MICHAEL DELACRUZ

MET 440

TEST 2

PURPOSE: DEVELOP TEMPERATURE PROFILE EQUATIONS FOR HEMISPHERE WITH INNER RADIUS OF a + OUTER RADIUS B GIVEN HEAT GENERATION

$$g = S_0\left(\frac{b^2}{r^2}\right)$$

AND THERMAL CONDUCTIVITY

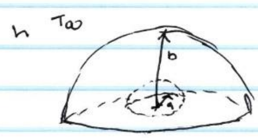
$$K = K_0\left(\frac{b^2}{r^2}\right)$$

AND BOUNDARY CONDITIONS

$r = a$ $\quad\quad$ $r = b$

$\frac{dT}{dr} = 0$ $\quad\quad$ $-K\frac{dT}{dr} = h(T - T_\infty)$

DRAWINGS + DIAGRAMS:



SOURCE: BAYAZITOGLU, Y., OZISIK, N. "A TEXTBOOK FOR HEAT TRANSFER FUNDAMENTALS", BEGEL HOUSE, INC (2012)

DESIGN CONSIDERATIONS:

- STEADY STATE
- 1 DIMENSIONAL
- CONSTANT PROPERTIES

DATA + VARIABLES:

PROCEDURE:

- USING DIFFERENTIAL EQUATION PROVIDED ON THE TEST
- INTEGRATE USING EQUATIONS OF THERMAL CONDUCTIVITY + HEAT GENERATION
- CALCULATE CONSTANTS
- EVALUATE USING BOUNDARY CONDITIONS

CALCULATIONS:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 k \frac{dT}{dr}\right) + g = 0 \qquad : \frac{dT}{dr} = 0 \ @ \ r = a$$

$$g = g_0 \frac{b^2}{r^2} \ W/m^3$$

$$k = k_0 \frac{b^2}{r^2} \ W/m \cdot k \qquad : -k\frac{dT}{dr} = h(T - T_\infty) \ @ \ r = b$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 k \frac{dT}{dr}\right) + g = 0$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 k_0 \frac{b^2}{r^2} \frac{dT}{dr}\right) + g_0 \frac{b^2}{r^2} = 0$$

$$\frac{1}{\cancel{r^2}}\frac{d}{dr}\left(\cancel{r^2} k_0 \frac{b^2}{\cancel{r^2}} \frac{dT}{dr}\right) = -g_0 \frac{b^2}{\cancel{r^2}}$$

$$\frac{d}{dr}\left(k_0 b^2 \frac{dT}{dr}\right) = -g_0 b^2$$

$$k_0 \cancel{b^2} \frac{d}{dr}\left(\frac{dT}{dr}\right) = -g_0 \cancel{b^2}$$

$$\frac{k_0}{k_0}\frac{d}{dr}\left(\frac{dT}{dr}\right) = \frac{-g_0}{k_0}$$

$$\frac{d}{dr}\left(\frac{dT}{dr}\right) = -\frac{g_0}{k_0}$$

$$d\left(\frac{dT}{dr}\right) = -\frac{g_0}{k_0} dr$$

INTEGRATE

$$\int d\left(\frac{dT}{dr}\right) = -\frac{g_0}{k_0}\int dr$$

$$* \quad \frac{dT}{dr} = -\frac{g_0}{k_0} r + C_1$$

INTEGRATE

$$\frac{dT}{dr} = -\frac{g_0}{k_0} r + C_1$$

$$dT = \left(\frac{-g_0}{k_0} r + C_1\right) dr$$

$$\cancel{dAW} \quad dT = \frac{-g_0}{k_0} r\,dr + C_1 dr$$

$$\int dT = \frac{-g_0}{k_0}\int r\,dr + C_1 \int dr$$

$$* \quad T = -\frac{g_0}{2k_0} r^2 + C_1 r + C_2$$

$$\frac{dT}{dr} = 0 \quad @ \quad r = a$$

$$\frac{dT}{dr} = -\frac{g_0}{K_0} r + C_1$$

$$\left.\frac{dT}{dr}\right|_{r=a} = -\frac{g_0}{K_0} a + C_1$$

$$0 = -\frac{g_0}{K_0} a + C_1$$

$$* \; C_1 = \frac{g_0}{K_0} a$$

$$-k \frac{dT}{dr} = h(T - T_\infty) \quad @ \quad r = b$$

$$-k \left.\frac{dT}{dr}\right|_{r=b} = h(T_{r=b} - T_\infty)$$

$$-k\left(-\frac{g_0}{K_0} r + C_1\right) = h\left(\left(-\frac{g_0}{2K_0} r^2 + C_1 r + C_2\right) - T_\infty\right) \qquad r = b$$

$$-k\left(-\frac{g_0}{K_0} b + C_1\right) = h\left(\left(-\frac{g_0}{2K_0} b^2 + C_1 b + C_2\right) - T_\infty\right)$$

$$-k\left(-\frac{g_0}{K_0} b + \frac{g_0}{K_0} a\right) = h\left(\left(\frac{-g_0}{2K_0} b^2 + \frac{g_0}{K_0} ab + C_2\right) - T_\infty\right)$$

$$\cancel{\frac{-k\left(\frac{-g_0}{K_0} b + \frac{g_0}{K_0} a\right)}{h} = h\left(\frac{-g_0}{2K_0} b^2 + \frac{g_0}{K_0} ab + C_2\right) - \frac{T_\infty(h)}{h}}$$

$$\cancel{\frac{-k}{h}\left(\frac{-g_0}{K_0} b + \frac{g_0}{K_0} a\right) = h\left(\frac{-g_0}{2K_0} b^2 + \frac{g_0}{K_0} ab + C_2\right) - T_\infty}$$

$$\cancel{+T_\infty \qquad\qquad +T_\infty}$$

$$\cancel{T_\infty - \frac{k}{h}\left(\frac{-g_0}{K_0} b + \frac{g_0}{K_0} a\right) = h\left(\frac{-g_0}{2K_0} b^2 + \frac{g_0}{K_0} ab + C_2\right)}$$

$$\cancel{T_\infty - \frac{k}{h}\left(\frac{-g_0}{K_0} b + \frac{g_0}{K_0} a\right) = -h \frac{g_0}{2K_0} b^2 + h \frac{g_0}{K_0} ab + C_2 h}$$

$$\cancel{T_\infty - \frac{k}{h}\left(\frac{-g_0}{K_0} b + \frac{g_0}{K_0} a\right) + h \frac{g_0}{2K_0} b^2 - h \frac{g_0}{K_0} ab = C_2 h}$$

$$-k\left(\frac{-g_0}{K_0} b + \frac{g_0 a}{K_0}\right) = h\left(\left(\frac{-g_0}{2K_0} b^2 + \frac{g_0}{K_0} ab + C_2\right) - T_\infty\right)$$

$$T_\infty - \frac{k}{h}\left(\frac{-g_0}{K_0} b + \frac{g_0}{K_0} \cdot a\right) = -\frac{g_0}{2K_0} \cdot b^2 + \frac{g_0}{K_0} ab + C_2$$

