Test 1
Solutions

Name: ____.

MET 330 Fluid Mechanics
Dr. Orlando Ayala
Fall 2024
Test 1

Take home – Due Wednesday September 25th, 2024, before midnight.

READ FIRST

1. RELAX!!!! DO NOT OVERTHINK THE PROBLEMS!!!! There is nothing hidden. The test was designed for you to pass and get the maximum number of points, while learning at the same time. <u>HINT:</u> THINK BEFORE TRYING TO USE/FIND EQUATIONS (OR EVEN FIND SIMILAR PROBLEMS)
2. The total points on this test are one hundred (100). Ten (10) points are from your HW assignments, and ten (10) other points are based on the basis of technical writing. The other eighty (80) points will come from the problem solutions. For the technical writing I will follow the attached rubric.
3. There are 2 main different parts, each one is worth 80/2 points.
4. What you turn in should be only your own work. You cannot discuss the exam with anyone, except me. Call me, text me, email me, come to my office, if you have any question.
5. I do not read minds. You should be explicit and organized in your answers. Use drawings/figures. If you make a mistake, do not erase it. Rather use that opportunity to explain why you think it is a mistake and show the way to correct the problem.
6. You have to turn in your test ON TIME and ONLY through CANVAS. You must submit the test solution in only one file, and it has to be a pdf file. You must also submit the excel spreadsheet. For the ePortfolio (which is optional) you are supposed to upload this artifact to your Google drive. I will provide more instructions later.
7. Do not start at the last minute so you can handle anything that could happen. Late tests will not be accepted. Test submitted through email will not be accepted either.
8. Cheating is completely wrong. The ODU Student Honor Pledge reads: "I pledge to support the honor system of Old Dominion University. I will refrain from any form of academic dishonesty or deception, such as cheating or plagiarism." By attending Old Dominion University, you have accepted the responsibility to abide by this code. This is an institutional policy approved by the Board of Visitors. It is important to remind you the following part of the Honor Code:

