39 PM Fri Sep 20 s3.us-east-1.amazonaws.com 29%

Problem solution rubric

	Exceeds Standard 4 10 points	Meets Standard 3 7 points	Approaches Standard 2 4 points	Needs Attention 1 0 points
1. Purpose 5%	The purpose of the section to be answered is clearly identified and stated.	The purpose of the section to be answered is identified, but is stated in a somewhat unclear manner.	The purpose of the section to be answered is partially identified, and is stated in a somewhat unclear manner.	The purpose of the section to be answered is erroneous or irrelevant.
2. Drawings & Diagrams 10%	Clear and accurate diagrams are included and make the section easier to understand. Diagrams are labeled neatly and accurately.	Diagrams are included and are labeled neatly and accurately.	Diagrams are included and are labeled.	Needed diagrams are missing OR are missing important labels.
3. Sources 5%	Several reputable background sources were used and cited correctly.	A few reputable background sources are used and cited correctly.	A few background sources are used and cited correctly, but some are not reputable sources.	Background sources are cited incorrectly.
4. Design considerations (assumptions, safety, cost, etc.) 10%	Design is carried out with applicable assumptions and full attention to safety and cost, etc.	Design is generally carried out with assumptions and attention to safety, cost, etc.	Design is carried out with some assumptions and some attention to safety, cost, etc.	Assumptions, safety and cost were ignored in the design.
5. Data and variables 5%	All data and variables are clearly described with all relevant details.	All data and variables are clearly described with most relevant details.	Most data and variables are clearly described with most relevant details.	Data and variables are not described OR the majority lack sufficient detail.
6. Procedure 25%	Procedure is described in clear steps. The step description is in a complete and easy to understand short paragraph.	Procedure is described in clear steps but the step description is not in a complete short paragraph.	Procedure is described in clear steps. The step description is in a complete short paragraph but it is difficult to understand.	Procedure is not described in clear steps at all.
7. Calculations 20%	All calculations are shown and the results are correct and labeled appropriately. The units of all values are shown.	Some calculations are shown and the results are correct and labeled appropriately.	Some calculations are shown and the results labeled appropriately.	No calculations are shown OR results are inaccurate or mislabeled.
8. Summary 5%	Summary describes the design, the relevant information and some future implications.	Summary describes the design and some relevant information.	Summary describes the design.	No summary is written.
9. Materials 5%	All materials used in the design are clearly and accurately described.	Almost all materials used in the design are clearly and accurately described.	Most of the materials used in the design are clearly and accurately described.	Many materials are described inaccurately OR are not described at all.
10. Analysis 10%	The design is discussed and analyzed. Argumentative predictions are made about what might happen in case of change in the operation and how the design could be change.	The design is discussed and analyzed. Argumentative predictions are made about what might happen in case of change in the operation.	The design is discussed and analyzed. No argumentative predictions are made about what might happen in case of change in the operation and how the design could be change.	The design is not discussed and analyzed.

Copper:
Type K: ·½ in nominal
·5/8 in OD
·0.528 in ID

Silver:
1 in thick

DATA & VARIABLES:

→ See Design Considerations

PROCEDURE:

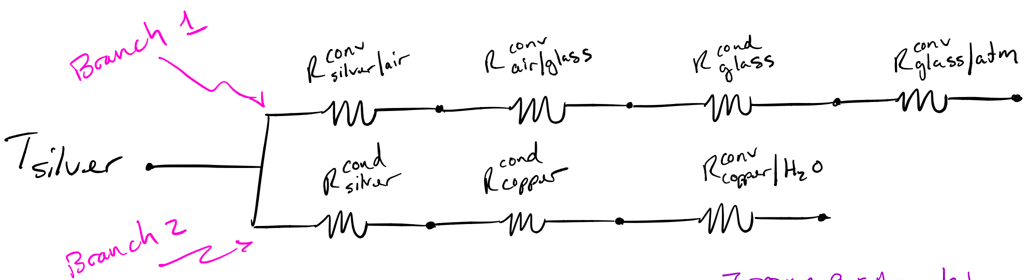
1-A) What is the amount of heat collected by the water in one of the tubes?