$$T_\infty - \frac{\cancel{k_0}}{h}\left(\frac{-g_0}{\cancel{K_0}} b + \frac{g_0}{\cancel{K_0}} a\right) + \frac{g_0}{2K_0} b^2 - \frac{g_0}{\cancel{K_0}} ab \pm C_2$$

$$T_\infty - \frac{g_0}{h}(a - b) + \frac{g_0 b}{K_0}\left(\frac{b}{2} - a\right) = C_2$$

$$* \; C_2 = T_\infty + \frac{g_0}{h}(b - a) + \frac{g_0 b}{K_0}\left(\frac{b}{2} - a\right)$$

$$T = -\frac{g_0}{2K_0} r^2 + C_1 r + C_2$$

$$C_1 = \frac{g_0}{K_0} a$$

$$C_2 = T_\infty + \frac{g_0}{h}(b-a) + \frac{g_0 b}{K_0}\left(\frac{b}{2} - a\right)$$

$$T = -\frac{g_0}{2K_0} r^2 + \frac{g_0}{K_0} a r + T_\infty + \frac{g_0}{h}(b-a) + \frac{g_0 b}{K_0}\left(\frac{b}{2} - a\right)$$

$$= -\frac{g_0}{2K_0} r^2 + \frac{g_0}{K_0} a r + T_\infty + \frac{g_0}{h}(b-a) + \frac{g_0 b^2}{2K_0} - \frac{g_0 b}{K_0} a$$

$$\boxed{T = \frac{g_0}{2K_0}(b^2 - r^2) - \frac{g_0 a}{K_0}(b - r) + \frac{g_0}{h}(b-a) + T_\infty}$$

HEAT TRANSFER

$Q = -KA \frac{dT}{dr}$ $\qquad r = b$

AREA OF SPHERE $A = 4\pi r^2$

AREA OF HEMISPHERE $= \frac{1}{2} 4\pi r^2$

$A = \frac{1}{2} 4\pi b^2$ $\qquad K_{r_\infty} = K_0 \frac{b^2}{b^2} = K_0$

$= 2\pi b^2$

$$\frac{dT}{dr} = \frac{-g_0 r}{K_0} + C_1$$

$$= -\frac{g_0}{K_0} r + \frac{g_0}{K_0} a$$

$$= -\frac{g_0}{K_0} b + \frac{g_0}{K_0} a = \frac{g_0}{K_0}(a-b)$$

$$Q = -\cancel{K_0}(2\pi b^2)\frac{g_0}{\cancel{K_0}}(a-b)$$

$$= (2\pi b^2)\, g_0 (a-b)$$

$$\boxed{Q = 2\pi b^2 g_0 (a-b)}$$

SUMMARY

TEMPERATURE PROFILE

$$T = \frac{g_0}{2k_0}(b^2 - r^2) - \frac{g_0 a}{k_0}(b - r) + \frac{g_0}{h}(b - a) + T_\infty$$

HEAT TRANSFER

$$Q = 2\pi b^2 g_0 (a - b)$$

MATERIALS : HEMISPHERE W/ INSULATED BOTTOM + HEAT GENERATION

ANALYSIS.

~~T~~HE HEMISPHERE IS PROPERLY INSULATED AT THE BOTTOM + AT $r = a$. THE SURFACE AT $r = b$ IS ENCOUNTERING HEAT TRANSFER. IF $g_0 = 0$, THE SURFACE WOULD BE THE ONE GENERATING HEAT. AT STEADY STATE THE AMBIENT WOULD EQUAL THE HEMISPHERE TEMPERATURE. THE TEMPERATURE PROFILE CHANGES RADIALLY.

EXAM 2 MET440

Report date	Oct 11, 2019 9:50:42 PM

Contents

1 Global Definitions

Date	Oct 10, 2019 1:24:10 PM

Global settings

Name	TEST 2 real.mph
Path	H:\TEST 2 real.mph
COMSOL version	COMSOL 5.1 (Build: 145)
Unit system	SI

Used products

COMSOL Multiphysics
CAD Import Module

1.1 Parameters 1

Parameters

Name	Expression	Value	Description
a	2[cm]	0.02 m	inside radius
b	5[cm]	0.05 m	outside radius
ho	200[W/m^2/K]	200 W/(m²·K)	convective coefficient
ko	10[W/m/K]	10 W/(m·K)	conductivity
Too	100[degC]	373.15 K	ambient temp
go	10^6[W/m^3]	1E6 W/m³	heat generation

2 Component 1

Date	Oct 8, 2019 4:13:11 PM

Component settings

Unit system	SI
Geometry shape order	automatic

2.1 Definitions

2.1.1 Coordinate Systems

Boundary System 1

Coordinate system type	Boundary system
Tag	sys1

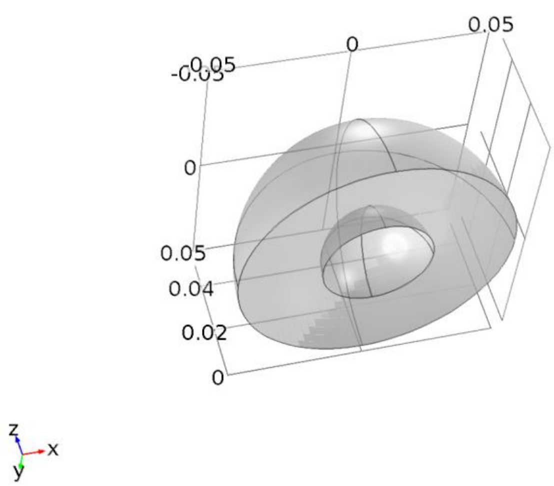
Coordinate names

First (t1)	Second (t2)	Third (n)
t1	t2	n

Settings

Description	Value
Create first tangent direction from	Global Cartesian

2.2 Geometry 1



Geometry 1

Units

Length unit	m
Angular unit	deg

Geometry statistics

Description	Value
Space dimension	3
Number of domains	1
Number of boundaries	9
Number of edges	16
Number of vertices	10

2.2.1 Sphere 1 (sph1)

Position

Description	Value
Position	{0, 0, 0}

Axis

Description	Value
Axis type	z - axis
Layers	

Size

Description	Value
Radius	b

2.2.2 Sphere 2 (sph2)

Position

Description	Value
Position	{0, 0, 0}

Axis

Description	Value
Axis type	z - axis
Layers	

Size

Description	Value
Radius	a

2.2.3 Block 1 (blk1)

Position

Description	Value
Position	{0, 0, -b}
Base	Center

Axis

Description	Value
Axis type	z - axis

Size and shape

Description	Value
Width	2*b
Depth	2*b
Height	2*b

Layers

Description	Value
Layers	

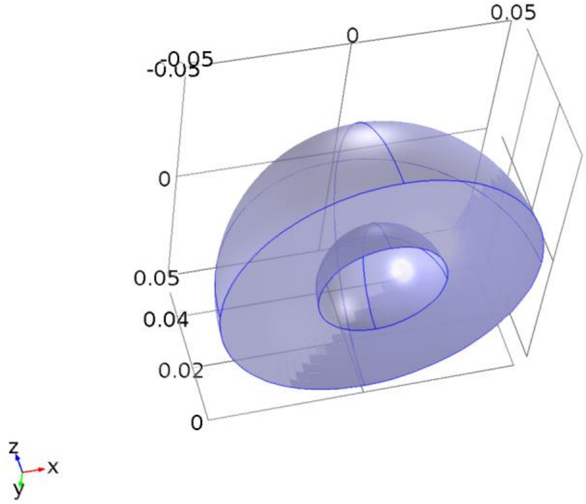
2.2.4 Form Assembly (fin)