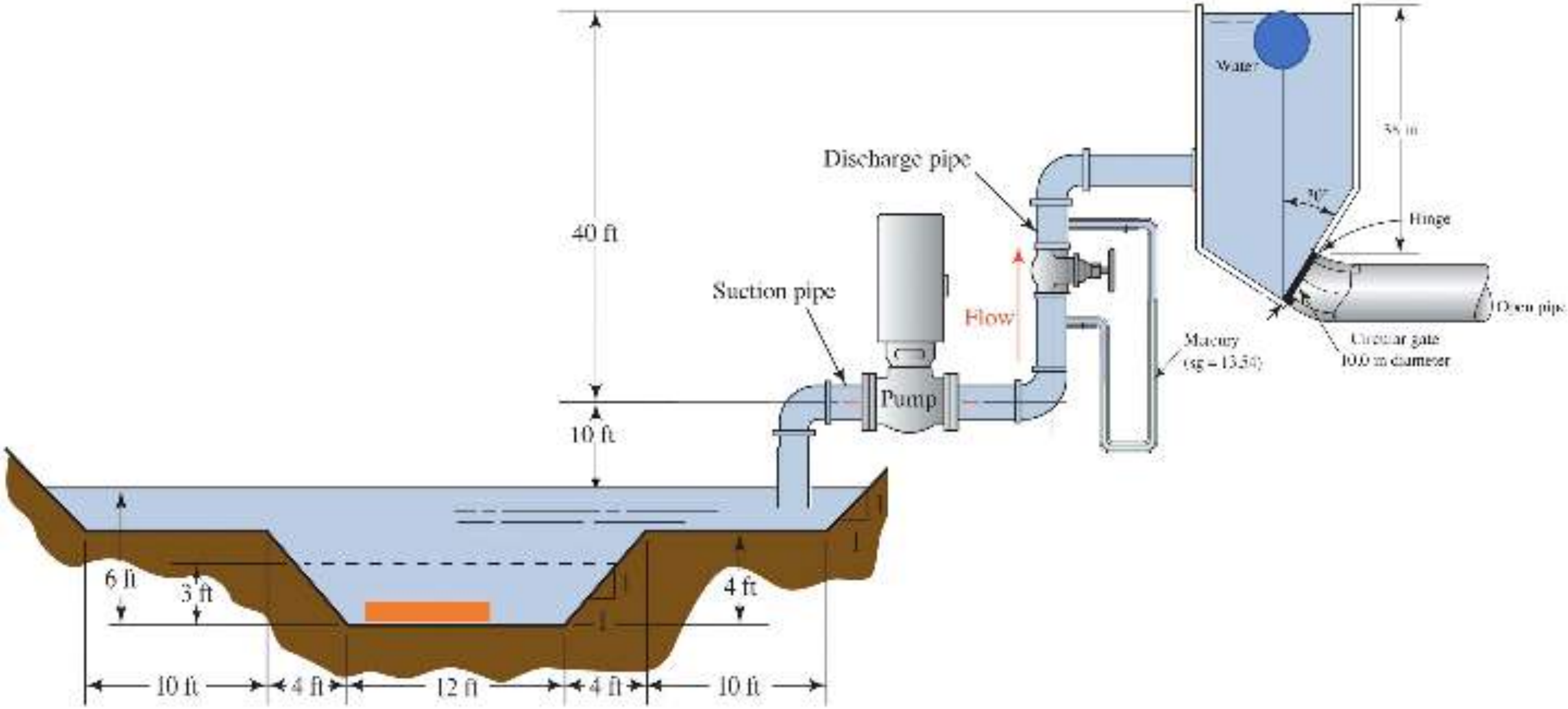
    IX. PROHIBITED CONDUCT
    A. Academic Integrity violations, including:
    1. *Cheating*: Using unauthorized assistance, materials, study aids, or other information in any academic exercise (Examples of cheating include, but are not limited to, the following: using unapproved resources or assistance to complete an assignment, paper, project, quiz or exam; collaborating in violation of a faculty member's instructions; and submitting the same, or substantially the same, paper to more than one course for academic credit without first obtaining the approval of faculty).

With that said, you are NOT authorized to use any online source of any type, unless is ODU related.

A company has hired you to design a system that delivers 60°F water from a large, open channel to another elevated one, as shown in the accompanying figure (NOTE: THE FIGURE IS NOT TO SCALE). The system must deliver 3.387 ft^3/s of water from the lower channel to the upper channel. The total length of the suction pipe is 11 feet, and the discharge pipe length is 2500 feet. For this task, you will focus on designing specific portions of the system.

a. To prevent spillage in the elevated channel, your client proposes the system shown in the figure. A circular gate seals the pipe to stop the flow, and when the water level reaches 38 inches, a fully submerged spherical buoy opens the gate.
- How large should the buoy be to perform this function? Ignore the weight of the circular gate and the buoy.
- Is the buoy stable while pulling the gate open? Why?
- You are required to perform all calculations by hand but must also create an Excel spreadsheet to automatically run the calculations. Ensure that the Excel results match your hand calculations. NOTE: When using trigonometric functions in Excel, be sure to use radians for angles.
- The company also wonders whether the 30° degree angle of the circular gate was an appropriate decision. Find the best angle which will allow the use of a smaller buoy. To facilitate the answer of this equation, use the spreadsheet, with it run the calculations for different angles and make a table with the different angles and the corresponding buoy diameter. Finally, plot buoy size (diameter) vs. the angle of the gate. Would the stability change because of a smaller (or larger) buoy? Why?
- Review your results and provide comments in the "analysis" section of your solution. Why do the results make sense?

b. The company needs to monitor valve performance, and they propose using an on-site U-tube manometer (shown in the figure) to measure the pressure difference across the valve. The valve's pressure difference at a flow rate of 3.387 ft^3/s is 3.393 psi. The distance between the two taps where the manometer is connected is 20 inches.
- What will the U-tube manometer reading (i.e., deflection) be?
- What would be the pressure difference between the two taps if there is no flow?
- You are required to perform all calculations by hand but must also create an Excel spreadsheet to automatically run the calculations. Ensure that the Excel results match your hand calculations.
- Use the spreadsheet, with it run the calculations for different valve pressure difference and make a table with the different valve pressure differences and the corresponding mercury deflection. Finally, plot mercury deflection vs. valve pressure difference.
- Review your results and provide comments in the "analysis" section of your solution. Why do the results make sense?