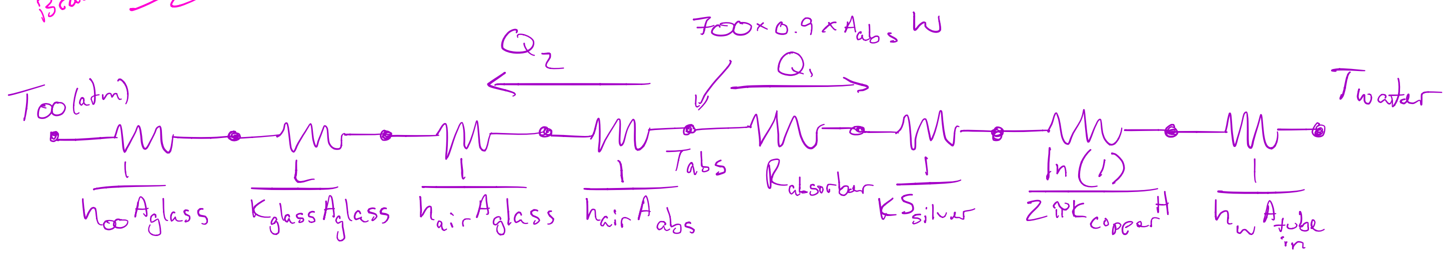
→ Silver is the starting point. This problem resembles the igloo problem given in class where heat transfers in two different directions.

→ The first thing that actually absorbs any heat is the silver, so I will start there. From there it goes up into the atmosphere, traveling through the absorber plate, the air pocket, and the cover glass.

→ Two shape factor eqns will be used to solve, since heat transfers in <u>two</u> directions

$\frac{1}{hA}$ = convection $\frac{L}{kA}$ = conduction





Solve for T_{silver} in the following equation, then plug T_{silver} back into the Q_2 equation to solve for the heat transferred to the water.

$$Q_1 = \frac{\Delta T}{R_{eq_1}} = \frac{T_{silver} - T_\infty}{\frac{1}{hA_{silver}} + \frac{1}{hA_{glass}} + \frac{L}{kA_{glass}} + \frac{1}{hA_{glass}}}$$

$$Q_2 = \frac{\Delta T}{R_{eq_2}} = \frac{T_{silver} - T_{water}}{\frac{L}{kA_{silver}} + \frac{L}{kA_{copper}} + \frac{1}{hA_{copper}}}$$

$(+) = Q_T$

this is L_{glass} in metric

$$R_{eq_1} = \frac{1}{(10W/(m^2\cdot{}^\circ K))(0.0254m^2)} + \frac{1}{(10W/(m^2\cdot{}^\circ K))(0.0254m^2)} + \frac{0.003175m}{(0.78W/(m\cdot{}^\circ K))(0.0254m^2)} + \frac{1}{(50W/(m^2\cdot{}^\circ K))(0.0254m^2)}$$

$$= 3.937 + 3.937 + 0.1603 + 0.7874$$

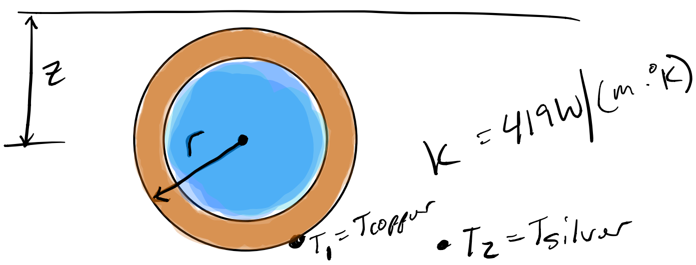
$$\boxed{R_{eq_1} = 8.82\ K/W}$$

$$Q_1 = \frac{\Delta T}{R_{eq_1}} =$$

We'll put R_{eq_1} where it belongs a little bit later

→ Now, we solve for R_{eq_2}. In order to do that, we must solve for the shape factors that are unknown in the R_{eq_2} equation.

Shape factor problem of silver to copper (case #6)



$r = (5/8\text{in})/2 = 0.3125\text{in} = .0079375m$

$z = 0.5\text{in} = .0127m$

$$S_1 = \frac{4\pi r}{1 + r/(2z)} = \frac{4\pi(0.3125\text{in})}{1 + (0.3125\text{in})/(2(0.5\text{in}))}$$