Settings

Description	Value
Action	Form an assembly

2.3 Materials

2.3.1 Material 1



Material 1

Selection

Geometric entity level	Domain
Selection	Domain 1

Material parameters

Name	Value	Unit
Thermal conductivity	ko*b^2/(x^2+y^2+z^2)	W/(m*K)
Density	1	kg/m^3
Heat capacity at constant pressure	1	J/(kg*K)

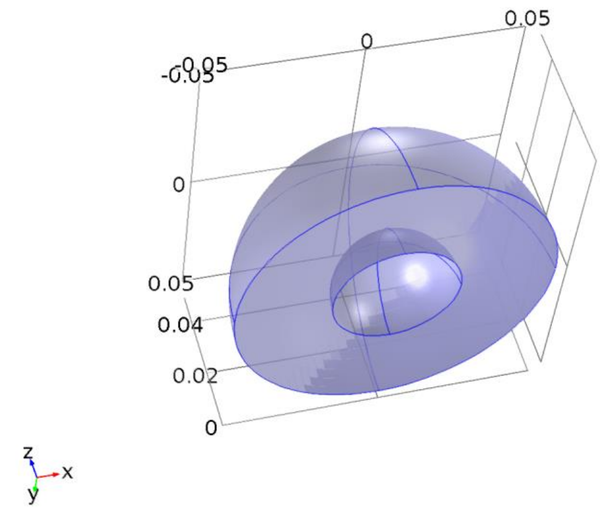
Basic Settings

Description	Value
Thermal conductivity	{{ko*b^2/(x^2 + y^2 + z^2), 0, 0}, {0, ko*b^2/(x^2 + y^2 + z^2), 0}, {0, 0, ko*b^2/(x^2 + y^2 + z^2)}}
Density	1
Heat capacity at constant pressure	1

2.4 Heat Transfer in Solids

Used products

COMSOL Multiphysics



Heat Transfer in Solids

Selection

Geometric entity level	Domain
Selection	Domain 1

Equations

$$\rho C_p \mathbf{u} \cdot \nabla T + \nabla \cdot \mathbf{q} = Q + Q_{\mathrm{ted}}$$
$$\mathbf{q} = -k \nabla T$$

Settings

Description	**Value**
Temperature	Quadratic
Compute boundary fluxes	On
Apply smoothing to boundary fluxes	On
Value type when using splitting of complex variables	Real
Surface-to-surface radiation	Off
Radiation in participating media	Off
Heat transfer in biological tissue	Off
Isothermal domain	Off
Heat transfer in porous media	Off
Streamline diffusion	On
Crosswind diffusion	On

Description	Value
Isotropic diffusion	Off

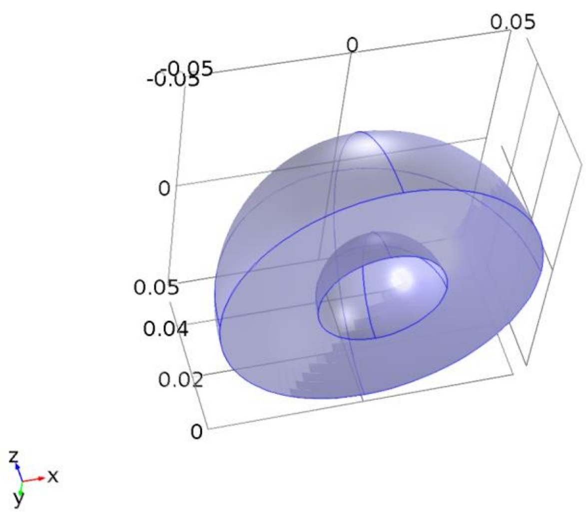
Variables

Name	Expression	Unit	Description	Selection
ht.q0	0	W/m^2	Inward heat flux	Boundaries 1–9
ht.Tu	T	K	Temperature	Boundaries 1–9
ht.Td	T	K	Temperature	Boundaries 1–9
ht.opaqueLayer	1		Thin layer opacity	Boundaries 1–9
ht.Tvar	T	K	Temperature	Domain 1
ht.Tvar	T	K	Temperature	Boundaries 1–9
ht.Tvar	T	K	Temperature	Edges 1–16
ht.Tvar	T	K	Temperature	Points 1–10
ht.addContinuityWeak	1		Boolean	Boundaries 1–9
ht.isConductiveLayer	1		Boolean	Domain 1
ht.d	1	1	Thickness	Domain 1
ht.nx	dnx	1	Normal vector, x component	Boundaries 1–9
ht.ny	dny	1	Normal vector, y component	Boundaries 1–9
ht.nz	dnz	1	Normal vector, z component	Boundaries 1–9
ht.nxmesh	root.dnxmesh	1	Normal vector (mesh), x component	Boundaries 1–9
ht.nymesh	root.dnymesh	1	Normal vector (mesh), y component	Boundaries 1–9

Name	Expression	Unit	Description	Selection
ht.nzmesh	root.dnzmesh	1	Normal vector (mesh), z component	Boundaries 1–9
ht.dnx	dnx	1	Normal vector down direction, x component	Boundaries 1–9
ht.dny	dny	1	Normal vector down direction, y component	Boundaries 1–9
ht.dnz	dnz	1	Normal vector down direction, z component	Boundaries 1–9
ht.unx	unx	1	Normal vector up direction, x component	Boundaries 1–9
ht.uny	uny	1	Normal vector up direction, y component	Boundaries 1–9
ht.unz	unz	1	Normal vector up direction, z component	Boundaries 1–9
ht.dEiInt	ht.intDom(d(ht.rho*ht.Ei,t)*ht.varIntSpa)	W	Total accumulated heat rate	Global
ht.dEi0Int	ht.intDom(d(ht.rho*ht.Ei0,t)*ht.varIntSpa)	W	Total accumulated energy rate	Global
ht.ntfluxInt	ht.intExtBnd(ht.ntflux*ht.varIntSpa)	W	Total net heat rate	Global
ht.ntefluxInt	ht.intExtBnd(ht.nteflux*ht.varIntSpa)	W	Total net energy rate	Global
ht.QInt	ht.intDom(ht.Qtot*ht.varIntSpa)-ht.intIntBnd((ht.ndflux_u+ht.ndflux_d)*ht.varIntSpa)	W	Total heat source	Global
ht.WnsInt	0	W	Total work source	Global