Water
Discharge pipe
Hinge
40 ft
Suction pipe
Flow
Open pipe
Circular gate
Pump
10 ft
6 ft
3 ft
4 ft
1
1
1
1
10 ft
4 ft
12 ft
4 ft
10 ft

Problem solution rubric

	Exceeds Standard 4 10 points	Meets Standard 3 7 points	Approaches Standard 2 4 points	Needs Attention 1 0 points
1. Purpose **5%**	The purpose of the section to be answered is clearly identified and stated.	The purpose of the section to be answered is identified, but is stated in a somewhat unclear manner.	The purpose of the section to be answered is partially identified, and is stated in a somewhat unclear manner.	The purpose of the section to be answered is erroneous or irrelevant.
2. Drawings & Diagrams **10%**	Clear and accurate diagrams are included and make the section easier to understand. Diagrams are labeled neatly and accurately.	Diagrams are included and are labeled neatly and accurately.	Diagrams are included and are labeled.	Needed diagrams are missing OR are missing important labels.
3. Sources **5%**	Several reputable background sources were used and cited correctly.	A few reputable background sources are used and cited correctly.	A few background sources are used and cited correctly, but some are not reputable sources.	Background sources are cited incorrectly.
4. Design considerations (assumptions, safety, cost, etc) **10%**	Design is carried out with applicable assumptions and full attention to safety and cost, etc.	Design is generally carried out with assumptions and attention to safety, cost, etc.	Design is carried out with some assumptions and some attention to safety, cost, etc.	Assumptions, safety and cost were ignored in the design.
5. Data and variables **5%**	All data and variables are clearly described with all relevant details.	All data and variables are clearly described with most relevant details.	Most data and variables are clearly described with most relevant details.	Data and variables are not described OR the majority lack sufficient detail.
6. Procedure **25%**	Procedure is described in clear steps. The step description is in a complete and easy to understand short paragraph.	Procedure is described in clear steps but the step description is not in a complete short paragraph.	Procedure is described in clear steps. The step description is in a complete short paragraph but it is difficult to understand.	Procedure is not described in clear steps at all.
7. Calculations **20%**	All calculations are shown and the results are correct and labeled appropriately. The units of all values are shown.	Some calculations are shown and the results are correct and labeled appropriately.	Some calculations are shown and the results labeled appropriately.	No calculations are shown OR results are inaccurate or mislabeled.
8. Summary **5%**	Summary describes the design, the relevant information and some future implications.	Summary describes the design and some relevant information.	Summary describes the design.	No summary is written.
9. Materials **5%**	All materials used in the design are clearly and accurately described.	Almost all materials used in the design are clearly and accurately described.	Most of the materials used in the design are clearly and accurately described.	Many materials are described inaccurately OR are not described at all.
10. Analysis **10%**	The design is discussed and analyzed. Argumentative predictions are made about what might happen in case of change in the operation and how the design could be change.	The design is discussed and analyzed. Argumentative predictions are made about what might happen in case of change in the operation.	The design is discussed and analyzed. No argumentative predictions are made about what might happen in case of change in the operation and how the design could be change.	The design is not discussed and analyzed.

