12.3   In the branched pipe system shown in Fig. 12.8, 850 L/min of water at 10°C is flowing in a DN 100 Schedule 40 pipe at A. The flow splits into two DN 50 Schedule 40 pipes as shown and then rejoins at B. Calculate (a) the flow rate in each of the branches and (b) the pressure difference pA - pB. Include the effect of the minor losses in the lower branch of the system. The total length of pipe in the lower branch is 60 m. The elbows are standard.
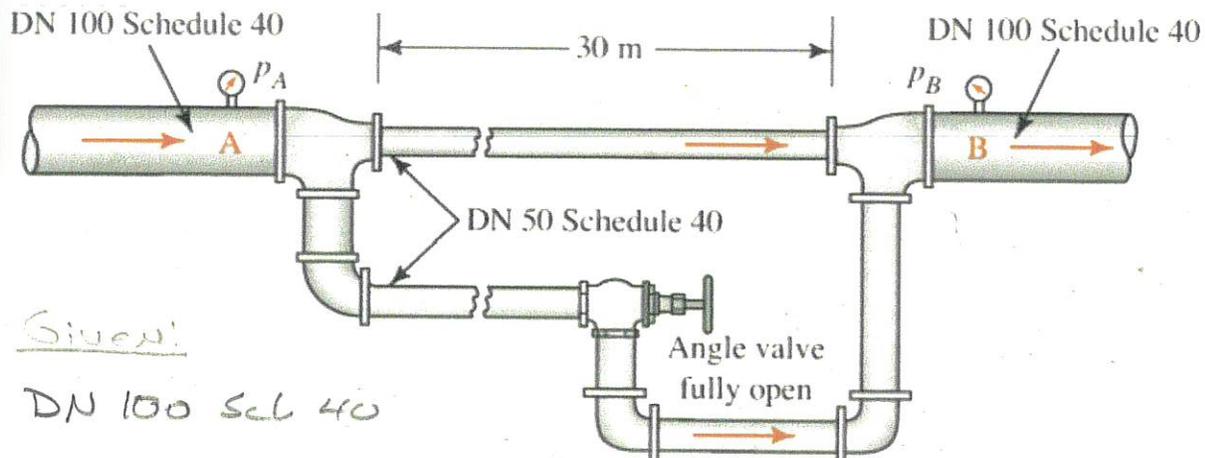
Given:

DN 100 Sch 40

$ID = 102.3mm \rightarrow 0.1023m : \dfrac{\pi (0.1023)^2}{4} = 8.2 \times 10^{-3} m^2$

$OD = 114.3mm \rightarrow 0.1143m$

DN 50 Sch 40

$ID = 52.5mm \rightarrow 0.0525m : \dfrac{\pi (0.0525)^2}{4} = 2.2 \times 10^{-3} m^2$

$OD = 60.3mm \rightarrow 0.0603m$

Flow Rate (Q)

$Q = 850 \dfrac{L}{min}$

$\gamma_w = 9.81 \dfrac{kN}{m^3}$

Top Branch

Pipe DN 50 Sch 40
Pipe Length 30m

Bottom Branch

Pipe DN 50 sch 40
Pipe Length 60 m

3 - Standard Elbow
1 - Angle Value

123

Top Branch

$$h_{La} = f \cdot \frac{30m}{0.0525m} \cdot \frac{V^2}{2g}$$

Friction factor
- Table 10.5

DN 50 sch 40
$$f = 0.019$$

DN 40 sch 40
$$f = 0.020$$

Bottom Branch

Pipe   Elbow   Angle Value

$$h_{LB} = f \frac{60m}{0.0525m} \frac{V^2}{2g} + 3 \cdot f \, 30 \frac{V^2}{2g} + f(150) \frac{V^2}{2g}$$

$$h_{La} = h_{LB}$$

$$f \frac{30m}{0.0525m} \frac{V^2}{2g} = f \frac{60m}{0.0525m} \frac{V^2}{2g} + 3 \cdot f \, 30 \frac{V^2}{2g} + f \, 150 \frac{V^2}{2g}$$

$$f \, 571.43 \frac{V^2}{2g} = f \, 1142.9 \frac{V^2}{2g} + f \, 240 \frac{V^2}{2g}$$

$$(0.02) 571.43 \frac{V^2}{2g} = (0.019) 1142.9 \frac{V^2}{2g} + (0.019) 240 \frac{V^2}{2g}$$

$$11.43 \frac{V^2}{2g} = \left(21.72 \frac{V^2}{2g} + 4.56 \frac{V^2}{2g} \right)$$

$$11.43 \frac{V^2}{64.4} = \frac{V^2}{2g} (21.72 + 4.56)$$

$$736.08 \, V^2 = 26.28 \cdot \frac{V^2}{2 \cdot (32.2)}$$
$$64.4$$

$$736.08 V^2 = 1692.43 \, V^2$$

$$V^2 = \sqrt{\frac{169243}{736.08}}$$

$$V_B = 1.516$$

12.3   Table F.1 - Flow Area

DN 50 Sch 40 $A = 2.168 \times 10^{-3} \, m^2$

$V_B = 1.516 \, V_B$

$Q_A = A_a V_a + A_b V_b$ $Q = 850 \, \frac{L}{min}$

$Q_A = A_a (1.516) + A_b V_b$ $Q = 0.0142 \, \frac{m^2}{sec}$

$Q_A = V_b (2.516) A_b$

$Q_A = V \cdot A \Rightarrow V_b = \dfrac{Q_A}{A_b} = \dfrac{0.0142 \frac{m^2}{sec}}{(2.516)(2.168 \times 10^{-3} m^2)}$