Name	Expression	Unit	Description	Selection
ht.WInt	0	W	Total work source	Global

2.4.1 Heat Transfer in Solids 1



Heat Transfer in Solids 1

Selection

Geometric entity level	Domain
Selection	Domain 1

Equations

$$\rho C_p \mathbf{u} \cdot \nabla T + \nabla \cdot \mathbf{q} = Q + Q_{\mathrm{ted}}$$
$$\mathbf{q} = -k \nabla T$$

Settings

Description	Value
Thermal conductivity	From material
Density	From material
Heat capacity at constant pressure	From material

Properties from material

Property	Material	Property group
Thermal conductivity	Material 1	Basic
Density	Material 1	Basic

Property	Material	Property group
Heat capacity at constant pressure	Material 1	Basic

Variables

Name	Expression	Unit	Description	Selection
domflux.Tx	-ht.k_effxx*Tx-ht.k_effxy*Ty-ht.k_effxz*Tz	W/m^2	Domain flux, x component	Domain 1
domflux.Ty	-ht.k_effyx*Tx-ht.k_effyy*Ty-ht.k_effyz*Tz	W/m^2	Domain flux, y component	Domain 1
domflux.Tz	-ht.k_effzx*Tx-ht.k_effzy*Ty-ht.k_effzz*Tz	W/m^2	Domain flux, z component	Domain 1
ht.WnsInt	ht.solid1.intDom(ht.pA*(d(ht.ux,x)+d(ht.uy,y)+d(ht.uz,z))*ht.solid1.varIntSpa)	W	Total work source	Global
ht.Q	0	W/m^3	Heat source	Domain 1
ht.Qtot	0	W/m^3	Total heat source	Domain 1
ht.kxx	model.input.k11	W/(m*K)	Thermal conductivity, xx component	Domain 1
ht.kyx	model.input.k21	W/(m*K)	Thermal conductivity, yx component	Domain 1
ht.kzx	model.input.k31	W/(m*K)	Thermal conductivity, zx component	Domain 1
ht.kxy	model.input.k12	W/(m*K)	Thermal conductivity, xy component	Domain 1
ht.kyy	model.input.k22	W/(m*K)	Thermal conductivity, yy component	Domain 1
ht.kzy	model.input.k32	W/(m*K)	Thermal conductivity, zy component	Domain 1

Name	Expression	Unit	Description	Selection
ht.kxz	model.input.k13	W/(m*K)	Thermal conductivity, xz component	Domain 1
ht.kyz	model.input.k23	W/(m*K)	Thermal conductivity, yz component	Domain 1
ht.kzz	model.input.k33	W/(m*K)	Thermal conductivity, zz component	Domain 1
ht.rho	model.input.rho	kg/m^3	Density	Domain 1
ht.Cp	model.input.Cp	J/(kg*K)	Heat capacity at constant pressure	Domain 1
ht.solid1.pRef	model.input.pRef	Pa	Reference pressure level	Domain 1
ht.alphap	-d(ht.rho,T)/(ht.rho+eps)	1/K	Isobaric compressibility coefficient	Domain 1
ht.pA	model.input.minput_pressure	Pa	Absolute pressure	Domain 1
ht.gradTmag	sqrt(ht.gradTx^2+ht.gradTy^2+ht.gradTz^2)	K/m	Temperature gradient magnitude	Domain 1
ht.kmean	(ht.k_effxx+ht.k_effyy+ht.k_effzz)/3	W/(m*K)	Mean effective thermal conductivity	Domain 1
ht.qs	0	W/(m^3*K)	Production/absorption coefficient	Domain 1
ht.Qmet	0	W/m^3	Metabolic heat source	Domain 1
ht.rhoInt	subst(ht.rho,root.comp1.ht.solid1.minput_pressure,ht.pA)	kg/m^3	Density for integration	Domain 1
ht.CpInt	subst(ht.Cp,root.comp1.ht.solid1.minput_pressure,ht.pA)	J/(kg*K)	Specific heat capacity for integration	Domain 1
ht.gammaInt	subst(ht.gamma,root.comp1.ht.solid1.minput_pressure,ht.pA)	1	Ratio of specific heats for integration	Domain 1
ht.TRef	298.15[K]	K	Reference temperature	Domain 1

Name	Expression	Unit	Description	Selection
ht.pRef	ht.solid1.pRef	Pa	Reference pressure level	Domain 1
ht.HRef	0	J/kg	Reference enthalpy	Domain 1
ht.DeltaH	integrate(subst(ht.CpInt,ht.pA,ht.pRef),T,ht.TRef,T)	J/kg	Sensible enthalpy	Domain 1
ht.H	ht.HRef+ht.DeltaH	J/kg	Enthalpy	Domain 1
ht.H0	ht.H	J/kg	Total enthalpy	Domain 1
ht.Ei	ht.H	J/kg	Internal energy	Domain 1
ht.Ei0	ht.Ei	J/kg	Total internal energy	Domain 1
ht.Qbtot	0	W/m^2	Total boundary heat source	Boundaries 1–9
ht.k_effxx	ht.kxx	W/(m*K)	Effective thermal conductivity, xx component	Domain 1
ht.k_effyx	ht.kyx	W/(m*K)	Effective thermal conductivity, yx component	Domain 1
ht.k_effzx	ht.kzx	W/(m*K)	Effective thermal conductivity, zx component	Domain 1
ht.k_effxy	ht.kxy	W/(m*K)	Effective thermal conductivity, xy component	Domain 1
ht.k_effyy	ht.kyy	W/(m*K)	Effective thermal conductivity, yy component	Domain 1
ht.k_effzy	ht.kzy	W/(m*K)	Effective thermal conductivity, zy component	Domain 1
ht.k_effxz	ht.kxz	W/(m*K)	Effective thermal conductivity, xz component	Domain 1
ht.k_effyz	ht.kyz	W/(m*K)	Effective thermal conductivity, yz component	Domain 1
ht.k_effzz	ht.kzz	W/(m*K)	Effective thermal conductivity, zz component	Domain 1

