Curtis Domingues Exam 2 March 22 2022

Problem 1:

> Purpose: a-g

a) Determine depth of given channel if $T = 2.309y$ and slope of channel is 0.1 percent. (use table 14.3)

b) Determine reaction forces acting upon pipe-elbow to obtain proper pipe support.

c) Determine maximum log size in channel. Log is known to have a square-cross section and is barley floating.
Next, determine if the largest possible log will be stable or not.

d) Determine the pressure drop across a nozzle used to measure flow, knowing the ratio of nozzle-pipe diameter is 0.5.

e) Determine the pressure increment produced if the gate valve is closed suddenly.
Next, determine if any change in cavitation occurs in the system.

f) Determine the largest drag force experienced by a log half the size of the "max log" found in part C.

g) Determine the force acting on the blind flange in the tank. given it's diameter is equal to pipe diameter.

Diagrams & Drawings:

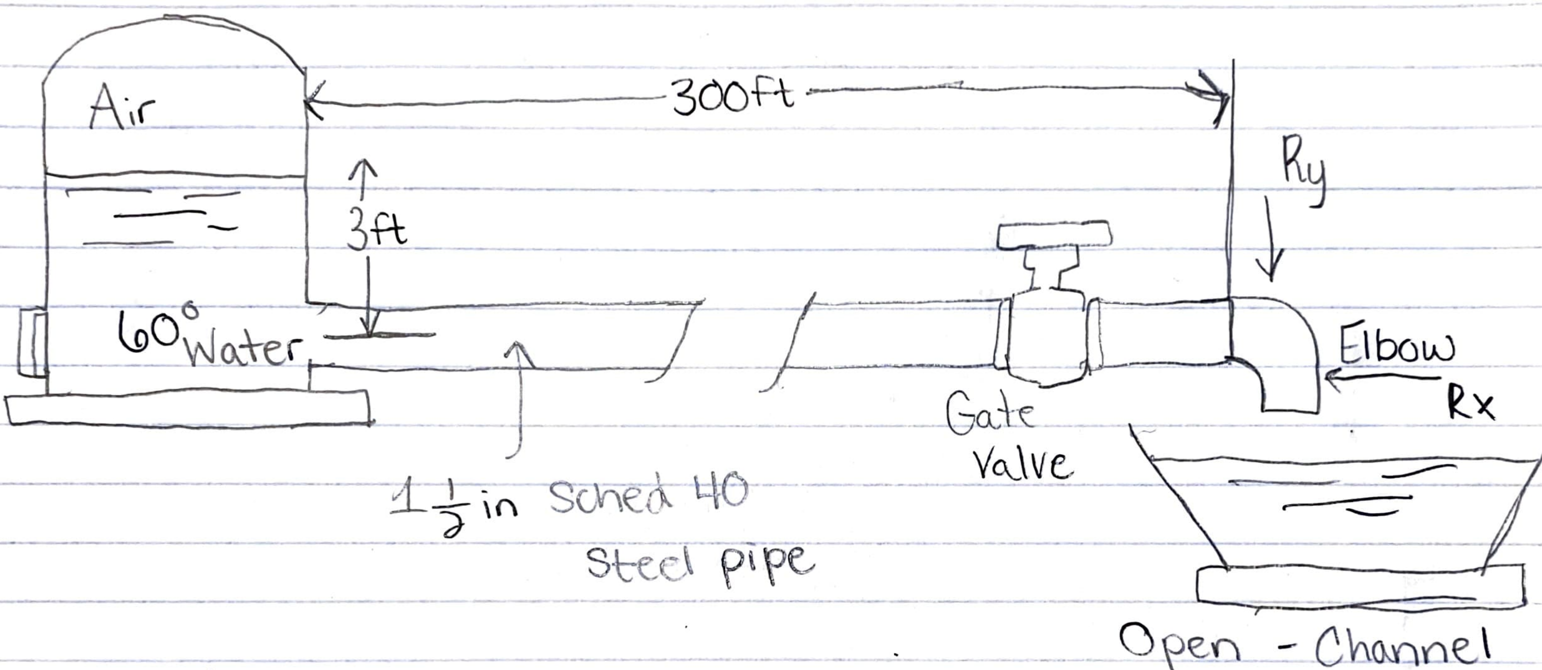
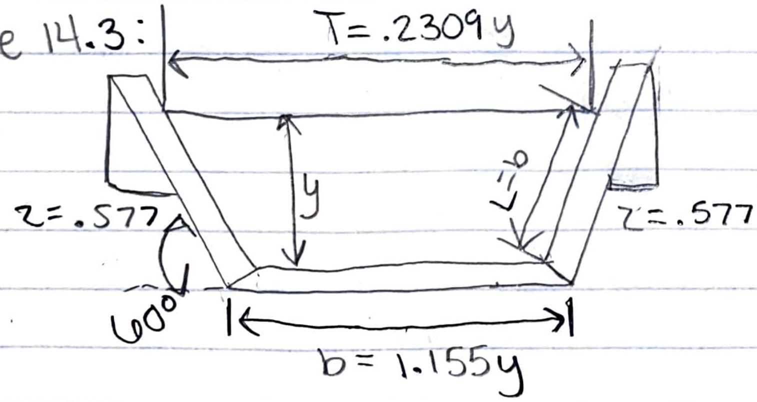


