Curtis Domingues Exam 3 April 10 (problem 2 chosen)

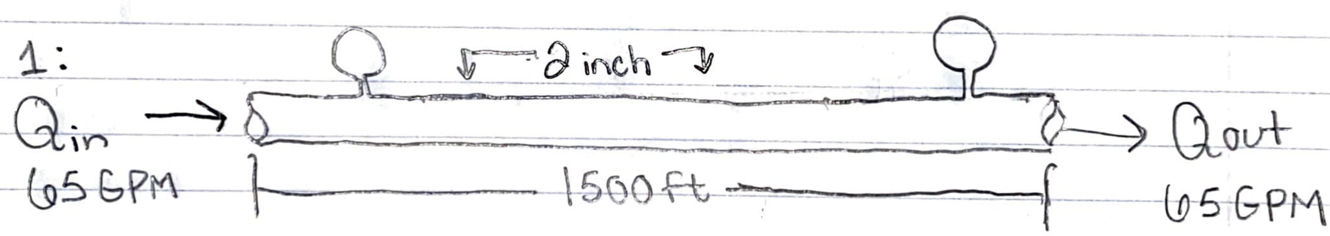
> Problem 2

> **Purpose:** Part 1: Determine the corresponding pressure drop associated with the 1500 ft 2-inch steel tubing given a flow rate of 65 GPM.

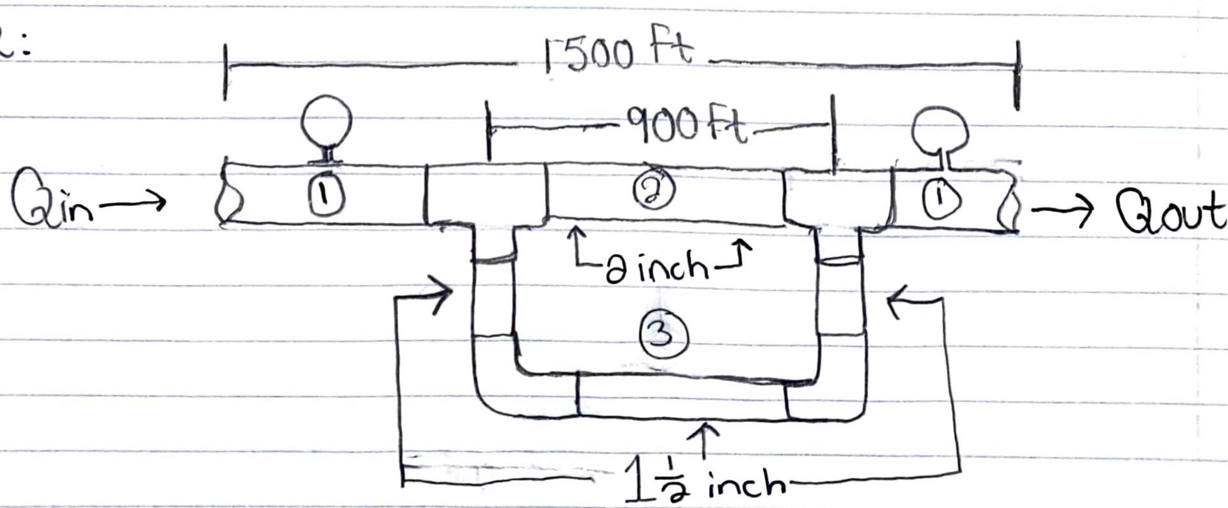
Part 2: Determine the expected increase in flow rate given a 900 ft loop of 1 ½" inch steel tubing is added to the system, and pressure drop is unchanged.

> **Drawings & Diagrams:**

Part 1:



Part 2:



> **Sources:** Mott & Untener. Applied Fluid Mechanics. 7th Edition. Pearson 2015

> Design Consideration:

1) Incompressible Fluid (water)
2) Constant "pressure change" (ΔP)
3) Isothermal process (70°F)
4) Steady State

> Data & Variables: Table G.1 pg. 502 Table A.2 Pg 489

Tubing

2-inch:
Flow area = 1.907×10^{-2} ft^2
Length = 1500 ft
Diameter = 1.870 in = .1558 ft
$\varepsilon = 5.0 \times 10^{-6}$ ft

1½ inch
Flow area = 1.024×10^{-2} ft^2
Length = 900 ft
Diameter = 1.370 in = .1142 ft

Water @ 70°F
Density = 1.94 slugs/ft^3 = 1.94 lb-s/ft^3
γ = 62.3 lb/ft^3
$\mu = 2.04 \times 10^{-5}$ lb-s/ft^2
$\nu = 1.05 \times 10^{-5}$ ft^2/s

> Materials:

Standard steel tubing
Water
Fittings

> Procedure & Calculations

Let us begin by applying bernoulli's equation to solve for the change in pressure for part 1.