Name	Expression	Unit	Description	Selection
ht.C_eff	ht.rho*ht.Cp	J/(m^3*K)	Effective volumetric heat capacity	Domain 1
ht.ux	0	m/s	Velocity field, x component	Domain 1
ht.uy	0	m/s	Velocity field, y component	Domain 1
ht.uz	0	m/s	Velocity field, z component	Domain 1
ht.gradTx	Tx	K/m	Temperature gradient, x component	Domain 1
ht.gradTy	Ty	K/m	Temperature gradient, y component	Domain 1
ht.gradTz	Tz	K/m	Temperature gradient, z component	Domain 1
ht.Qltot	0	W/m	Total line heat source	Edges 1–16
ht.Qptot	0	W	Total point heat source	Points 1–10
ht.alphaTdxx	ht.k_effxx/ht.C_eff	m^2/s	Thermal diffusivity, xx component	Domain 1
ht.alphaTdyx	ht.k_effyx/ht.C_eff	m^2/s	Thermal diffusivity, yx component	Domain 1
ht.alphaTdzx	ht.k_effzx/ht.C_eff	m^2/s	Thermal diffusivity, zx component	Domain 1
ht.alphaTdxy	ht.k_effxy/ht.C_eff	m^2/s	Thermal diffusivity, xy component	Domain 1
ht.alphaTdyy	ht.k_effyy/ht.C_eff	m^2/s	Thermal diffusivity, yy component	Domain 1
ht.alphaTdzy	ht.k_effzy/ht.C_eff	m^2/s	Thermal diffusivity, zy component	Domain 1
ht.alphaTdxz	ht.k_effxz/ht.C_eff	m^2/s	Thermal diffusivity, xz component	Domain 1
ht.alphaTdyz	ht.k_effyz/ht.C_eff	m^2/s	Thermal diffusivity, yz component	Domain 1

Name	Expression	Unit	Description	Selection
ht.alphaTdzz	ht.k_effzz/ht.C_eff	m^2/s	Thermal diffusivity, zz component	Domain 1
ht.alphaTdMean	ht.kmean/ht.C_eff	m^2/s	Mean thermal diffusivity	Domain 1
ht.dfluxx	-ht.k_effxx*Tx-ht.k_effxy*Ty-ht.k_effxz*Tz	W/m^2	Conductive heat flux, x component	Domain 1
ht.dfluxy	-ht.k_effyx*Tx-ht.k_effyy*Ty-ht.k_effyz*Tz	W/m^2	Conductive heat flux, y component	Domain 1
ht.dfluxz	-ht.k_effzx*Tx-ht.k_effzy*Ty-ht.k_effzz*Tz	W/m^2	Conductive heat flux, z component	Domain 1
ht.dfluxMag	sqrt(ht.dfluxx^2+ht.dfluxy^2+ht.dfluxz^2)	W/m^2	Conductive heat flux magnitude	Domain 1
ht.trlfluxx	0	W/m^2	Translational heat flux, x component	Domain 1
ht.trlfluxy	0	W/m^2	Translational heat flux, y component	Domain 1
ht.trlfluxz	0	W/m^2	Translational heat flux, z component	Domain 1
ht.trlfluxMag	sqrt(ht.trlfluxx^2+ht.trlfluxy^2+ht.trlfluxz^2)	W/m^2	Translational heat flux magnitude	Domain 1
ht.cfluxx	0	W/m^2	Convective heat flux, x component	Domain 1
ht.cfluxy	0	W/m^2	Convective heat flux, y component	Domain 1
ht.cfluxz	0	W/m^2	Convective heat flux, z component	Domain 1
ht.cfluxMag	sqrt(ht.cfluxx^2+ht.cfluxy^2+ht.cfluxz^2)	W/m^2	Convective heat flux magnitude	Domain 1
ht.tfluxx	ht.dfluxx+ht.trlfluxx+ht.cfluxx	W/m^2	Total heat flux, x component	Domain 1
ht.tfluxy	ht.dfluxy+ht.trlfluxy+ht.cfluxy	W/m^2	Total heat flux, y component	Domain 1
ht.tfluxz	ht.dfluxz+ht.trlfluxz+ht.cfluxz	W/m^2	Total heat flux, z component	Domain 1

Name	Expression	Unit	Description	Selection
ht.tfluxMag	sqrt(ht.tfluxx^2+ht.tfluxy^2+ht.tfluxz^2)	W/m^2	Total heat flux magnitude	Domain 1
ht.tefluxx	ht.dfluxx	W/m^2	Total energy flux, x component	Domain 1
ht.tefluxy	ht.dfluxy	W/m^2	Total energy flux, y component	Domain 1
ht.tefluxz	ht.dfluxz	W/m^2	Total energy flux, z component	Domain 1
ht.tefluxMag	sqrt(ht.tefluxx^2+ht.tefluxy^2+ht.tefluxz^2)	W/m^2	Total energy flux magnitude	Domain 1
ht.rflux	0	W/m^2	Radiative heat flux	Boundaries 1–9
ht.chflux	0	W/m^2	Boundary convective heat flux	Boundaries 1–9
ht.ntrlflux	mean(ht.trlfluxx)*ht.nx+mean(ht.trlfluxy)*ht.ny+mean(ht.trlfluxz)*ht.nz	W/m^2	Normal translational heat flux	Boundaries 1–9
ht.ncflux	mean(ht.cfluxx)*ht.nx+mean(ht.cfluxy)*ht.ny+mean(ht.cfluxz)*ht.nz	W/m^2	Normal convective heat flux	Boundaries 1–9
ht.ndflux	-dflux_spatial(T)/ht.d	W/m^2	Normal conductive heat flux	Boundaries 1–9
ht.ntflux	ht.ndflux+ht.ntrlflux+ht.ncflux	W/m^2	Normal total heat flux	Boundaries 1–9
ht.nteflux	mean(ht.tefluxx)*ht.nx+mean(ht.tefluxy)*ht.ny+mean(ht.tefluxz)*ht.nz-mean(ht.dfluxx)*ht.nx-mean(ht.dfluxy)*ht.ny-mean(ht.dfluxz)*ht.nz+ht.ndflux	W/m^2	Normal total energy flux	Boundaries 1–9
ht.solid1.dEiInt	ht.solid1.intDom(d(ht.rho*ht.Ei,t)*ht.solid1.varIntSpa)	W	Total accumulated heat rate	Global
ht.solid1.dEi0Int	ht.solid1.intDom(d(ht.rho*ht.Ei0,t)*ht.solid1.varIntSpa)	W	Total accumulated energy rate	Global
ht.solid1.ntfluxInt	ht.solid1.intExtBnd(ht.ntflux*ht.solid1.varIntSp	W	Total net heat rate	Global

Name	Expression	Unit	Description	Selection
	a)+ht.solid1.intExtBndUp(ht.ntflux_u*ht.solid1.varIntSpa)+ht.solid1.intExtBndDown(ht.ntflux_d*ht.solid1.varIntSpa)			
ht.solid1.ntefluxInt	ht.solid1.intExtBnd(ht.nteflux*ht.solid1.varIntSpa)+ht.solid1.intExtBndUp(ht.nteflux_u*ht.solid1.varIntSpa)+ht.solid1.intExtBndDown(ht.nteflux_d*ht.solid1.varIntSpa)	W	Total net energy rate	Global
ht.solid1.QInt	ht.solid1.intDom(ht.Qtot*ht.solid1.varIntSpa)-ht.solid1.intIntBnd((ht.ndflux_u+ht.ndflux_d)*ht.solid1.varIntSpa)	W	Total heat source	Global
ht.solid1.WnsInt	ht.solid1.intDom(ht.pA*(d(ht.ux,x)+d(ht.uy,y)+d(ht.uz,z))*ht.solid1.varIntSpa)	W	Total work source	Global
ht.solid1.WInt	0	W	Total work source	Global
ht.gamma	1	1	Ratio of specific heats	Domain 1
ht.helem	h	m	Element size	Domain 1
ht.res_T	-ht.k_effxx*Txx-ht.k_effxy*Txy-ht.k_effxz*Txz-ht.k_effyx*Tyx-ht.k_effyy*Tyy-ht.k_effyz*Tyz-ht.k_effzx*Tzx-ht.k_effzy*Tzy-ht.k_effzz*Tzz-(ht.qs+ht.qs_oop)*T+ht.rho*ht.Cp*(ht.ux*Tx+ht.uy*Ty+ht.uz*Tz)-ht.Q-ht.Qoop	W/m^3	Equation residual	Domain 1

