

Test 2

My Work
Parts A - C

Spencer Reed
MET 330

Test 2

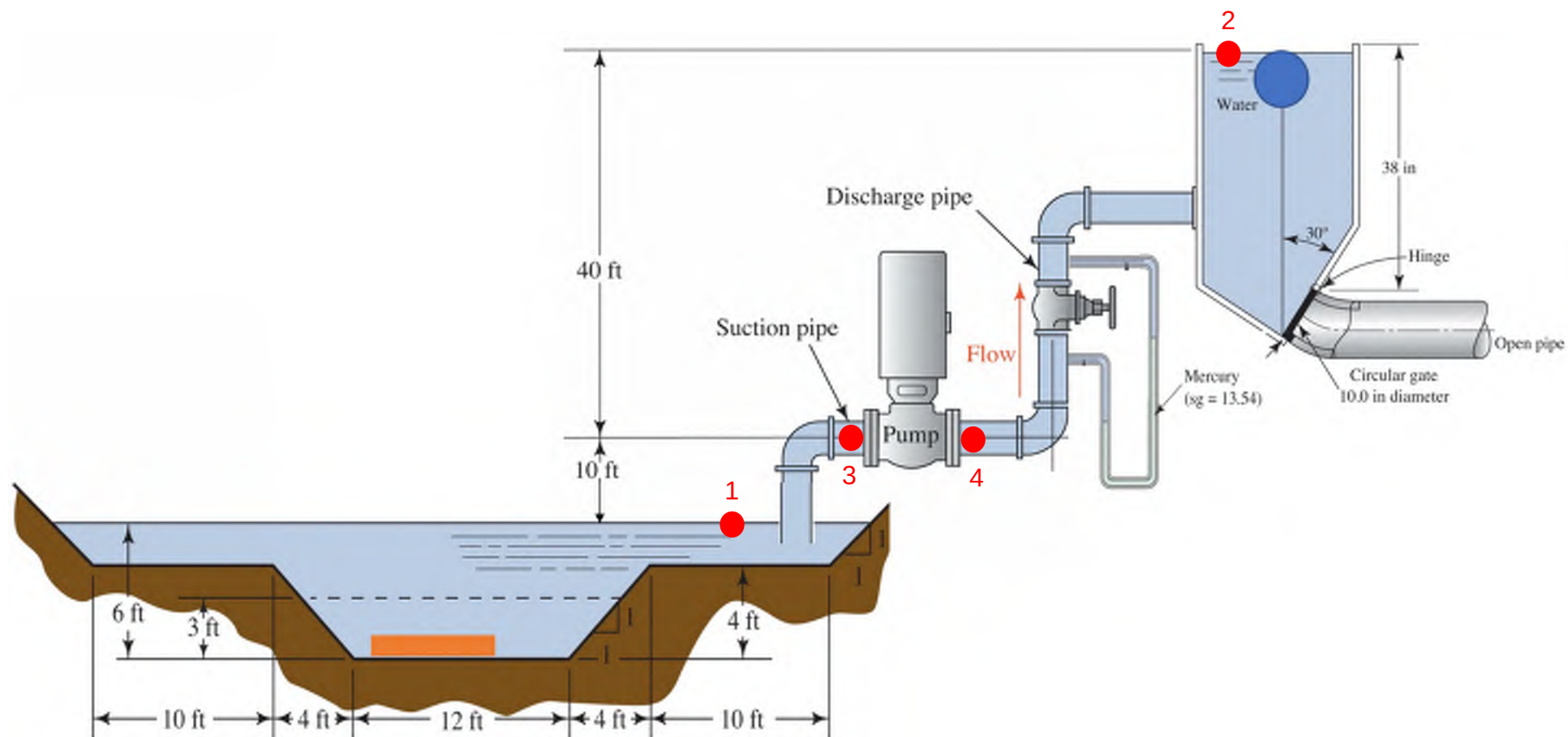
Part A Purpose:

To Continue designing a water delivery system that pumps 60°F water from a lower channel to an upper open channel at a rate of $3.387 \text{ ft}^3/\text{s}$.