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_{L_{1\to 2}} \qquad (1)$$

As height and velocity are constant, bernoulli's can be rewritten as:

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = h_{L_{1\to 2}} \quad \text{or} \quad \Delta p = \gamma\left[h_{L_{1-2}}\right] \qquad (2)$$

Thus, change in pressure is equal to γ_{water} times the total energy loss within the pipe.
To solve for h_L, we will need to calculate f, which will require solving for Reynolds number and D/ε.

$$D/\varepsilon = \frac{.1558\text{ ft}}{5.0\times 10^{-6}\text{ ft}} = \boxed{31160} \qquad N_R = \frac{VD}{\nu} \qquad V = \frac{Q}{A}$$

$$Q = 65\text{GPM} \cdot \frac{1\text{ ft}^3/\text{s}}{449\text{ GPM}} = .144\text{ ft}^3/\text{s}$$

$$V = \frac{.144\text{ ft}^3/\text{s}}{.01907\text{ ft}^2} = 7.59\text{ ft/s} \qquad N_R = \frac{(7.59)(.1558)}{1.05\times 10^{-5}}$$

$$\boxed{N_R = 112621.14}$$

↳ Solve for f

> Calculations Continued...

$$f = \frac{.25}{\left[\log\left(\frac{1}{3.7(D/\varepsilon)} + \frac{5.74}{N_R^{.9}}\right)\right]^2} = \frac{.25}{\left[\log\left(\frac{1}{3.7(31160)} + \frac{5.74}{112621.14^{.9}}\right)\right]^2}$$

$$f = \frac{.25}{\left[\log(8.6736E^{-6} + 1.631E^{-4})\right]^2} \qquad f = \frac{.25}{14.1755} \qquad \boxed{f = .0176}$$

Now that f is known, we shall plug the known values into "Darcey's Equation" and solve for total energy loss.

$$h_L = f \times \frac{L}{D} \times \frac{v^2}{2g} = (.0176) \times \frac{1500\,\cancel{ft}}{.1558\,\cancel{ft}} \times \frac{7.59\,ft/s^2}{2(32.2\,ft/s^2)}$$

$$h_L = 169.448 \times \frac{57.61\,ft^2/\cancel{s^2}}{64.4\,ft/\cancel{s^2}}$$

$$\boxed{h_L = 151.58\,ft}$$

Now that h_L is calculated, we will plug into Bernoulli's and solve for the change in pressure.

$$\Delta p = \gamma[h_{L_{1-2}}] = 62.3\,lb/ft^3\,[151.58\,ft] = 9443.43\,lb/ft^2$$

Thus, Δp is equal to $\boxed{9443.43\text{ PSF or }65.58\text{ PSI}}$

ANS

Calculations & Procedure Continued...

Part 2:

To determine the expected increase in flow rate, we will determine the minor losses added to the system and derive the 3 flow rates generated by the 3 sections of the pipe system. Since $Q_{in} = Q_{out}$, summing the two new flow rates will result in the new total flow. Thus,

$$Q_1 = Q_2 + Q_3 \qquad \text{(I)}$$

Next, we will manipulate bernoulli's Equation to solve for Q_2 and Q_3. The two resulting equations will be in terms of Q_1, which is also unknown, therefore Q_1 must be assumed and solved through iteration.