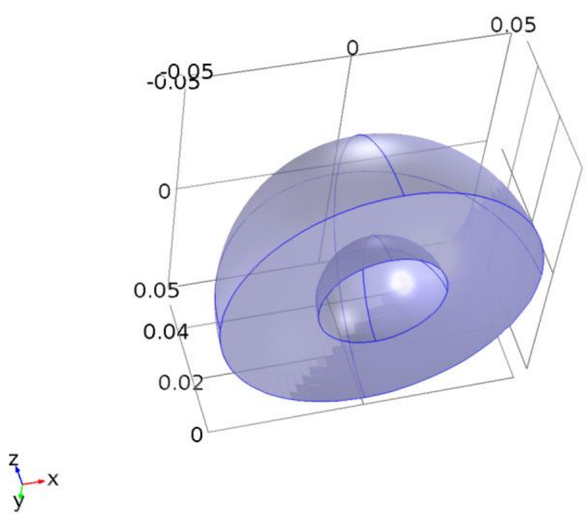
Shape functions

Name	Shape function	Unit	Description	Shape frame	Selection
T	Lagrange (Quadratic)	K	Temperature	Material	Domain 1

Weak expressions

Weak expression	Integration frame	Selection
(-(ht.k_effxx*Tx+ht.k_effxy*Ty+ht.k_effxz*Tz)*test(Tx)-(ht.k_effyx*Tx+ht.k_effyy*Ty+ht.k_effyz*Tz)*test(Ty)-(ht.k_effzx*Tx+ht.k_effzy*Ty+ht.k_effzz*Tz)*test(Tz))*ht.d	Material	Domain 1
-ht.rho*ht.Cp*(ht.ux*Tx+ht.uy*Ty+ht.uz*Tz)*test(T)*ht.d	Material	Domain 1
ht.streamline	Material	Domain 1

2.4.2 Initial Values 1



Initial Values 1

Selection

Geometric entity level	Domain
Selection	Domain 1

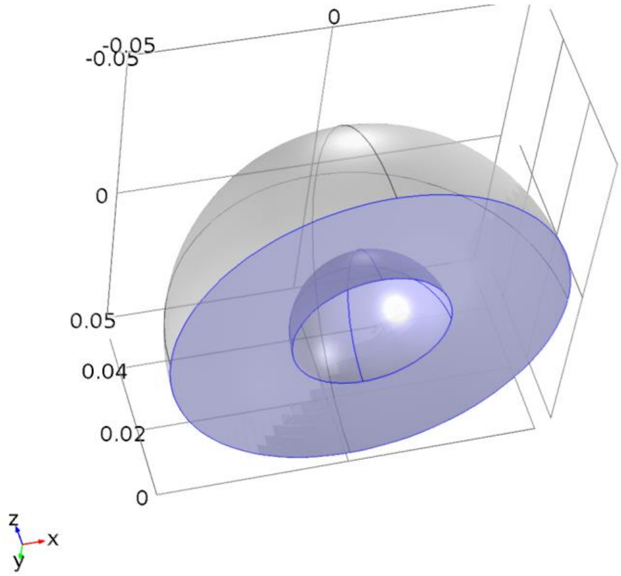
Settings

Description	Value
Temperature	293.15[K]

Variables

Name	Expression	Unit	Description	Selection
ht.Tinit	293.15[K]	K	Temperature	Domain 1

2.4.3 Thermal Insulation 1



Thermal Insulation 1

Selection

Geometric entity level	Boundary
Selection	Boundaries 2, 4–5, 7–8

Equations

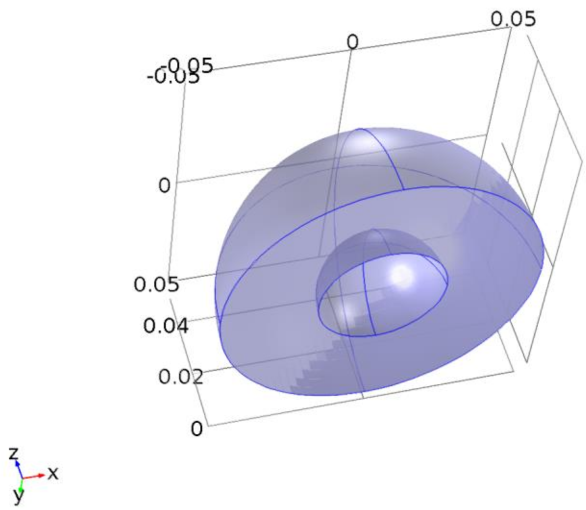
$$-\mathbf{n} \cdot \mathbf{q} = 0$$

Variables

Name	Expression	Unit	Description	Selection
ht.ins1.ntfluxInt	ht.ins1.intExtBnd(ht.ntflux*ht.ins1.varIntSpa)	W	Total net heat rate	Global
ht.ins1.ntefluxInt	ht.ins1.intExtBnd(ht.nteflux*ht.ins1.varIntSpa)	W	Total net energy rate	Global
ht.ins1.ntfluxInt_u	ht.ins1.intIntBnd(ht.ntflux_u*ht.ins1.varIntSpa)	W	Total net heat rate, upside	Global
ht.ins1.ntefluxInt_u	ht.ins1.intIntBnd(ht.nteflux_u*ht.ins1.varIntSpa)	W	Total net energy rate, upside	Global

Name	Expression	Unit	Description	Selection
ht.ins1.ntfluxInt_d	ht.ins1.intIntBnd(ht.ntflux_d*ht.ins1.varIntSpa)	W	Total net heat rate, downside	Global
ht.ins1.ntefluxInt_d	ht.ins1.intIntBnd(ht.nteflux_d*ht.ins1.varIntSpa)	W	Total net energy rate, downside	Global
ht.ins1.Tave	if(ht.ins1.intBnd(ht.ins1.varIntSpa*ht.rho*ht.Cp*(ht.ux*ht.nx+ht.uy*ht.ny+ht.uz*ht.nz))==0,ht.ins1.intBnd(ht.ins1.varIntSpa*T)/ht.ins1.intBnd(ht.ins1.varIntSpa),ht.ins1.intBnd(ht.ins1.varIntSpa*ht.rho*ht.Cp*T*(ht.ux*ht.nx+ht.uy*ht.ny+ht.uz*ht.nz))/ht.ins1.intBnd(ht.ins1.varIntSpa*ht.rho*ht.Cp*(ht.ux*ht.nx+ht.uy*ht.ny+ht.uz*ht.nz)))	K	Weighted average temperature	Global