[A]

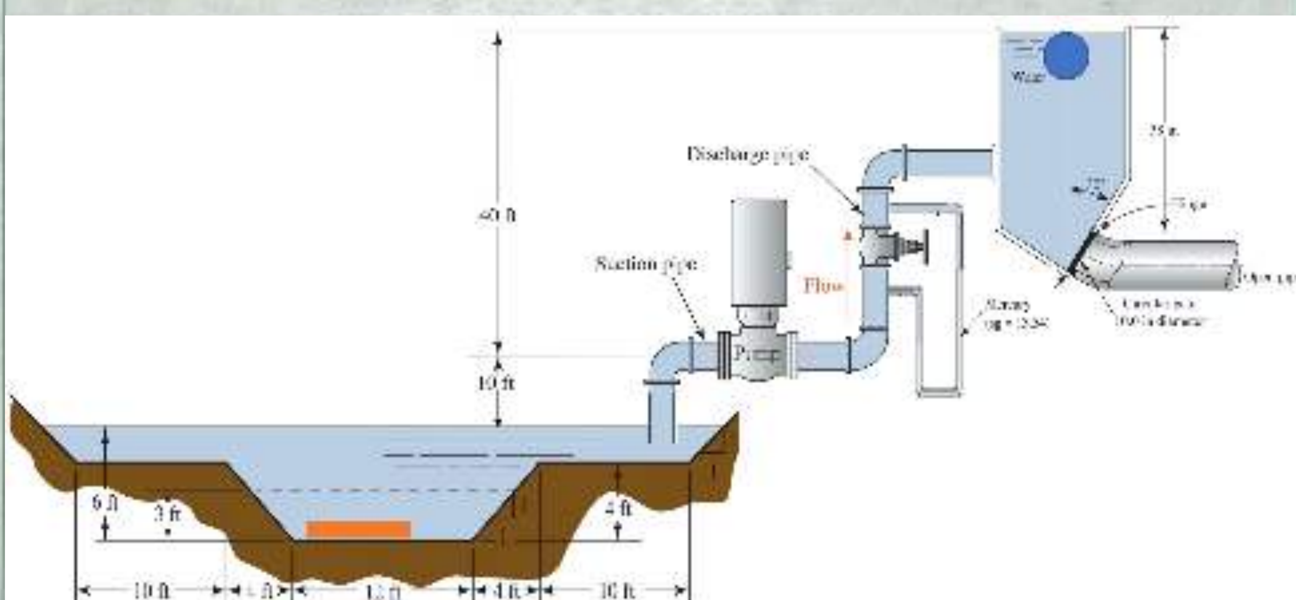
PURPOSE

Review a design of a pumped system that delivers water from a lower open channel to an elevated open channel. For now, the review includes:

a) Design a buoy system to make sure the water level in the elevated open channel does not go above 38 inches.

b) Design the U-tube manometer by calculating the reading that measures the pressure difference across the valve.

DRAWINGS & DIAGRAMS



SOURCES

Mott, R, Untener, J.A., "Applied Fluid Mechanics" 7the Edition, Pearson Education Inc (2015)

DESIGN CONSIDERATIONS

- Constant properties
- Incompressible fluid
- Isothermal conditions
- Steady State
- Newtonian fluid

MATERIALS

- Water @ 60°F
- Steel pipe

DATA & VARIABLES

- $L_{dis} = 2,000$ ft
- $L_{suction} = 11$ ft
- All dimension in drawings
- $\nu = 1.21 \times 10^{-5}$ ft²/s
- $\gamma = 62.4$ lb/ft³
- $\gamma_{Hg} = 844.9$ lb/ft³