$\underline{V_b = 2.60 \, \frac{m}{sec}}$

Top Branch

Velocity   Flow Rate

$V_a = 1.516 \, V_b$ $Q_a = A_a V_a$

$V_a = 1.516 \cdot 2.55$ $\quad = (2.168 \times 10^{-3} m^2)(3.866 \frac{m}{sec})$

$\underline{V_a = 3.866 \, \frac{m}{sec}}$ $Q_a = 0.0084 \, \frac{m^3}{sec}$

                                  $\underline{Q_a = 8.4 \times 10^{-3} \, \frac{m^3}{sec}}$

12.3 $\qquad\qquad\qquad\qquad\qquad\qquad \gamma = 9.81 \frac{kN}{m^3}$

→ Reynold's # Top Branch

$$R_e = \frac{V_a \cdot D_a}{V} = \frac{\left(3.866 \frac{m}{sec}\right)\left(0.0525 m\right)}{1.30 \times 10^{-6}}$$

$$R_e = 1.56 \times 10^5$$

DN 50 sch 40   Pipe Roughness

$$\frac{D}{\varepsilon} = \frac{0.0525}{4.6 \times 10^{-5}} = 1141$$

Table 8.2 p.183

$f_a = 0.021$

Look up on Re Table

pg 185

→ Bottom Branch

$$R_{e_{Bottom \; Branch}} = \frac{V_b \, D_b}{V} = \frac{(2.60)(0.0525)}{1.30 \times 10^{-6}}$$

$$R_e = 1.05 \times 10^5$$

Re Table p.185

$$\frac{D}{\varepsilon} = \text{SAME } 1141 \qquad f_b = 0.021$$

Conversion

ⓐ $Q_a = V_A A_a \rightarrow \left(3.866 \frac{m}{sec}\right)\left(2.168 \times 10^{-3} m^2\right) \dfrac{60000 \frac{L}{min}}{1 \frac{m^3}{sec}}$

$$Q_a = \underline{\underline{502.89 \frac{L}{min}}}$$

$Q_b = V_b A_b \rightarrow \left(2.60 \frac{m}{sec}\right)\left(2.168 \times 10^{-3} m^2\right) \dfrac{60000 \frac{L}{min}}{1 \frac{m^3}{sec}}$

$$Q_b = \underline{\underline{338.2 \frac{L}{min}}}$$

ⓑ $P_a - P_b = \gamma h_L \frac{v^2}{2g} \rightarrow \left(9.81 \frac{kN}{m^3}\right)(57.43)(0.021)\frac{(3.866)^2}{2 \cdot 9.81} = \underline{\underline{90.61 \, kPa}}$

1)

12.5 A 160-mm pipe branches into a 100-mm and a 50-mm pipe as shown in Fig. 12.10. Both pipes are hydraulic copper tubing and 30 m long. (The fluid is water at 10°C.) Determine what the resistance coefficient K of the valve must be to obtain equal volume flow rates of 500 L/min in each branch.
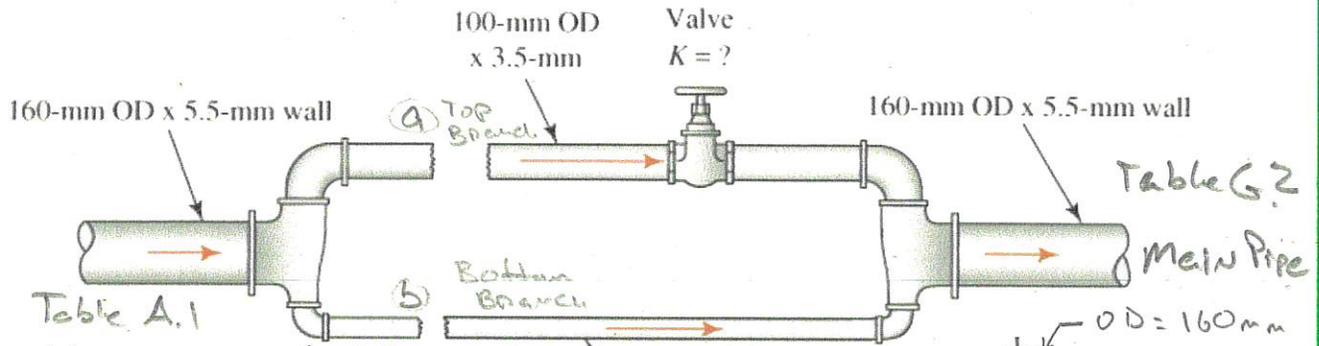