2.4.4 Heat Source 1



Heat Source 1

Selection

Geometric entity level	Domain
Selection	Domain 1

Equations

$$Q = Q_0$$

Settings

Description	Value
Heat source	General source
Heat source	User defined
Heat source	go*b^2/(x^2 + y^2 + z^2)

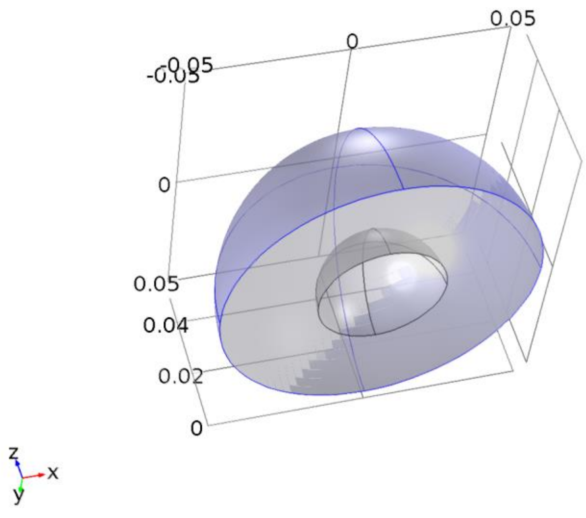
Variables

Name	Expression	Unit	Description	Selection
ht.Q	ht.hs1.Q	W/m^3	Heat source	Domain 1
ht.Qtot	ht.hs1.Q	W/m^3	Total heat source	Domain 1
ht.hs1.Q	go*b^2/(x^2+y^2+z^2)	W/m^3	Heat source	Domain 1

Weak expressions

Weak expression	Integration frame	Selection
ht.hs1.Q*test(T)*ht.d	Material	Domain 1

2.4.5 Heat Flux 1



Heat Flux 1

Selection

Geometric entity level	Boundary
Selection	Boundaries 1, 3, 6, 9

Equations

$$-\mathbf{n}\cdot\mathbf{q}=q_0$$

Settings

Description	Value
Heat flux	Convective heat flux
Heat transfer coefficient	User defined
Heat transfer coefficient	ho
External temperature	Too

Variables

Name	Expression	Unit	Description	Selection
ht.q0	ht.hf1.q0	W/m^2	Inward heat flux	Boundaries 1, 3, 6, 9
ht.hf1.h	ho	W/(m^2*K)	Heat transfer coefficient	Boundaries 1, 3, 6, 9
ht.hf1.Text	Too	K	External temperature	Boundaries 1, 3, 6, 9
ht.hf1.q0	ht.hf1.h*(ht.hf1.Text-ht.hf1.Tvar)	W/m^2	Boundary convective heat flux	Boundaries 1, 3, 6, 9
ht.hf1.Tvar	ht.Tu	K	Temperature	Boundaries 1, 3, 6, 9
ht.hf1.ntfluxInt	ht.hf1.intExtBnd(ht.ntflux*ht.hf1.varIntSpa)	W	Total net heat rate	Global
ht.hf1.ntefluxInt	ht.hf1.intExtBnd(ht.nteflux*ht.hf1.varIntSpa)	W	Total net energy rate	Global
ht.hf1.ntfluxInt_u	ht.hf1.intIntBnd(ht.ntflux_u*ht.hf1.varIntSpa)	W	Total net heat rate, upside	Global
ht.hf1.ntefluxInt_u	ht.hf1.intIntBnd(ht.nteflux_u*ht.hf1.varIntSpa)	W	Total net energy rate, upside	Global
ht.hf1.ntfluxInt_d	ht.hf1.intIntBnd(ht.ntflux_d*ht.hf1.varIntSpa)	W	Total net heat rate, downside	Global
ht.hf1.ntefluxInt_d	ht.hf1.intIntBnd(ht.nteflux_d*ht.hf1.varIntSpa)	W	Total net energy rate, downside	Global
ht.hf1.Tave	if(ht.hf1.intBnd(ht.hf1.varIntSpa*ht.rho*ht.Cp*(ht.ux*ht.nx+ht.uy*ht.ny+ht.uz*ht.nz))==0,ht.hf1.intBnd(ht.hf1.varIntSpa*T)/ht.hf1.intBnd(ht.hf1.va	K	Weighted average temperature	Global

Name	Expression	Unit	Description	Selection
	rIntSpa),ht.hf1.intBnd(ht.hf1.varIntSpa*ht.rho*ht.Cp*T*(ht.ux*ht.nx+ht.uy*ht.ny+ht.uz*ht.nz))/ht.hf1.intBnd(ht.hf1.varIntSpa*ht.rho*ht.Cp*(ht.ux*ht.nx+ht.uy*ht.ny+ht.uz*ht.nz)))			

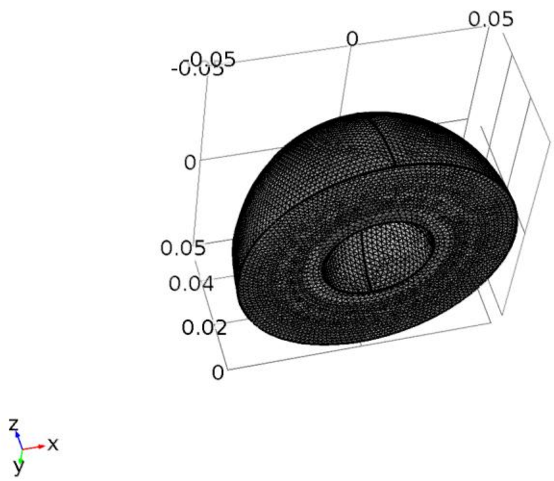
Weak expressions

Weak expression	Integration frame	Selection
ht.hf1.q0*test(ht.hf1.Tvar)*ht.d	Material	Boundaries 1, 3, 6, 9

2.5 Mesh 1

Mesh statistics

Description	Value
Minimum element quality	0.1966
Average element quality	0.7744
Tetrahedral elements	525731
Triangular elements	16270
Edge elements	448
Vertex elements	10



Mesh 1

2.5.1 Size (size)

Settings

Description	Value
Maximum element size	0.002
Minimum element size	2.0E-5
Curvature factor	0.2
Maximum element growth rate	1.3
Predefined size	Extremely fine

2.5.2 Free Tetrahedral 1 (ftet1)

Selection

Geometric entity level	Remaining

3 Study 1

Computation information

Computation time	43 s
CPU	Intel(R) Xeon(R) CPU E5-2680 v4 @ 2.40GHz, 2 cores
Operating system	Windows 8

3.1 Stationary

Study settings

Description	Value
Include geometric nonlinearity	Off

Physics and variables selection

Physics interface	Discretization
Heat Transfer in Solids (ht)	physics

Mesh selection

Geometry	Mesh
Geometry 1 (geom1)	mesh1

3.2 Solver Configurations

3.2.1 Solution 1

Compile Equations: Stationary (st1)