$\Delta P = 3.393$ psi $= 488.592 \frac{lb}{ft^2}$

$Q = 3.387$ ft³/s

$\Delta z_{tanks} = 20$ in $= 1.667$ ft

$D = 10$ in

$H = 38$ in

PROCEDURE

For the 2 main parts, I will do the following

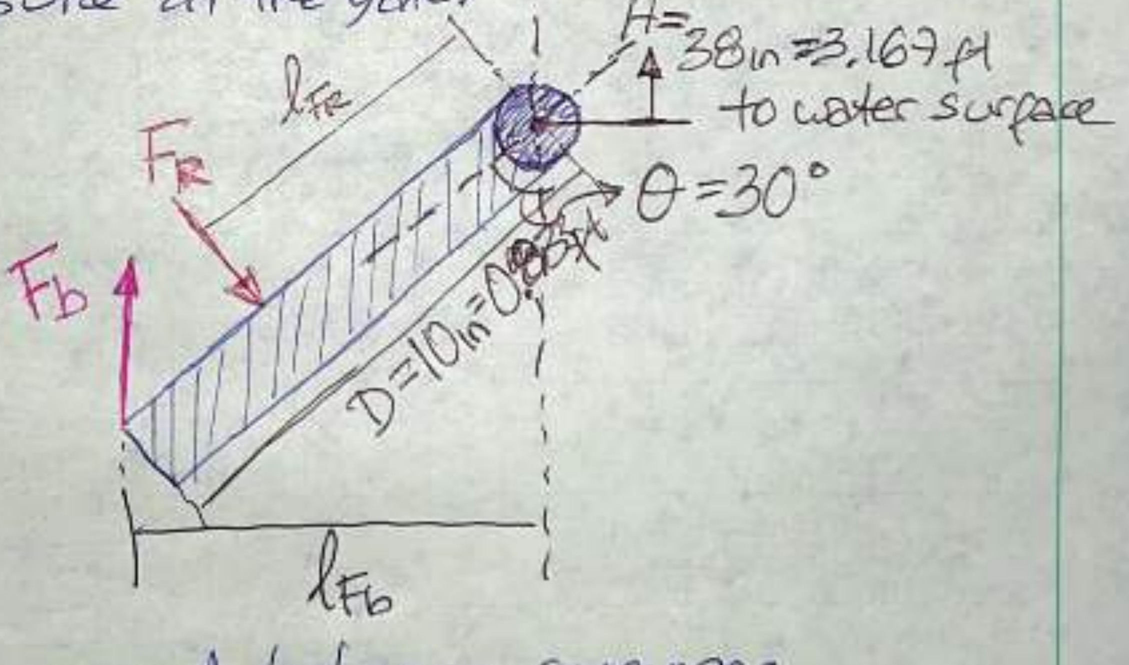
a) The buoy needs to be big enough to produce a force on the circular gate so it opens. This happens when the moment due to the buoyancy force on the gate is larger than the moment due to the force of the water pressure. So, I will use the moment balance equation.

$\boxed{C}$

b) I will use the "gamma × h" equation starting from one of the U-tube manometer connections (taps) move along the U-tube until I get to the other connection. I need to recognize that the distances between the connections are important to simplify the equation.

CALCULATIONS

a) The circular gate opens because of a moment. And it happens when the moment due to buoyancy is greater than the moment due to hidrostatic pressure on the gate.



The moment balance gives me:

$$F_b \times l_{F_b} = F_R \times l_{F_R}$$

from which: $F_b = F_R \dfrac{l_{F_R}}{l_{F_b}}$

D

I need now F_R, l_{F_R}, and l_{F_b}.

For F_R:

$$F_R = \gamma h_c A$$

$$* \, h_c = H + \frac{D \times \cos\theta}{2}$$

$$= 3.167\,ft + \frac{0.833\,ft \times \cos 30^\circ}{2} = 3.528\,ft$$

and

$$* \, A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.833\,ft)^2 = 0.545\,ft^3$$

Now, $F_R = 62.4 \frac{lb}{ft^3} \times 3.528\,ft \times 0.544\,ft^3$

$$F_R = 119.98\,lb$$

For l_{F_b}:

$$l_{F_b} = D \sin\theta$$

$$l_{F_b} = 0.4165\,ft$$

For l_{F_R}:

$$l_{F_R} = L_p - \frac{H}{\cos\theta}$$

E

We need L_p:

$$L_p = L_c + \frac{I_c}{L_c A}$$

$$\cdot L_c = \frac{h_c}{\cos\theta} = \frac{3.528\,ft}{\cos 30^\circ} = 4.074\,ft$$

$$\cdot I_c = \frac{\pi D^4}{64} = 2.36\times10^{-2}\,ft^4$$

$$L_p = 4.074\,ft + \frac{2.36\times10^{-2}\,ft^4}{4.074\,ft \times 0.545\,ft^2}$$

$$L_p = 4.085\,ft$$

So,

$$l_{FR} = 4.085\,ft + \frac{3.167\,ft}{\cos 30^\circ}$$

$$l_{FR} = 0.427\,ft$$

With those numbers I can now calculate F_b

$$F_b = 119.98\,lb\,\frac{0.427\,ft}{0.4165\,ft}$$

$$\boxed{F_b = 123.004\,lb}$$

This buoyancy force is produced by the spherical buoy, so follow this equation:

F

$$F_b = \gamma_f \forall_{disp}$$

From $\forall_{disp}$, we can get the buoy diameters:

$$F_b = \gamma_f \frac{1}{6} \pi D^3$$

So,

$$D = \sqrt[3]{\frac{6 F_b}{\gamma_f \pi}} = \sqrt[3]{\frac{6 \times 123.004\,lb}{62.4 \frac{lb}{ft^3} \times \pi}}$$

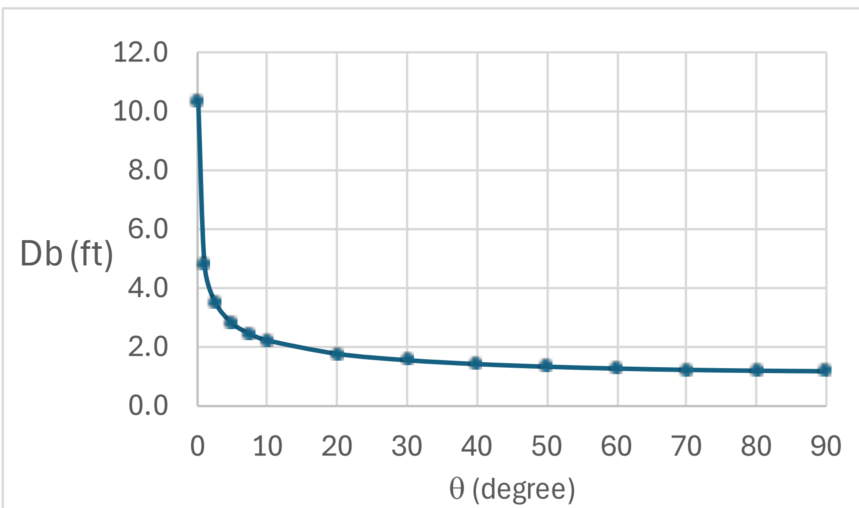
$$\cancel{\boxed{D = 1.556\,ft}}$$

- Regarding whether the buoy is stable or not, given that it is fully submerged and that it is made uniformly with same material, the center of buoyancy and the center of mass are at exactly same locations. Thus it follows the theory of stable submerged bodies: "center of mass should be located under the center of buoyancy."

- I created a spreadsheet and as it can be seen in the highlighted row, the excel results match my hand calculations.
Excel file is included