$= .00794$; $= .0127$

$$\boxed{S_1 = 2.99\text{in}} \Rightarrow 0.0759m ✓$$

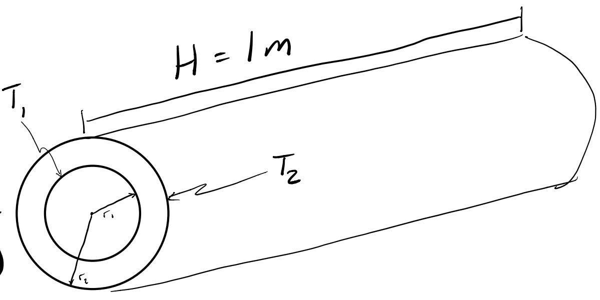
$$S_1 = \frac{4\pi r}{1 + r/(2\cdot z)} = \frac{4\pi(.0079375m)}{1 + (.0079375/(2(.0127)))} = 0.076m$$

$$\boxed{S_1 = 0.076m}$$

Shape factor problem of copper to water (case #2)

$$S_2 = \frac{2\pi H}{\ln(r_2/r_1)}$$

$r_1 = \frac{0.528\text{in}}{2} = 0.264\text{in} = 0.0067056\text{m}$



$r_2 = (5/8\text{in})/2 = 0.3125\text{in} = 0.0079375\text{m}$

$$S_2 = \frac{2\pi(1\text{m})}{\ln(.0079375\text{m}/.0067056\text{m})}$$

$$\boxed{S_2 = 37.25\text{m}}$$

→ Now we can solve for R_{eq_2}

$$R_{eq_2} = \frac{1}{S_2 k_{silver}} + \frac{1}{S_2 k_{copper}} + \frac{1}{h_{H_2O} A_{normal}}$$

$$R_{eq_2} = \frac{1}{(0.00759\text{m})(419\text{W}/(\text{m}\cdot°\text{K}))} + \frac{1}{(37.25\text{m})(386\text{W}/(\text{m}\cdot°\text{C}))} + \frac{1}{(1000\text{W}/(\text{m}^2\cdot°\text{K}))(0.0254\text{m}^2)}$$

$$= 0.314445 + 0.00007 + 0.03937$$

$$\boxed{R_{eq_2} = 0.353885\ °\text{K}/\text{W}}$$

$h_{water} = 1000\text{W}/(\text{m}^2\cdot°\text{K})$

$h_{air} = 10\text{W}/(\text{m}^2\cdot°\text{K})$

$h_{glass-air} = 50\text{W}/(\text{m}^2\cdot°\text{K})$

$k_{silver} = 419\text{ W}/(\text{m}\cdot°\text{K})$

$k_{copper} = 386\text{ W}/(\text{m}\cdot°\text{K})$

$k_{glass} = 0.78\text{ W}/(\text{m}\cdot°\text{K})$

Copper:

TYPE K
- 1/2in nominal
- 5/8in OD
- 0.15875m OD
- 0.528in ID
- 0.134112 ID

$\frac{OD}{2} - \frac{ID}{2} = L_{copper} = 0.0123\text{m}$

Silver:

1in thick = 0.254m

$$Q_1 = \frac{\Delta T}{R_{eq_1}} = \frac{T_{silver} - T_\infty}{\frac{1}{hA_{silver}} + \frac{1}{hA_{glass}} + \frac{L}{kA_{glass}} + \frac{1}{hA_{glass}}}$$

$$Q_2 = \frac{\Delta T}{R_{eq_2}} = \frac{T_{silver} - T_{water}}{\frac{L}{kA_{silver}} + \frac{L}{kA_{copper}} + \frac{1}{hA_{copper}}}$$

$$(+) = Q_T$$

These two have been changed with the shape factor solutions

$$Q_1 = \frac{\Delta T}{R_{eq_1}} = \frac{T_{silver} - T_\infty}{\frac{1}{hA_{silver}} + \frac{1}{hA_{glass}} + \frac{L}{kA_{glass}} + \frac{1}{hA_{glass}}}$$

$$Q_2 = \frac{\Delta T}{R_{eq_2}} = \frac{T_{silver} - T_{water}}{\frac{1}{S_2 k_{silver}} + \frac{1}{S_2 k_{copper}} + \frac{1}{hA_{copper}}}$$

$$(+) = Q_T$$

$$Q_T = (700W)(0.90) = 630W = Q_1 + Q_2 = \frac{T_{silver} - 298\,K}{8.82\,K/W} + \frac{T_{silver} - 293\,K}{0.353885\,K/W}$$

$$Q_T = 630W = \frac{T_{silver} - 298}{8.82\ ^1/W} + \frac{T_{silver} - 293}{0.353885\ ^1/W}$$