Study and step

Description	Value
Use study	Study 1
Use study step	Stationary

Dependent Variables 1 (v1)

General

Description	Value
Defined by study step	Stationary

Initial values of variables solved for

Description	Value
Solution	Zero

Values of variables not solved for

Description	Value
Solution	Zero

Temperature (comp1.T) (comp1_T)

General

Description	Value
Field components	comp1.T

Stationary Solver 1 (s1)

General

Description	Value
Defined by study step	Stationary

Log

```
Stationary Solver 1 in Solution 1 started at 10-Oct-2019 14:32:05.
Linear solver
Number of degrees of freedom solved for: 718081 (plus 33444 internal DOFs).
Symmetric matrices found.
Format not changed since SOR line uses nonsymmetric storage.
Scales for dependent variables:
Temperature (comp1.T): 2.9e+002
Orthonormal null-space function used.
Iter      SolEst      Damping    Stepsize #Res #Jac #Sol LinIt   LinErr   LinRes
   1        0.46    1.0000000        0.46    1    1    1     1  0.00017    0.005
Stationary Solver 1 in Solution 1: Solution time: 34 s
                                   Physical memory: 2.61 GB
                                   Virtual memory: 2.85 GB
```

Fully Coupled 1 (fc1)

General

Description	Value
Linear solver	Iterative 1

Method and termination

Description	Value
Initial damping factor	0.01
Minimum damping factor	1.0E-6
Maximum number of iterations	50

Iterative 1 (i1)

General

Description	Value
Factor in error estimate	20

Multigrid 1 (mg1)

General

Description	Value
Use hierarchy in geometries	Geometry 1

Presmoother (pr)

SOR Line 1 (sl1)

Main

Description	Value
Relaxation factor	0.2

Secondary

Description	Value
Number of secondary iterations	2
Relaxation factor	0.5

Postsmoother (po)

SOR Line 1 (sl1)

Main

Description	Value
Relaxation factor	0.2

Secondary

Description	Value
Number of secondary iterations	2
Relaxation factor	0.5

Coarse Solver (cs)

Direct 1 (d1)

General

Description	Value
Solver	PARDISO

Information 1 (prob1)

Warnings 1 (warning1)

Log

```
There was a warning message from the linear solver.
Ill-conditioned preconditioner. Increase factor in error estimate.
```

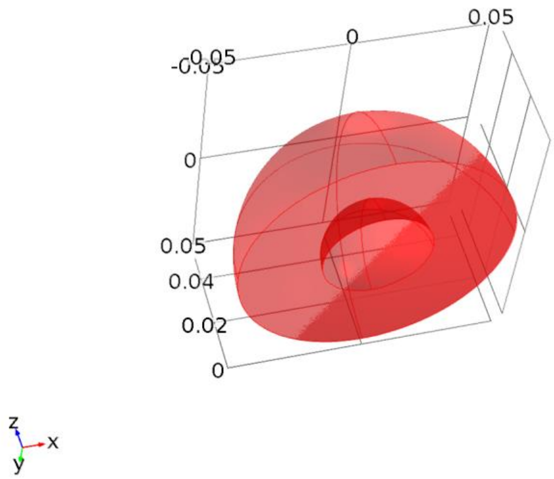
4 Results

4.1 Data Sets

4.1.1 Study 1/Solution 1

Solution

Description	Value
Solution	Solution 1
Component	Save Point Geometry 1
Frame	Mesh (x, y, z)



Data set: Study 1/Solution 1

4.1.2 Cut Line 3D 1

Data

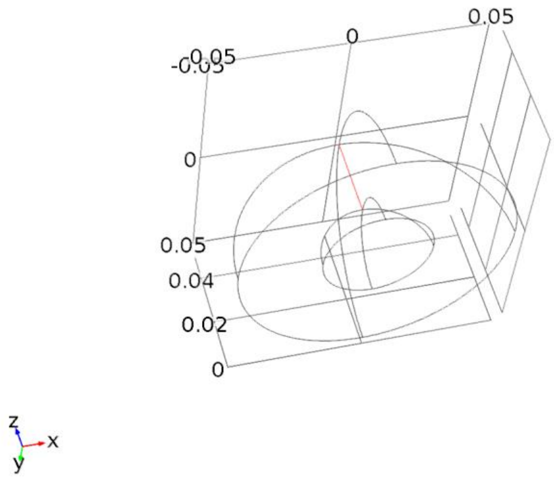
Description	Value
Data set	Study 1/Solution 1

Line data

Description	Value
Line entry method	Two points
Points	{{0, 0, 0}, {0, 0, 1}}

Advanced

Description	Value
Space variable	cln1x



Data set: Cut Line 3D 1

4.2 Derived Values

4.2.1 Surface Integration 1

Selection

Geometric entity level	Boundary
Selection	Boundaries 1, 3, 6, 9

Data

Description	Value
Data set	Study 1/Solution 1

Expression

Description	Value
Expression	ht.ndflux
Unit	W
Description	Normal conductive heat flux

4.3 Tables

4.3.1 Table 1

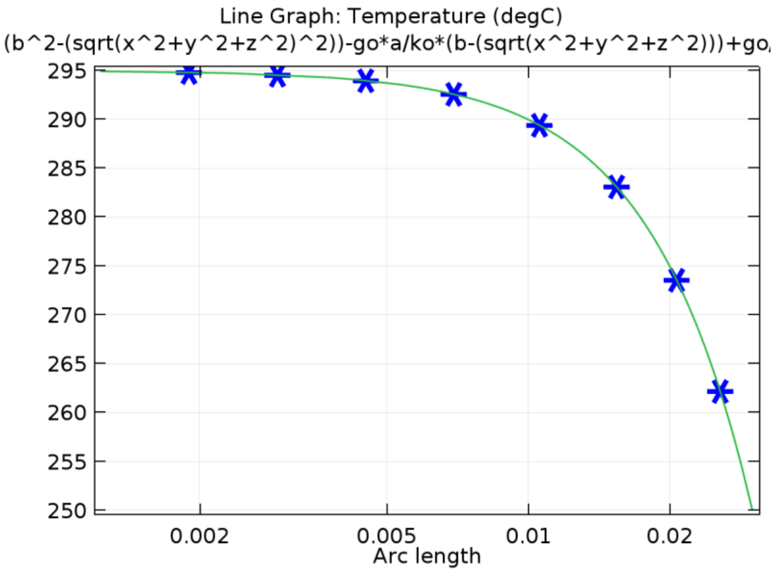
Surface Integration 1 (ht.ndflux)

Table 1

Normal conductive heat flux (W)
471.61

4.4 Plot Groups

4.4.1 1D Plot Group 1



Line Graph: Temperature (degC) Line Graph: go/2/ko(b^2-(sqrt(x^2+y^2+z^2)^2))-go*a/ko*(b-(sqrt(x^2+y^2+z^2)))+go/ho*(b-a)+Too (degC)*