Table 14.3:



> **Sources:** "Mott, R., Utener, J.A., "Applied Fluid Mechanics," 7th Edition.
Pearson Education, INS (2015)

> **Design Considerations:** a-g

- Constant properties
- pressurized tank
- Incompressible fluid
- Constant Temperature

> **Data & Variables:** a-g

> For Trapezoidal open channel: (Table 14.3)
$A = 1.73y^2$ $W_p = 3.46y$ $R = y/2$ $R = \frac{A}{W_p}$

> Given Data:
Water @ 60° $Q = 75$ gpm
Pipe $L = 300$ ft Pipe Size = 1 1/2 in.
$\rho_{\text{Hickory wood}} = 830\ kg/m^3$

Nozzle ratio = .5

"Manning's n" = .017 (Table 14.1)

> Sched 40 steel pipe (pg.50
Flow area = .01414 ft^2
ID = .1342 ft
E = 200 GPa
δ = .0120 ft

> Textbook variables:
$f = .02$ (Table 10.5 pg 242)
$K_{elbow} = 20 f_t$ (Fig 10.24)
$K_{valve} = 8 f_t$ (Table 10.4)

> Water
$\gamma_{water} = 62.4\ lb/ft^3$
E_o water = 316,000 psi
$V_k = 1.21 \times 10^{-5}$

> Conversions:

$1\ \frac{slug}{ft^3} = 1\ \frac{lb \cdot s^2}{ft^4}$

1 psi = 144 PSF

$1\ lb \cdot f/s^2 = .138N$

1 GPa = 145038 Psi

Procedure: a-g

a) To find the depth of the channel I will use the known Flow rate, Slope, and "manning's n" to solve for the term "$AR^{2/3}$".
Next, I will use the trapezoidal channel from table 14.2 to get all unknown variables in terms of "y". Then I will rearange the flow rate equation so that Q is isolated. Lastly, I will assume a depth "y" and compare the results of Q_{calc} and Q_{actual}. I will repeat this process through iteration until Q_{calc} equals the known value of Q. This will determine the true depth of the channel.

b) For part b I will use Bernoulli's equation to determine the pressure at the entrance of the tank. NOTE: since the elbow is open to the atmosphere, pressure, and velocity. at the elbow will cancel. Next, I will use the sum of forces equations to determine the forces exerted on the pipe-elbow. Thus, I will solve for R_x and R_y

c) To find the largest log the channel can carry, I will use the channel depth found in part a, as well as the given density & cross-section of hickory wood. Furthermore, since the log is known to be "barley floating", I will assume Weight = F_b. This will allow me to assume the volume of the log is nearly equal to it's volume displaced. Effectivly, I will set the volume of the log equal to the volume of water displaced, allowing me to create a ratio between V and V_d. Lastly, I will manipulate volumetric dimensions to solve for the "height" of the log.