Q_2) $$\frac{\Delta P}{\gamma} = \left(f_1 \frac{L_1}{D_1} + 2K_{Tee\,(run)}\right)\frac{8Q_1^2}{g\pi^2 D_1^4} + \left(f_2 \frac{L_2}{D_2}\right)\frac{8Q_2^2}{g\pi^2 D_2^4}$$

Q_3) $$\frac{\Delta P}{\gamma} = \left(f_1 \frac{L_1}{D_1} + 2K_{Tee\,(branch)}\right)\frac{8Q_1^2}{g\pi^2 D_1^4} + \left(f_3 \frac{L_3}{D_3} + 2K_{Elbow}\right)\frac{8Q_3^2}{g\pi^2 D_3^4}$$

NOTE: This procedure neglects the presence of "reducers" as they are not present in the problem statement.

Solving for Q_2 & Q_3 yields:

$$Q_2 = \sqrt{\frac{\frac{\Delta P}{\gamma} - \left(f_1\frac{L_1}{D_1}\right)\frac{8Q_1^2}{g\pi^2 D_1^4}}{\left(2K_{Tee_{run}} + f_2\frac{L_2}{D_2}\right)\frac{8}{g\pi^2 D_2^4}}} \qquad \text{(II)}$$

$$Q_3 = \sqrt{\frac{\frac{\Delta P}{\gamma} - \left(f_1\frac{L_1}{D_1}\right)\frac{8Q_1^2}{g\pi^2 D_1^4}}{\left(2K_{Tee_{brnch}} + 2K_{Elb} + f_3\frac{L_3}{D_3}\right)\frac{8}{g\pi^2 D_3^4}}} \qquad \text{(III)}$$

Part 2 Calculations Continued...

I will start the iteration process, NOTE: Full iteration will be in the form of an excell sheet and attatched to this document.

Assume						Assume				solve
Q_1	NR1	D/ε1	F1	Ft1	Numer	F2	D/ε2	Ft2	DEN2	Q_2
.15	1.19E5	3.18E4	1.74E-2	9.73E-3	9.28E1	.02	3.18E4	9.73E3	4.5E3	.143
						.0175			3.9E3	.153
						.0173				.154
↓	↓	↓	↓	↓	↓	.0173				.1541
						.0173	↓	↓	↓	

		New		Assume				solve		
V2	NR2	F2	% Diff	F3	D/ε3	Ft3	DEN3	Q3	V3	NR3
7.53	1.14E5	1.7E-2	-3%	.02	2.28E4	1.03E-2	2.35E4	.0628	6.16	6.7E4
8.03	1.21E5			1.9E-2			2.31E4	.0633	6.21	6.76E4
8.08	1.22E5			1.96E-2				.0634	6.22	
8.08	1.22E5	1.735E-2	-.01%	1.96E-2	↓	↓	↓	.0634	6.22	↓

New F3	% Diff	New Q1	% Diff "Q_1"
1.97E-2	-2%	.218	45%
1.96E-2			
↓	0.0%		

NOTE: This represents one complete iteration. Variables assumed: Q1, F_2 and F_3. As N_R is dependant on velocity which is dependant on Q. Thus, assuming F_2 and F_3 and reiterrating will result in solving for Q2 & Q3 which sum to equal Q_1 new.

Calculations Continued...

As shown in the attatched excell sheet, this iteration was repeated 6 times, until a sufficent level of precision was reached. After 6 iterations Q_1 "new" was found to be $\boxed{.183\ ft^3/s}$ with a percent error of .07%

Thus, $Q_2 = .1295\ ft^3/s$ and $Q_3 = .0532\ ft^3/s$.

plugging into equation I $.1295 + .0532 = .183\ ft^3/s = Q_1$

Finally, given the original flow rate from part 1 was $.144\ ft^3/s$, the expected increase of flow rate is:

$.183 - .144 = \boxed{.039\ ft^3/s}$ increase in flow rate

or

$\boxed{17.5\ GPM}$ ANS

Summary:

The change in pressure from part 1 was calculated to be 9443.43 PSF. Using this same pressure change in part 2, Q_1 was assumed to be .15 ft^3/s, but through iteration it was proved to actually be .183 ft^3/s. Thus, the expected increase in flow rate is roughly 17.5 GPM.

Analysis:

The values obtained seem to be justified in that they do make sense within the system given the slight modifications made. It should be noted however, this procedure did not account for reduction fittings within the system. Therefore, in real-world applications the expected increase would be slightly lower as there would be more energy loss within the system. Since no reduction fittings were present in the problem, I assumed they were negligable. (This could be easily fixed by adding in the two reducers).