γ=	62.4	lb/ft3		
H=	38	in	3.167	ft
D=	10	in	0.833	ft

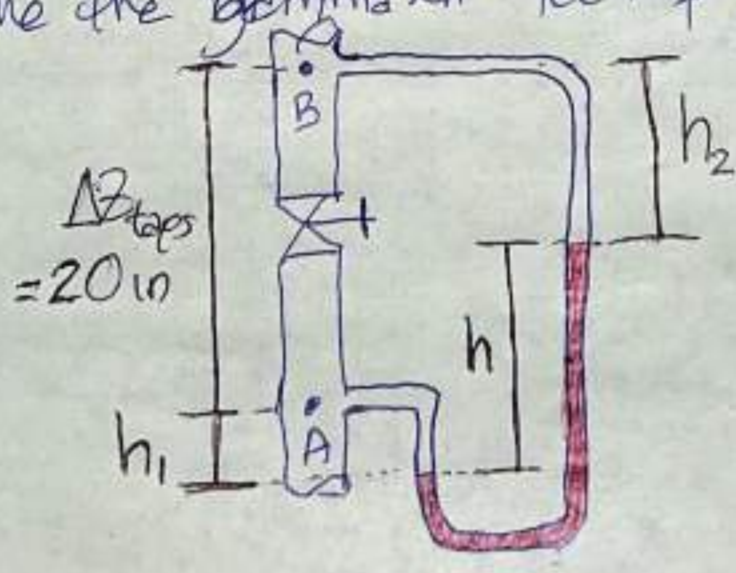
θ (degree)	θ (rad)	hc (ft)	A (ft2)	F_R (lb)	l_F_b (ft)	Lc (ft)	Ic (ft4)	Lp (ft)	l_F_R (ft)	Fb (lb)	Db (ft)
0.1	0.002	3.583	0.545	121.955	0.001	3.583	0.024	3.595	0.429	35953.136	10.324
1	0.017	3.583	0.545	121.953	0.015	3.584	0.024	3.596	0.429	3595.418	4.792
2.5	0.044	3.583	0.545	121.941	0.036	3.586	0.024	3.598	0.429	1438.388	3.531
5	0.087	3.582	0.545	121.901	0.073	3.595	0.024	3.608	0.429	719.589	2.803
7.5	0.131	3.580	0.545	121.834	0.109	3.611	0.024	3.623	0.429	480.166	2.449
10	0.175	3.577	0.545	121.739	0.145	3.632	0.024	3.644	0.429	360.588	2.226
20	0.349	3.558	0.545	121.100	0.285	3.787	0.024	3.798	0.428	181.906	1.772
30	0.524	3.528	0.545	120.055	0.417	4.073	0.024	4.084	0.427	123.125	1.556
40	0.698	3.486	0.545	118.637	0.536	4.550	0.024	4.560	0.426	94.396	1.424
50	0.873	3.434	0.545	116.889	0.638	5.343	0.024	5.351	0.425	77.781	1.335
60	1.047	3.375	0.545	114.864	0.722	6.750	0.024	6.756	0.423	67.340	1.273
70	1.222	3.309	0.545	112.624	0.783	9.675	0.024	9.680	0.421	60.571	1.228
80	1.396	3.239	0.545	110.237	0.821	18.653	0.024	18.655	0.419	56.281	1.199
89.99	1.571	3.167	0.545	107.777	0.833	18144.080	0.024	18144.080	0.417	53.889	1.182



G

I included in the spreadsheets calculations for a number of gate angles. I plotted the buoy diameter vs this angle. The plot shows the largest buoy for an angle of 0 degree (vertical) and the smallest buoy size occurs when the gate is fully horizontal ($\theta = 90°$).
The stability will not change because the buoy is constantly submerged regardless of the gate angle. Thus, the criteria described before remains unchanged.

b) As I said, I will use the "gamma×h" technique. As explained in class and in the solved problems posted in CANVAS, any unknown distance will be labeled and carry on in the exercise of the the "gamma×h" techique. So, drawing is importante



Observing the geometry we notice:

$$\Delta z_{tapes} + h_1 = h_2 + h$$

Using the "gamma*h" techique:

$$P_A + \gamma_w h_1 - \gamma_{Hg} h - \gamma_w h_2 = P_2$$

Manipulating:

$$\underbrace{P_A - P_B}_{\text{This is the } \Delta P} = \gamma_{Hg} \times h + \gamma_w \underbrace{(h_2 - h_1)}_{\text{This can be obtaining from the geometry relation we observed:}}$$

$$h_2 - h_1 = \Delta z_{tapes} - h$$

So:

$$\frac{\Delta P}{\gamma_w} = \frac{\gamma_{Hg}}{\gamma_w} h + (\Delta z_{tapes} - h)$$

$$= \left(\frac{\gamma_{Hg}}{\gamma_w} - 1\right) h + \Delta z_{tapes}$$

Solving for "h":

$$h = \frac{\left(\frac{\Delta P}{\gamma_w} - \Delta z_{tapes}\right)}{\left(\frac{\gamma_{Hg}}{\gamma_w} - 1\right)}$$