$$\left(\frac{0.353885}{0.353885}\right)\left(\frac{T_{silver} - 298}{8.82}\right) + \left(\frac{8.82}{8.82}\right)\left(\frac{T_{silver} - 293}{0.353885}\right)$$

$$630 = \frac{0.353885T_s - 105.458}{3.12127} + \frac{8.82T_s - 2584.26}{3.12127}$$

$$630 = \frac{9.17389T_s - 2689.72}{3.12127}$$

$$630 = 2.93915T_s - 861.739$$

$$1491.74 = 2.93915\, T_s$$

$$T_s = 507.541\,K = 234.391^\circ C$$ ← This is the silver temperature due to the total heat absorption

$$Q_1 = \frac{T_{silver} - T_\infty}{R_{eq_1}} = \frac{234.391^\circ C - 25^\circ C}{8.82^\circ K/W}$$

$$Q_1 = 23.7405\,W$$

$$Q_2 = \frac{T_{silver} - T_{H_2O}}{R_{eq_2}} = \frac{234.391^\circ C - 20^\circ C}{0.353885^\circ K/W} = 605.821\,W$$

$$Q_2 = 605.821\,W$$ ANSWER

1-B) what is the air space temperature?

$$Q = \frac{\Delta T}{R}$$

$$\frac{T_{silver} - T_{air\,pocket}}{R^{conv}_{silver \to air}} = \frac{234.391°C - T_{air}}{\left(\frac{1}{(10\,W/(m^2 \cdot °K))(0.0254\,m^2)}\right)} = 23.7405\,W$$

$$T_{air} = 234.391 - (23.7405)(3.93701)$$

$$T_{air} = 140.924°C$$

ANSWER

1-c) If the outlet temperature of the water is 45°C, what is the flow rate?
Assume the specific heat of the water to be 4179 J/kg·°K

$$\left[Q_{H_2O} = \dot{m}\, C_{v_{H_2O}} \Delta T \right]$$

$$Q_{H_2O} = Q_2 = Q_{\text{into } H_2O} = 605.821\text{ W} = \dot{m}\, C_{v_{H_2O}} \overset{45°C - 20°C}{(T_1 - T_2)}$$

$$\dot{m} = \frac{605.821\text{ W}}{(4179\text{ J/kg}\cdot°\text{K})(45°C - 20°C)}$$

$$\dot{m} = 0.006\text{ kg/s}$$

ANSWER

1-D) The collector efficiency (η) is defined as the ratio of the useful heat collected to the rate at which solar energy is incident on the collector. What is the value of η?

before the percentage of absorption

$$\eta = \frac{Q_{H_2O}}{Q_i} = \frac{605.821\,W}{700\,W} =$$

$$\eta = 0.87$$

ANSWER

ANALYSIS:

1-A) Units cancelled nicely, and the correct units of W is in place. This is the amount of heat that was transferred from the ambient air outside the collector into the water flowing in the pipes.

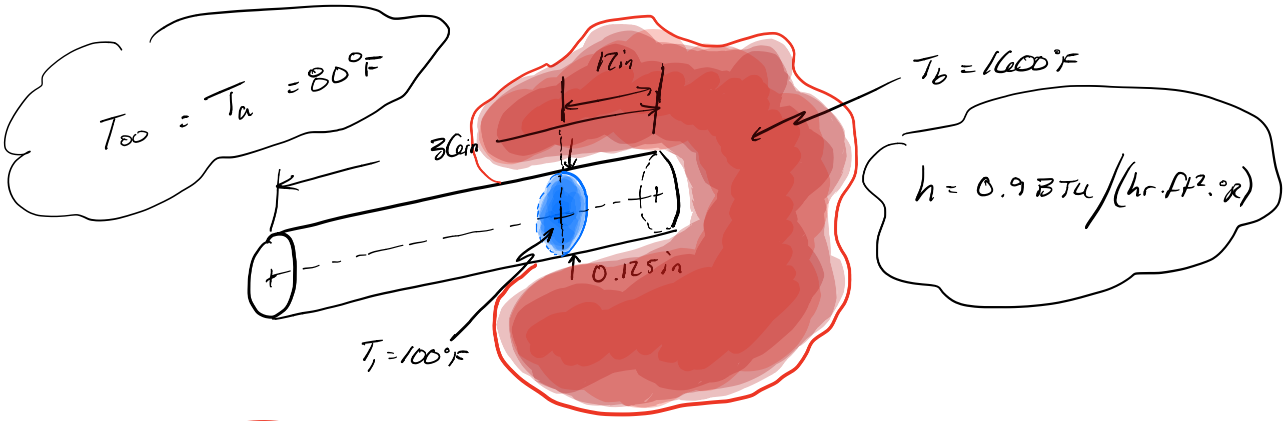
1-B) This value is the temperature of the air pocket between the collector and the glass. When it's coming in, the heat passes through the air and collector and is essentially fully absorbed by the silver. From there (the silver), heat overflows back into the airspace between the collector and the glass