- (A1) Find a pipe diameter from table F-1 so that fluid velocity is close to 3 m/s .
- (A2) Determine pump power in HP with 60% efficiency. Suction pipe is 11ft and discharge pipe is 2500ft. No losses due to fittings.
- (A3) Find pressure at pump inlet and pump outlet. Points 3 & 4.
- (A4) Build an excel to autorun calculations based on different pipe diameters. Then plot pump power vs pipe diameter and inlet/outlet pump pressures.

Sources:

Mott, R & Untener Jr, "Applied Fluid Mechanics, 8th edition digital, Pearson Education Inc., 2002"



A-1) Data Given:

$$Q = 3.387 \text{ ft}^3/\text{s} \quad v \approx 3 \text{ m/s} \times \frac{3.28 \text{ ft/s}}{1 \text{ m/s}} \\ T_w = 60^\circ\text{F} \quad \approx 9.84 \text{ ft/s}$$

Solve for estimated Area of pipe

$$A = \frac{Q}{v} = \frac{3.387 \text{ ft}^3/\text{s}}{9.84 \text{ ft/s}} = 0.344 \text{ ft}^2$$

Table F-1 shows the Flow Area of an 8" sch 40 steel pipe to be 0.3472 ft².

Using flow area from our pipe to solve for pipe velocity we get.

$$Q = AV \quad V = \frac{Q}{A} = \frac{3.387 \text{ ft}^3/\text{s}}{0.3472 \text{ ft}^2} = 9.755 \text{ ft/s} \\ \text{or } 2.97 \text{ m/s.}$$

$$A = \frac{\pi D^2}{4} \rightarrow D = \sqrt{\frac{4A}{\pi}} \rightarrow \sqrt{\frac{4(0.3472)}{\pi}} = 0.6649 \text{ ft}$$

Closest pipe given in Table F-1 for the inside diameter of 0.6649 ft and a flow area of 0.344 ft² is the 8" sch. 40 steel pipe with a flow area of 0.3472 ft² and an inside diameter of 0.6651 ft

8" sch. 40 steel Flow Area = 0.3472 ft²
Inside Diameter = 0.6651 ft

A-2) Given + Solved Data:

$$Q = 3.387 \text{ ft}^3/\text{s} \quad V = 9.755 \text{ ft/s}$$

$$A = 0.3472 \text{ ft}^2 \quad ID = 0.6651 \text{ ft}$$

$$\eta = 60\% \quad L_s = 11 \text{ ft} \quad L_D = 250 \text{ ft}$$

$$T_w = 60^\circ\text{F} \quad \Delta h = 50 \text{ ft}$$

$$\gamma = 62.415 \text{ lb/ft}^3 \text{ table A.2}$$

To determine the pump power needed with a 60 percent efficiency we must start with Bernoulli's eq.,

$$h_A + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_{L_{12}}$$

Since point 1 is surface level and open to the atmosphere the pressure and velocity are both Zero. Since point 1 is our base point and is below point 2 then Z_1 is also 0. Since point 2 is surface level and open to the atmosphere the pressure and velocity are Zero as well. Thus:

$$h_A + \frac{P_1^0}{\gamma} + \frac{V_1^0}{2g} + Z_1^0 = \frac{P_2^0}{\gamma} + \frac{V_2^0}{2g} + Z_2 + h_{L_{12}}$$

$$h_A = Z_2 + h_{L_{12}}$$

$$h_L = f \frac{LV^2}{2gD}$$

to find f we need Re
and Relative Roughness.

$$h_{L12} = f \frac{LV^2}{2gD}$$

$$Re = \frac{VD}{\nu}$$

$$\text{Relative Roughness} = \frac{D}{E}$$

$$\frac{D}{E} = \frac{0.6651 \text{ ft}}{1.5 \times 10^{-4} \text{ ft}} = 4434$$

$$Re = \frac{9.755 \text{ ft/s} \times 0.6651 \text{ ft}}{1.21 \times 10^{-5} \text{ ft}^2/\text{s}}$$