100-mm OD x 3.5-mm
Valve K = ?
160-mm OD x 5.5-mm wall
160-mm OD x 5.5-mm wall
ⓐ Top Branch
ⓑ Bottom Branch
All flow elements are copper tubing
50-mm OD x 1.5-mm wall

Table A.1

$$\nu = 1.30 \times 10^{-6} \frac{m^2}{sec}$$

$$Q = 500 \frac{L}{MIN} \text{ Convert}$$

$$Q = 8.33 \times 10^{-3} \frac{m^3}{sec}$$

$$\gamma = 9.81 \frac{kN}{m^3} \quad @ \; 10°C$$

$$V_a = \frac{Q}{A_a} = \frac{8.33 \times 10^{-3} \frac{m^3}{sec}}{6.793 \times 10^{-3} m^2}$$

$$V_a = 1.23 \frac{m}{sec}$$

$$V_b = \frac{Q}{A_b} = \frac{8.33 \times 10^{-3} \frac{m^3}{sec}}{1.735 \times 10^{-3} m^2}$$

$$V_b = 4.80 \frac{m}{sec}$$

$$Re_a = \frac{V_a \, D_a}{\nu}$$

$$Re_a = \frac{(1.23 \frac{m}{sec})(0.093 \, m)}{1.30 \times 10^{-6} \frac{m^2}{sec}}$$

$$Re_a = 8.79 \times 10^{4}$$

Table G.2
Main Pipe
OD = 160 mm
−5.5 mm
−5.5 mm
ID = 149 mm

Flow Area
$1.744 \times 10^{-3} m^2$

ⓐ
Flow Area
$6.793 \times 10^{-3} m^2$
OD = 100 mm
−3.5 mm
−3.5 mm
ID = 93 mm

ⓑ Flow Area
$1.735 \times 10^{-3} m^2$
OD = 50 mm
−1.5 mm
−1.5 mm
ID = 47 mm

→ Table 8.2 p.183 (ε)
@ $\frac{D}{\varepsilon} = \frac{0.093}{1.5 \times 10^{-6}} = 62 K$

→ ⓑ Table 8.2 p.183 (ε)
$\frac{D}{\varepsilon} = \frac{0.047}{1.5 \times 10^{-6}} = 31.33 K$

→ Table Re P.185
ⓐ Frictional Factor
$f_a = 0.00875$

ⓑ Frictional Factor
$f_b = 0.0093$

→ Head Loss

Top Branch

$f_a = 0.00875$
$f_b = 0.0093$ } Moody's chart

$$h_{L_a} = \underset{\text{friction}}{\text{Pipe}} + 2\,\text{Elbow} + \text{Valve}$$

$$= \underset{\text{Pipe}}{f_a \frac{L_a}{D_a} \frac{V_a^2}{2g}} + \underset{\text{Valve}}{k \frac{V_a^2}{2g}} + \underset{\text{Elbows}}{f_a \cdot 2 \cdot 30 \frac{V^2}{2g}}$$

$$= (f_a) \frac{30_m}{0.093_m} \frac{V^2}{2g} + k \frac{V_c^2}{2g} + f_a\,60 \frac{V^2}{2g}$$

$$= f_a\,322.58 \frac{V^2}{2g} + k \frac{V_a^2}{2g} + f_a\,60 \frac{V^2}{2g}$$

$$= (0.00875)\,322.85 \frac{V_c^2}{2g} + k + (0.00875)(60) \frac{V^2}{2g}$$

$$= \left[2.83 + k + 0.525 \right] \frac{V^2}{2g}$$

$$h_{L_a} = (3.355 + k) \frac{V^2}{2g}$$

$$h_{L_b} = \text{Pipe} + 2\,\text{Elbow}$$

$$= f_b \frac{L_b}{D_b} \frac{V_b^2}{2g} + f_b\,2 \cdot 30 \frac{V^2}{2g}$$

$$= f_b \frac{30_m}{0.047_m} \frac{V^2}{2g}$$

$$= (0.0093)\,638.30 \frac{V^2}{2g} + (0.0093)\,60 \frac{V^2}{2g}$$

$$= 5.94 \frac{V^2}{2g} + 0.558 \frac{V^2}{2g}$$

$$h_{L_b} = 6.498 \frac{V^2}{2g}$$

$$Q_a = Q_b$$

$$Q_a = V_a A_a \qquad Q_b = V_b A_b$$

$$V_a A_a = V_b A_b$$

$$V_b = V_a \left(\frac{A_a}{A_b}\right)^2$$

$$V_b = V_a \left(\frac{0.093}{0.047}\right)^2 = 3.92 \, V_a$$

$$V_b = 3.92 \, V_a \implies V_b^2 = 15.4 \, V_a^2$$

$$h_{La} = h_{Lb}$$

$$(3.36 + K)\frac{V_a^2}{2g} = 6.498 \frac{V_b^2}{2g}$$

$$3.36 + K = 6.498 \cdot \frac{15.4 \, V_a}{2g}$$

$$K = 99.85 - 3.36$$

$$K = \underline{\underline{96.49}}$$