c) ... Continued

To determine the stability of the found log, I will use the found channel depth from part a. Since stability is dependent on I & V_d, I will solve for both. I will also use the known channel and log dimensions to solve for yCG and yCB. NOTE: Since Vd requires length to solve, I will assume a length of the log. NOTE: This assumption will not effect result as it will cancell out mathematically. Lastly, I will determine MB and compare it to yCG to verify the stability of the log.

d) To determine the pressure drop across the nozzle I will manipulate the "Nozzle Equation" to solve for Δp. Next, I will determine unknown variable C by solving for N_R (β is given as .5). Once C is known, I will evaluate the equation and solve for the Δp.

e) To determine wave velocity (c) I will use the associated equation. First, I will convert all known variable to common units. NOTE: In order to achieve units of f/s I will convert slugs/ft^3 to $lb \cdot s^2/ft^4$. Once all units agree I will evaluate and solve for c. Once c is known I will multiply it by density and velocity to determine Δp.

f) For part f I will assume a log half the size of the max log found in part C. To determine the largest drag force said log will experience, I will use the drag Force equation, inputing known variables. Note: C_D is determined from (Table 17.1). Lastly, I will evaluate and solve for F_D

g) To determine the force acting on the flange, I will use the h_L determined in part b. Using Bernoulli's Equation I will determine the pressure at the fluid surface in the tank (starting from the elbow since the tank is pressurized). Once, said pressure is known, I will use the "gamma h" equation to solve for the pressure 3 feet below the surface. Since the Flange is circular, I will assume the force location to be distributed about it's center.

Calculations Part A:

$Q = \left(\frac{1.49}{n}\right) AR^{2/3} S^{1/2}$ $\quad Q = 75\,GPM \cdot \frac{1\,Ft^3/s}{449\,gpm} = .167\,Ft^3/s$

• Rearrange to solve for $AR^{2/3}$

$AR^{2/3} = \frac{nQ}{1.49 S^{1/2}} = \frac{.017(.167\,Ft^3/s)}{1.49\sqrt{.001}} = \frac{.0028\,ft^3/s}{.04711} = .06025\,ft^3/s$

Since Q is known, and all variable are in terms of y, assume y value and re-iterate until Q calc = Q actual

$\therefore R = \frac{A}{W_p}$ $\quad \boxed{A = 1.73y^2 \quad W_p = 3.46y \quad R = \frac{y}{2}}$

$.167\,ft^3/s = \left(\frac{1.49}{.017}\right) AR^{2/3} S^{1/2}$ For $y = .5\,ft$:

$Q = 87.647 \cdot .4325\,ft^2 \cdot (.39685\,ft) \cdot (.0316)$
$Q = .475\,ft^3$

Reiterate: For $y = .25\,ft$
$Q = 87.647 \cdot .108125\,ft^2 \cdot (.25\,ft) \cdot (.0316)$
$Q = .075\,ft^3$

Reiterate to find $Q_{calc} = Q_{act}$

(See excell Sheet)

After iteration: For $y = .388\,ft$:
$Q = 87.647 \cdot .197642\,ft^2 \cdot (.3056\,ft)(.0316)$
$Q_{calc} = .167\,Ft^3 = Q_{actual}$ ✓

$\boxed{\text{Thus, the depth of the channel (y) is .388 ft or 4.06 in.}}$

ANS

Calculations Part B:

Find horizontal & vertical forces pipe-elbow:

$F = \frac{P}{A}$

Bernoulies Equation:

$$\frac{P_1}{\gamma} + \cancel{z_1} + \cancel{\frac{V_1^2}{2g}} = \cancel{\frac{P_2}{\gamma}} + \cancel{z_2} + \cancel{\frac{V_2^2}{\gamma}} + h_L$$

$$\frac{P_1}{\gamma} = h_L \therefore P_1 = h_L \cdot \gamma$$

$f = .02$ (Table 10.5 pg. 242)