Table A.2 Kinematic
Viscosity
for 60°F
water

$$Re = 536202 \text{ or } 5.36 \times 10^5$$

$$f = \frac{0.25}{\left(\log \left(\frac{1}{3.7Dk} + \frac{5.74}{Re^{0.9}} \right) \right)^2}$$

$$f = \frac{0.25}{\left(\log \left(\frac{1}{3.7 \times 4434} + \frac{5.74}{536202^{0.9}} \right) \right)^2}$$

$$f = 0.0157$$

Moody's Chart $f \approx 0.0157$

(Attached to end of test)

$$h_{L12} = f \frac{LV^2}{2gD} \rightarrow 0.0157 \frac{2511 \times 9.755^2}{2 \times 32.2 \times 0.6651} = 87.5846 \text{ ft}$$

Using Bernoulli's Eq from earlier:

$$h_A = Z_2 + h_{L12}$$

$$h_A = 50 \text{ ft} + 87.5846 \text{ ft}$$

$$h_A = 137.5846 \text{ ft}$$

$$P_A = \gamma Q h_A$$

$$P_m = \frac{P_A}{\eta}$$

$$P_A = 62.4 \text{ lb/ft}^3 \times 3.387 \text{ ft}^3/\text{s} \times 137.5846 \text{ ft}$$

$$P_A = 29078.3401 \text{ lb} \cdot \text{ft}/\text{s}$$

Convert to HP from table K.1 for Power

$$29078.3401 \text{ lb} \cdot \text{ft}/\text{s} \times \frac{1 \text{ HP}}{550 \text{ lb} \cdot \text{ft}/\text{s}} = 52.8697 \text{ HP}$$

$$P_A = 52.8679 \text{ HP}$$

$$P_m = \frac{52.8679 \text{ HP}}{0.60} = 88.1161 \text{ HP}$$

$$P_m = 88.1161 \text{ HP}$$

A-3) To compute the pressures at the pump inlet and pump outlet, Bernoulli's eq. must be used.

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3 + h_{L13}$$

Since point one is on the surface of an open channel ^{and is the lowest point} we don't have pressure, velocity or Z at that point. V_3 is the velocity inside the pipe which has been determined to be 9.755 ft/s from earlier.

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3 + h_{L13}$$

$$= \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3 + h_{L13} \rightarrow \frac{P_3}{\gamma} = -\frac{V_3^2}{2g} - Z_3 - h_{L13}$$

$$\rightarrow P_3 = \gamma \times \left(-\frac{V_3^2}{2g} - Z_3 - h_{L13} \right)$$

h_{L13} is the only unknown so we solve for it. $h_L = f \frac{LV^2}{2gD}$

$\frac{D}{4}$ from earlier is 4434

Re from earlier is 536202 or 5.36×10^5

f from earlier is 0.0157

$$h_{L13} = 0.0157 \frac{11 \times 9.755^2}{2 \times 32.2 \times 0.6651}$$

$$h_{L13} = 0.3837$$

Eg. from earlier: $P_3 = \gamma \left(\frac{V_3^2}{2g} - Z_3 - h_{L13} \right)$

$$P_3 = 62.4 \text{ lb/ft}^3 \times \left(-\frac{9.75^2}{2 \times 32.246} - 10 \text{ ft} - 0.3837 \text{ ft} \right)$$

$$P_3 = -740.1476 \text{ lb/ft}^2$$

Converting to Pa using Table K1 for pressure then to kPa

$$P_3 = -740.1476 \text{ lb/ft}^2 \times \frac{47.88 \text{ Pa}}{1 \text{ lb/ft}^2} \times \frac{1 \text{ kPa}}{1000 \text{ Pa}} =$$

$$P_3 = -35.4383 \text{ kPa}$$

To solve for P_4 we start with Bernoulli's using points 4 and 2 and solve for h_{L42} .

$$\frac{P_4}{\gamma} + \frac{V_4^2}{2g} + Z_4 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_{L42}$$

Since Z_4 is our starting point Z is zero.