1-C) This is the mass flow rate of the water flowing through the pipes embedded in the silver. Given the knowledge of the specific heat of the fluid, and what temperature it heats to, we can pair it with the calculated heat (in Watts) transferred to it in 1-A and then solve for the mass flow rate.

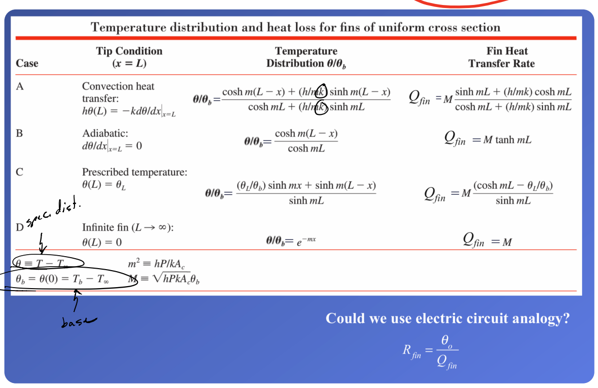
1-D) Efficiency is always unitless, as is my answer.

2. (25 points) The brazing operation is a metal-joining process that is done by heating a base metal to a high temperature and applying a brazing material to the heated joint. The heated base metal melts the brazing alloy, which fills the joint, and then solidifies when allowed to cool. The brazing material is a copper alloy in the shape of a rod of 1/8 in. diameter and 3 ft length. The end of the brazing rod (where it is being melted) reaches an average temperature of 1600 oF. What is the conductivity of the brazing material if at a distance of 1 ft the temperature is 100 oF? The ambient temperature is 80 oF, and the convection coefficient is 0.9 BTU/(hr.ft2.oR).

TREAT THIS AS A "FIN" PROBLEM



$k = ?$ ⟸ solve for k



Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx\vert_{x=L}$	$\theta/\theta_b = \dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$Q_{fin} = M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic: $d\theta/dx\vert_{x=L} = 0$	$\theta/\theta_b = \dfrac{\cosh m(L-x)}{\cosh mL}$	$Q_{fin} = M\tanh mL$
C	Prescribed temperature: $\theta(L) = \theta_L$	$\theta/\theta_b = \dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	$Q_{fin} = M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$
D	Infinite fin ($L \to \infty$): $\theta(L) = 0$	$\theta/\theta_b = e^{-mx}$	$Q_{fin} = M$

$\theta \equiv T - T_\infty$ (spec. dist.)　　$m^2 \equiv hP/kA_c$
$\theta_b = \theta(0) = T_b - T_\infty$ (base)　　$M \equiv \sqrt{hPkA_c}\,\theta_b$

Could we use electric circuit analogy?

$R_{fin} = \dfrac{\theta_o}{Q_{fin}}$

use the following equations:

$$\theta/\theta_b = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL} = \frac{T - T_\infty}{T_b - T_\infty}$$

$$m = \sqrt{hP/kA_c}$$

LHS = RHS

$T = 100°F$

$T_\infty = 80°F$

$T_b = 1600°F$

$$\frac{100 - 80}{1600 - 80} = \frac{20}{1520} = 0.0132$$

$$\frac{\theta}{\theta_b} = \frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh\sqrt{hP/kA_c}\,(L-x) + \left(h/(\sqrt{hP/kA_c} \times k)\right)\sinh\sqrt{hP/kA_c}\,(L-x)}{\cosh\sqrt{hP/kA_c}\,L + \left(h/(\sqrt{hP/kA_c} \times k)\right)\sinh\sqrt{hP/kA_c}\,L}$$

→ Set up excel file to solve for k (see attached submission).

I tried to use Excel's GoalSeek function to find the value of k by setting the "RHS" equal to the "LHS". The closest value of k that was computed was $k = 6120$ BTU/(h·ft·°F)