For straight pipe (300ft)

$$h_L = f \times \frac{L}{D} \times \frac{V^2}{2g} = .02 \cdot \frac{300ft}{.125ft} \cdot \frac{11.81 ft/s^2}{64.4 ft/s^2} = \underline{103.95 ft}$$

gate valve:

$$h_L = f \times 8 \times \frac{V^2}{2g} = (.02)(8)\left(\frac{11.81 ft/s^2}{64.4 ft/s^2}\right) = \underline{.3464 ft}$$

elbow:

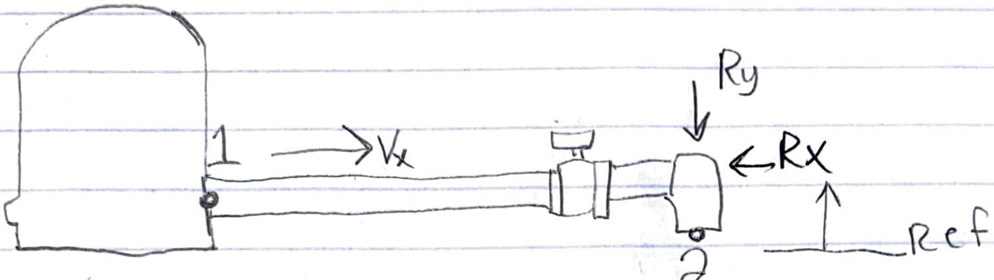
$$h_L = f \times 20 \times \frac{V^2}{2g} = (.02)(20)\left(\frac{11.81 ft/s^2}{64.4 ft/s^2}\right) = \underline{.866 ft}$$

$$F_x = \rho Q \Delta V_x = \rho Q(-V)$$

$$F_y = \rho Q \Delta V_y = \rho Q(-V)$$

> Bernoulies Equation

Calculations Part B continued:



$$\frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} = \cancel{\frac{P_2}{\gamma}} + \cancel{\frac{V_2^2}{2g}} + h_L$$ Velocities are equal

$> \quad P_1 = \gamma h_L \qquad 62.4\,\frac{lb}{ft^3}\,(105.1624\,ft) = 6562.13\,lb/ft^3$

Sum of forces $= \Sigma F_x = \rho Q \Delta V_x + P_1 + R_x = 0 \qquad \Delta V = V_2 - V_1 : V_2 = 0$

$\Sigma F_y = \rho Q \Delta V_x + R_y = 0$

Force in x-Direction:

$\Sigma F_x = 1.94\,\frac{slug}{ft^3}\left(.167\,\frac{ft^3}{s}\right)(-11.81\,ft/s) + (6562.13\,lb/ft^3)(.01414\,ft^2) + Rx$

$\Sigma F_x: \; 1.94\,\frac{slug}{ft^3} = 1.94\,\frac{lb \cdot s^2}{ft^3}\left(-1.97\,\frac{ft^4}{s^2}\right) + (6562.13\,lb/ft^3)(.01414\,ft^2) + Rx$

$\Sigma F_x: \; -3.826\,lb + 92.78\,lb + Rx = 0 \qquad \boxed{Rx = 88.954\,lb \leftarrow}$

ANS

Force in y-Direction:

$\Sigma F_y: \; 1.94\,\frac{slug}{ft^3}\left(.167\,\frac{ft^3}{s}\right)\left(-11.81\,\frac{ft}{s}\right) + Ry = 0$

$1.94\,\frac{lb \cdot s^2}{ft^4} \cdot \left(-1.9706\,\frac{ft^4}{s^2}\right) = -Ry \qquad \boxed{Ry = 3.822\,lb \downarrow}$