Since Point 2 is on the surface of an open channel it has no pressure or velocity.

$$\frac{P_4}{\gamma} + \frac{V_4^2}{2g} + \overset{0}{Z_4} = \frac{\overset{0}{P_2}}{\gamma} + \frac{\overset{0}{V_2^2}}{2g} + Z_2 + h_{L42}$$

$$\frac{P_4}{\gamma} + \frac{V_4^2}{2g} = Z_2 + h_{L42} \rightarrow \frac{P_4}{\gamma} = Z_2 + h_{L42} - \frac{V_4^2}{2g}$$

$$\rightarrow P_4 = \gamma \left(Z_2 + h_{L42} - \frac{V_4^2}{2g} \right)$$

Solve for h_{L42}

$$h_{L42} = f \frac{L V^2}{2gD} \rightarrow 0.0157 \times \frac{2500 \times 9.755^2}{2 \times 32.2 \times 0.6651}$$

$$= 87.2009 \text{ ft}$$

$$P_4 = 62.4 \text{ lb/ft}^3 \times (40 \text{ ft} + 87.2009 \text{ ft} - \frac{9.755^2 \text{ ft/s}^2}{2 \times 32.2 \text{ ft/s}^2})$$

$$= 7845.2259$$

Convert to Pa per table K1 then to KPa

$$P_4 = 7845.2259 \text{ lb/ft}^2 \times \frac{47.88 \text{ Pa}}{1.11 \text{ lb/ft}^2} \times \frac{1 \text{ KPa}}{1000 \text{ Pa}}$$

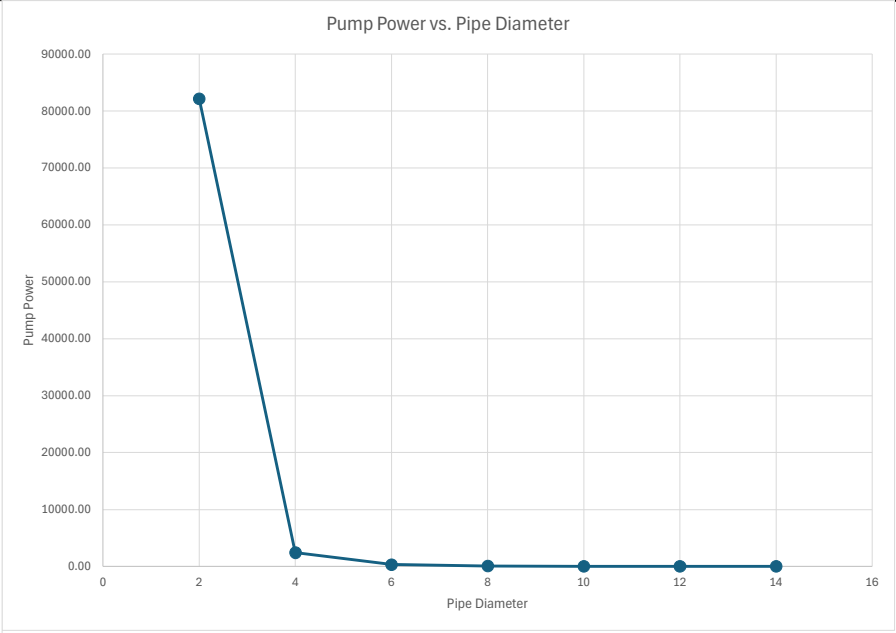
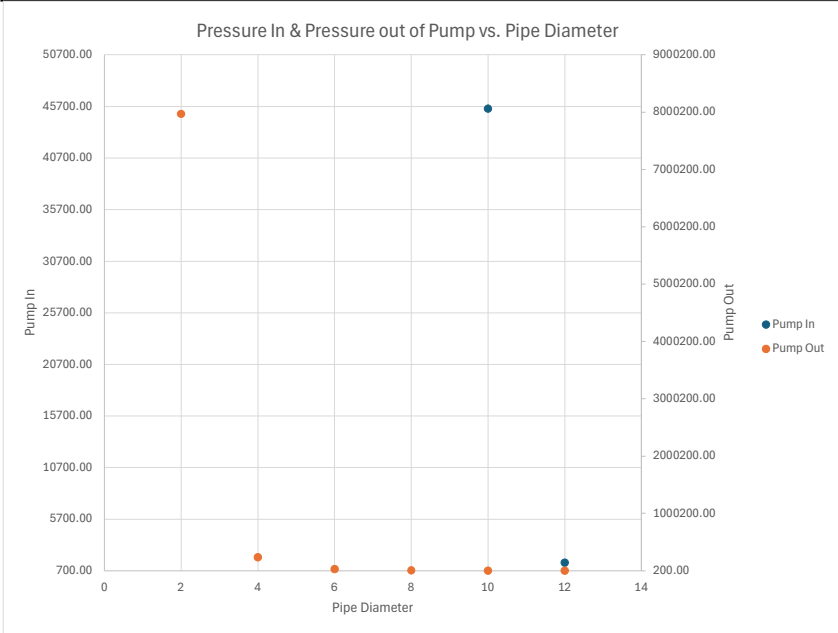
$$P_4 = 375.629 \text{ KPa}$$