Substituing values:

$$h = \frac{\left(\frac{488.592\ lb/ft^2}{62.4\ lb/ft^3} - 1.667\ ft\right)}{\left(\frac{844.9\ lb/ft^3}{62.4\ lb/ft^3} - 1\right)}$$

$$\boxed{h = 0.492\ ft = 5.89\ in}$$

▶ If there were no flow, I can also apply "gamma-h" technique through the pipe to go from A to B:

$$P_A - \gamma_w \Delta z_{taps} = P_B$$

So,

$$P_A - P_B = \gamma_w \Delta z_{taps} = 62.4\ \frac{lb}{ft^3} \times 1.667\ ft$$

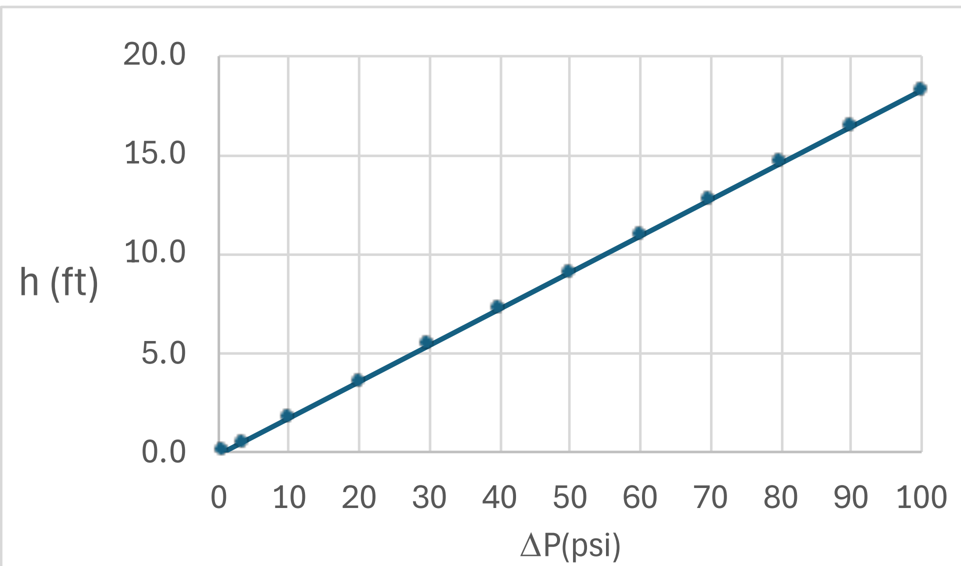
$$\Delta P = 0.722\ psi = 104.0\ lb/ft^2$$

▶ I created a spreadsheet and as it can be seen in the highlighted row, the excel results match all my hand calculations.

Excel file is included.

Δz=	20	in	1.667	ft
γ_w=	62.4	lb/ft3		
γ_Hg=	844.9	lb/ft3		

ΔP (psi)	ΔP (lb/ft2)	h (ft)
0.722	104.000	0.0000
3.393	488.592	0.4915
10	1440.000	1.7073
20	2880.000	3.5476
30	4320.000	5.3879
40	5760.000	7.2281
50	7200.000	9.0684
60	8640.000	10.9086
70	10080.000	12.7489
80	11520.000	14.5891
90	12960.000	16.4294
100	14400.000	18.2696



J

I included in the spreadsheet calculations for a number of pressure differences. I plotted manometer deflection vs. pressure difference. The plot shows that the deflection increases linearly with the increase of pressure difference.

SUMMARY

The buoy must be of 1.556 ft in diameter so the circular gate opens when the water level in the elevated channel reaches 38 in. The buoy will always be stable.

The manometer reading to measure the pressure difference across the valve for the 3.387 ft^3/s flow rate is 5.89 in

ANALYSIS

All results make sense.

We could use a smaller buoy if the circular gate is fully horizontal.

Note that the length of the tubes of the manometer must be larger than the reading value "h". Otherwise the mercury will get into the system