ANS

Part C calculations

Channel depth found to be .338 ft = .103 m

Density of hickorywood = $830 \frac{kg}{m^3}$

$\gamma_{Hickory} = 830 \frac{kg}{m^3} \cdot 9.81 \frac{m}{s^2} = 8142.3 \frac{kg \cdot m}{m^3 s^2}$

$\gamma_{Hickory} = 8142.3\ N/m^3$ or $8.14\ kN/m^3$

$\gamma_{water} = 9.81\ kN/m^3$

Since Log is "barley floating" Weight = F_b

> Determine depth of immersion:

Weight = $\gamma_{Hickory} V$, $F_b = \gamma_{water} V_d$ $\therefore$ $\gamma_{Hickory} V = \gamma_{water} V_d$

Volume ≈ Volume displaced $\frac{\gamma_{Hickory}}{\gamma_{water}} = \frac{V_d}{V}$ $\frac{8.14\ kN/m^3}{9.81\ kN/m^3} = .829$

$\frac{V_d}{V} = .829$

volume $= (L \times W \times h) = (L \times W^2)$
$V_d = (L \times W \times y) = (LWy)$

$\frac{\cancel{L}\cancel{W}y}{\cancel{L}W^2}$ $\frac{y}{W} = .829$

$\frac{.103m}{W} = .829$

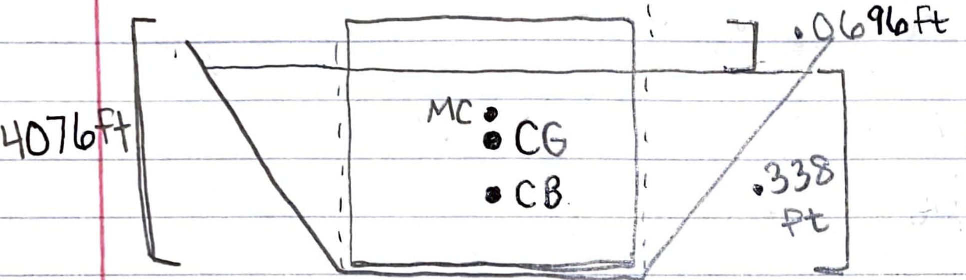
$\frac{.103m}{.829} = W = .124m$

$.124m \cdot \frac{1ft}{.3048m} = .4076ft$

$$\boxed{\text{Therefore, maximum Log size is } .124m \text{ or } .4076 ft}$$ ANS

> Determine Stability

Part C Calculations Continued...



Channel depth = .338 ft
Max log size = .4076 ft

$y_{CG} = .2038\text{ ft}$

$y_{CB} = .169\text{ ft}$

$y_{mc} = y_{CB} + MB$

$MB = \frac{I}{V_d}$ $\quad V_d = L \times B \times .338$

☆ Assume Log length of 2 ft ☆

$V_d = 2\text{ft} \times .4076\text{ft} \times .338\text{ ft}$

$V_d = .2755\text{ ft}^3$

$I = \frac{LB^3}{12} = \frac{2\text{ ft} \times (.4076\text{ft})^3}{12}$

$I = .0112\text{ Ft}^4$

$MB = \frac{.0112\text{ ft}^4}{.2755\text{ ft}^3}$

$MB = .041\text{ ft}$

$y_{mc} = .169\text{ft} + .041\text{ft}$

$y_{mc} = .21\text{ft}$

$.21\text{ft} > .2038\text{ ft}$

Thus, Mc is slightly higher than Center of gravity.
∴ The log is stable

ANS

Calculations part D:

Find pressure drop across Nozzle

Nozzle to pipe ratio = .5

Thus Nozzle is .75 inch or .0625 ft Diameter

known: $Q = .167\ ft^3/s$ $\quad \mathcal{V}_k = 1.21 \times 10^{-5}$ $\quad A_1 = .01414\ ft^2$