Analysis:

The pump power is proportion to the h_A and Q as it goes into the pump. H_L is effected by the full systems energy loss. In this problem we only accounted for pipe friction and elevation change. Pressure at the inlet was negative and the outlet was positive. Both points 1 + 2 saw 0 pressure and velocity while point 1 saw no Z value. They had no P or V due to being open channels to the ATM.

GIVEN:			Part A1			Part A2			Part A3		
Q =	3.3870	ft^3/s	V =	9.7552	ft/s	hA =	137.3574	ft	f =	0.0157	
γ =	62.4100	lb/ft^3	8in Pipe Flow Area =	0.3472	ft^2	hL2 =	87.3574	ft	Re =	536212.6528	
n - 60% =	0.6000					f =	0.0157		D/E =	4434.0000	
V =	9.8400	ft/s				Re =	536212.6528		V3 =	9.7552	ft/2
Flow Area of 8" Pipe =	0.3472	ft^2				D/E =	4434.0000		hL3 =	0.3827	ft
A =	0.3442	ft^2				PA =	29034.9635	lb*ft/s	P3 Pin =	-740.2066	lb/ft^2
g =	32.2000	ft/s^2					52.7908	HP		-35.44109184	kPa
Suction Pipe =	11.0000	ft				Pm =	87.9847	HP	V4 =	9.7552	ft/2
Discharge Pipe =	2500.0000	ft							hL4 =	86.9747	ft
Total Pipe Length	2511.0000	ft							P4 Pout =	7832.2656	lb/ft^2
ID 8" Pipe =	0.6651	ft								375.0089	kPa
v =	0.0000121	ft^2/s									
Epsilon =	0.0002	ft									

Part A4																	
Pipe Size	Internal Diameter (ft)	Flow Area (ft^2)	Q (ft^3/s)	V (ft/s^2)	Re	D/E	f	hL 2 (full system) (ft)	hA	PA (HP)	Pm (HP)	hL3 (Pre Pump System) (ft)	Inlet Pump Pressure (lb/ft^2)	Inlet Pump Pressure (kPa)	hL4 (Post Pump System) (ft)	Outlet Pump Pressure (lb/ft^2)	Outlet Pump Pressure (kPa)
2	0.1616	0.0205	3.3870	#####	#####	1077.3333	0.0195	#####	#####	49304.90	82174.83	561.7719	-33465747.99	-1602340.014	#####	#####	381633.6858
4	0.3188	0.0799	3.3870	#####	#####	2125.3333	0.0170	#####	#####	1460.07	2433.45	16.4232	-277442.26	-13283.93555	#####	#####	11268.68781
6	0.4801	0.1810	3.3870	#####	#####	3200.6667	0.0161	457.5440	507.5440	195.07	325.11	2.0044	-1325.14	-63.44789762	455.5396	30834.40	1476.351248
8	0.6651	0.3472	3.3870	9.7552	#####	4434.0000	0.0157	87.3574	137.3574	52.79	87.98	0.3827	-740.21	-35.44109184	86.9747	7832.27	375.0088792
10	0.9480	0.7056	3.3870	4.8002	#####	6320.0000	0.0156	14.7385	64.7385	24.88	41.47	0.0646	45479.97	2177.580996	14.6739	3319.98	158.9605122
12	1.0420	0.8521	3.3870	3.9749	#####	6946.6667	0.0156	9.2151	59.2151	22.76	37.93	0.0404	1495.56	71.60719902	9.1747	2976.77	142.5277776
14	1.1740	1.0389	3.3870	3.2601	#####	7826.6667	0.0156	5.5023	55.5023	21.33	35.55	0.0241	-1325.14	-63.44789762	5.4782	2746.07	131.4819333