$D = 1.5\ in = .125\ ft$ $\quad 2g = 64.4\ ft/s^2$ $\quad A_2 = .003\ ft^2$

$D/2 = .75\ in = .0625\ ft$ $\quad V_1 = 11.81\ ft/s$

$d = .75\ in = .0625\ ft$ $\quad \gamma_{water} = 62.4\ lb/ft^3$

Nozzle Equation:

$$V_1 = C\sqrt{\frac{2g(\Delta P)/\gamma}{(A_1/A_2)^2 - 1}}$$

- Manipulate to solve for Δp

$$\Delta p = \frac{\gamma\left((A_1/A_2)^2 - 1\right)\left(\frac{V_1}{C}\right)^2}{2g}$$

- solve for unknowns: C, N_R

$$C = .9975 - 6.53\sqrt{\beta/N_R} \quad \text{when } \beta = d/D = .5$$

$$N_R = \frac{VD}{\mathcal{V}} = \frac{(11.81\ ft/s)(.125\ ft)}{1.21 \times 10^{-5}\ ft^2/s} = 122004.1322$$

$$C = .9975 - 6.53\sqrt{.5/122004.1322} = .984$$

$$\Delta p = \frac{62.4\ lb/ft^3\left((.01414\ ft^2/.003\ ft^2)^2 - 1\right)(12.00\ ft/s)^2}{64.4\ ft/s^2}$$

$$= \frac{62.4\ lb/ft^3\,(22.215 - 1)(144\ ft^2/s^2)}{64.4\ ft/s^2}$$

part D Calculations Continued:

$$\Delta p = \frac{62.4\ lb/ft^3\ (21.215)(144\ ft^2/s^2)}{64.4\ ft/s^2}$$

$$\Delta p = \frac{1323.82\ lb/ft^3 \cdot 144\ ft^2/s^2}{64.4\ ft/s^2}$$

$$\frac{1323.82\ lb}{\cancel{ft^3}} \cdot \frac{144\ \cancel{ft^2}}{\cancel{s^2}} \cdot \frac{1\ \cancel{s^2}}{64.4\ ft}$$

$$= 2960\ lb/ft^2 \cdot \frac{1\ lb/in^2}{144\ lb/ft^2} = \boxed{20.5\ psi}$$

ANS

Part E Calculations:

Find pressure increment

$F = \rho C A V \quad \therefore \quad \Delta p = \rho C V \qquad P_{max} = P_{op} + \Delta p$

$$C = \frac{\sqrt{E_0/\rho}}{\sqrt{1 + \frac{E_0 D}{E \delta}}}$$

E_0 = Bulk Modulus$_{Fluid}$ = 316,000 psi

ρ = Fluid density = 981 slugs/ft^3

D = pipe diameter = .1342 ft

E = elastic Modulus of pipe = 200 GPa

δ = pipe thickness = .0120 ft

Convert: Psi = 144 Psf

$E_0 = 45504000$ psf $\quad \rho = 981 \frac{lb \cdot s^2}{ft^4} \quad E = (200 GPa)\frac{145038 Psi}{1 GPa} = 29007600 psi$

E = 4177094400 PSF

> solve for wave velocity

$$C = \frac{\sqrt{\frac{45504000 \cancel{lb}/ft^2}{981 \cancel{lb} \cdot s^2/ft^4}}}{\sqrt{1 + \frac{(45504000 lb/ft^2)(.1342 ft)}{(4177094400 lb/ft^2)(.0120 ft)}}} = \frac{\sqrt{46385.32 \frac{ft^2}{s^2}}}{\sqrt{1 + 1.218}}$$

Note: units in denominator cancel (as they must)

$$C = \frac{215.37 ft/s}{2.28} = 94.46 ft/s$$

$\Delta p = \rho C V = \left(981 \frac{lb \cdot s^2}{ft^4}\right)\left(94.46 \frac{ft}{s}\right)\left(11.81 \frac{ft}{s}\right)$

$\Delta p = 1094376.721 lb/ft^2 = \boxed{7599.83 Psi}$ ANS

Part F Calculation:

Max log size (part C) = .4076 ft
New log = .2038 ft

$F_D = C_D\left(\frac{\rho v^2}{2}\right)A$

CD = Drag Coeff.
ρ = fluid density
V = Channel Velocity
A = projected Area

$\frac{Q}{A} = \frac{.167\ ft^3/s}{.198\ ft^2} = .843\ ft/s$

$A_p = .2038^2 = .0415\ ft^2$

$F_D = 1.16\frac{\left(1.94\ slugs/ft^3\right)\left(.843\ ft/s\right)^2}{2} \cdot .0415\ ft^2$

$F_D = \frac{\left(62.42\ lb/ft^3\right)\left(.843\ ft/s\right)^2}{2}(1.16)(.0415\ ft^2)$

$= \frac{\left(62.42\ lb/ft^3\right)\left(.7106\ ft^2/s^2\right)}{2}(1.16)(.0415\ ft^2)$

$= \frac{44.36\ lb/ft \cdot s^2}{2} = 22.18\ lb/ft \cdot s^2 (1.16) = \frac{25.73\ lb}{ft \cdot s^2}$

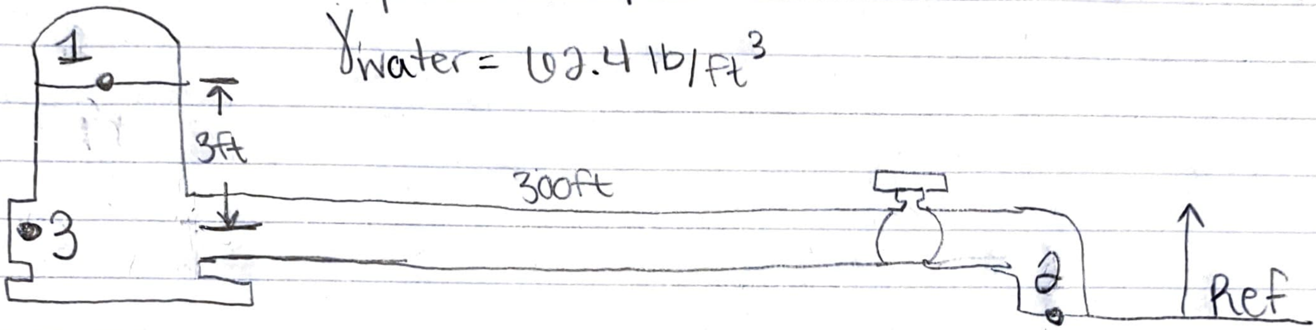
$\frac{25.73\ lb}{ft \cdot s^2}(.0415)ft^2 = 1.07\ lb \cdot ft/s^2 = \boxed{.1384\ N}$

Part G calculations

> Compute force acting on flange: $\text{Diameter}_{\text{flange}} = D_{pipe}$

from part b: h_L = pipe + valve + elbow = 105.1624 ft

$\gamma_{water} = 62.4\ \text{lb/ft}^3$



> Find pressure in tank (Air) using bernoulli's

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \qquad P_1 = 0,\ V_1 = 0,\ z_1 = 0$$

$$\frac{P_1}{\gamma} = \frac{V_2^2}{2g} + h_L - z_1$$

$$\frac{P_1}{\gamma} = \frac{(11.81\ \text{F/s})^2}{64.4\ \text{ft/s}^2} + 105.16\ \text{ft} - 3\ \text{ft} = 139.48\ \text{ft} + 105.16\ \text{ft} - 3\ \text{ft}$$

$$P_1 = \gamma(241.64) = 62.4\ \text{lb/ft}^3(241.64\ \text{ft})$$

$$P_1 = 15078.3\ \text{lb/ft}^2$$

> Find pressure 3 ft below water surface to find pressure @ point 3 using gamma h equation:

$$P_3 = \gamma h = 62.4\ \text{lb/ft}^3(3\ \text{ft}) = 187.2\ \text{lb/ft}^2$$

> Pressure felt by flange = $P_1 + P_3$

$\therefore$ $P_1 + P_3 = 15078.3\ \text{lb/ft}^2 + 187.2\ \text{lb/ft}^2 = 15265.5\ \text{lb/ft}^2$

Part G calculations Continued

$F = \frac{P}{A}$, total pressure at flange was found to be

15265.5 lb/ft^3

To find force, divide pressure by area of flange, $D_{\text{flange}} = D_{\text{Pipe}}$

$$A_{\text{flange}} = \frac{\pi D^2}{4} = \frac{\pi (.125)^2}{4} = 1.77 \text{ ft}^2$$

$$\frac{15265.5 \text{ PSF}}{1.77 \text{ ft}^2} = \boxed{8624.6 \text{ lb force}}$$ ANS

Where is the location of the force?
> since flange is circular, force is exerted on the center of the flange

ANS

> Summary: a-g

a) After itteration, channel depth was determined to be .388 ft or 4.06 in. (see excell sheet)

b) Total h_L was found to be 105.16 ft. Sum of force equation yield a horizontal force of 88.95 lb and a vertical force of 3.822 lb.

c) The largest log was found to be .4076 ft or .124 m. Calculating MB yielded a y_{mc} of .21 ft. Since Mc is slightly higher than the center of gravity, the log used is indeed stable. (Barely stable)

d) The pressure drop across the nozzle was found to be 20.5 psi.

e) Solving the wave velocity equation yielded a wave velocity of 94.46 ft/s. plugging this value into the pressure equation yielded a Δp of 7599.83 PSi. Therefore, no cavitation occurs in the system since cavitation requires extreme pressure drop.

f) The drag force experienced by a log half the size of maximum size was calculated to be .1384 N

g) The pressure at the fluid surface of the tank was found to be 15078.3 lb/ft^3, and at the flange was found to be 15265.5 lb/ft^3. Dividing by the area of the flange yielded a force of 8624.6 lb. Location of force was assumed to be centered on the flange.

> **Materials:**

- 300 ft Sched 40 steel pipe 1 1/2 inch
- Water @ 60°
- Hickory Wood log(s)
- Open channel to carry logs

> **Analysis:** a-g

a) The key to solving this problem was to realize the flow rate equation used could be manipulated to solve for the unknowns using the "left hand - right hand" rule. Furthermore, understanding that since all variables were in terms of "y", except for Q, y could be assumed and iterated until $Q_{act} = Q_{calc}$, proving "y" assumed correctly.

b) A crucial part of solving this part was manipulating Bernoulli's to solve for P_1. In addition, determining the losses in the pipe and elbow were essential to define the forces acting on the system. As expected, the forces in the horizontal were much greater than that of the vertical. Such information should assist the Civil Engineer in creating adequete support for the pipe.

c) The key to solving part c was realizing the relationship between V and Vd under the given conditions. Once max log size was found, it was very straightforward to determine stability. It should be noted that although the log was found to be stable, it was BARELY so. Therefore the logs are not stable beyond all doubt. Such is to be expected for a square-cross section.

d) Part d was simply a matter of locating required constants and variables. In addition, it required the manipulation of the nozzle equation to isolate Δp. A key part of solving this part was realizing β was given and N_R could be calculated and used to solve for C. The found answer of 20.5 psi seems to make sense given large size of the nozzle.

e) This part was very straight forward once units were made to match. Plugging in wave velocity to the Δp equation a high pressure, which is to be expected under extreme conditions of a waterhammer. Furthermore, no cavitation occured since it requires the pressure to drop below vapor pressure.

f) Drag force of the half-sized log was found to be relativley small at .1384 N, which makes sense given the log size and fluid velocity creating said drag force.

g) The key to part g was to reckognize the pressure within the tank, thus the need to use Bernoulli's to determine tank pressure. However, the force exerted on the flange was found to be 8624.6 lb. which seems slightly high for a tank storing 3 ft of pressurized fluid.