Part B: To determine the pressure drop across a nozzle with a ratio of 0.5 for our 8" pipe diameter.

Given: $D = 0.665 \text{ ft}$ from Part A-2

$$d = 0.5 \times 0.665 \text{ ft} = 0.3325 \text{ ft}$$

$$B = \frac{d}{D} \rightarrow \frac{0.3325 \text{ ft}}{0.665 \text{ ft}} = 0.4999 \text{ or } 0.5 \text{ ratio.}$$

B-1) Calculate pressure drop across nozzle.

$$V_1 = \frac{C \sqrt{2g(P_1 - P_2)/\gamma}}{\sqrt{(A_1/A_2)^2 - 1}} \quad \text{Solve for } P_1 - P_2$$

$$\left(\frac{V_1}{C}\right)^2 = \frac{2g(P_1 - P_2)/\gamma}{(A_1/A_2)^2 - 1} \rightarrow \left(\frac{V_1}{C}\right)^2 \times ((A_1/A_2)^2 - 1) = \frac{2g(P_1 - P_2)}{\gamma}$$

$$\rightarrow \frac{\gamma V_1^2}{C^2} \times ((A_1/A_2)^2 - 1) = 2g(P_1 - P_2)$$

$$\rightarrow \frac{\gamma V_1^2}{2gC^2} \times ((A_1/A_2)^2 - 1) = (P_1 - P_2)$$

We know everything in the equation except C and A_2

$$A_2 = \frac{\pi d^2}{4} \quad d = 0.3325 \text{ ft} \quad A_2 = \frac{\pi (0.3325)^2}{4}$$

$$A_2 = 0.0868 \text{ ft}^2$$

$$C = 0.9975 - 6.53 \sqrt{0.5/536202}$$

$$C = 0.9911$$

$$P_1 - P_2 = \frac{\gamma V_1^2}{2gC^2} \left[\left(\frac{A_1}{A_2} \right)^2 - 1 \right]$$

$$\frac{62.4 \text{ lb/ft}^2 \times (9.755 \text{ ft/s})^2}{2 \times 32.2 \times (0.9911)^2} \times \left[\left(\frac{0.3472 \text{ ft}^2}{0.0868 \text{ ft}^2} \right)^2 - 1 \right]$$

$$P_1 - P_2 = 1408.0224 \text{ lb/ft}^2 \text{ to kPa}$$

$$1408.0224 \text{ lb/ft}^2 \times \frac{47.88 \text{ Pa}}{1 \text{ lb/ft}^2} \times \frac{1 \text{ kPa}}{1000 \text{ Pa}} = 67.4161 \text{ kPa}$$

$$P_{\text{total}} = 67.4161 \text{ kPa}$$

With no loss

$$B-2) \frac{62.4 \text{ lb/ft}^2 \times 9.755 \text{ ft/s}^2}{2 \times 32.2 \times 1^2} \times \left[\left(\frac{0.3472 \text{ ft}^2}{0.0868 \text{ ft}^2} \right)^2 - 1 \right]$$

$$= 1383.0711 \text{ lb/ft}^2 \times \frac{47.88 \text{ Pa}}{1 \text{ lb/ft}^2} \times \frac{1 \text{ kPa}}{1000 \text{ Pa}} = 66.2214 \text{ kPa}$$

$$\Delta P_{\text{loss}} = \Delta P_{\text{total}} - \Delta P \rightarrow 67.4161 - 66.2214$$

$$\Delta P_{\text{loss}} = 1.1947 \text{ kPa}$$

P drop across the nozzle equals 1.1947 kPa

Analysis:

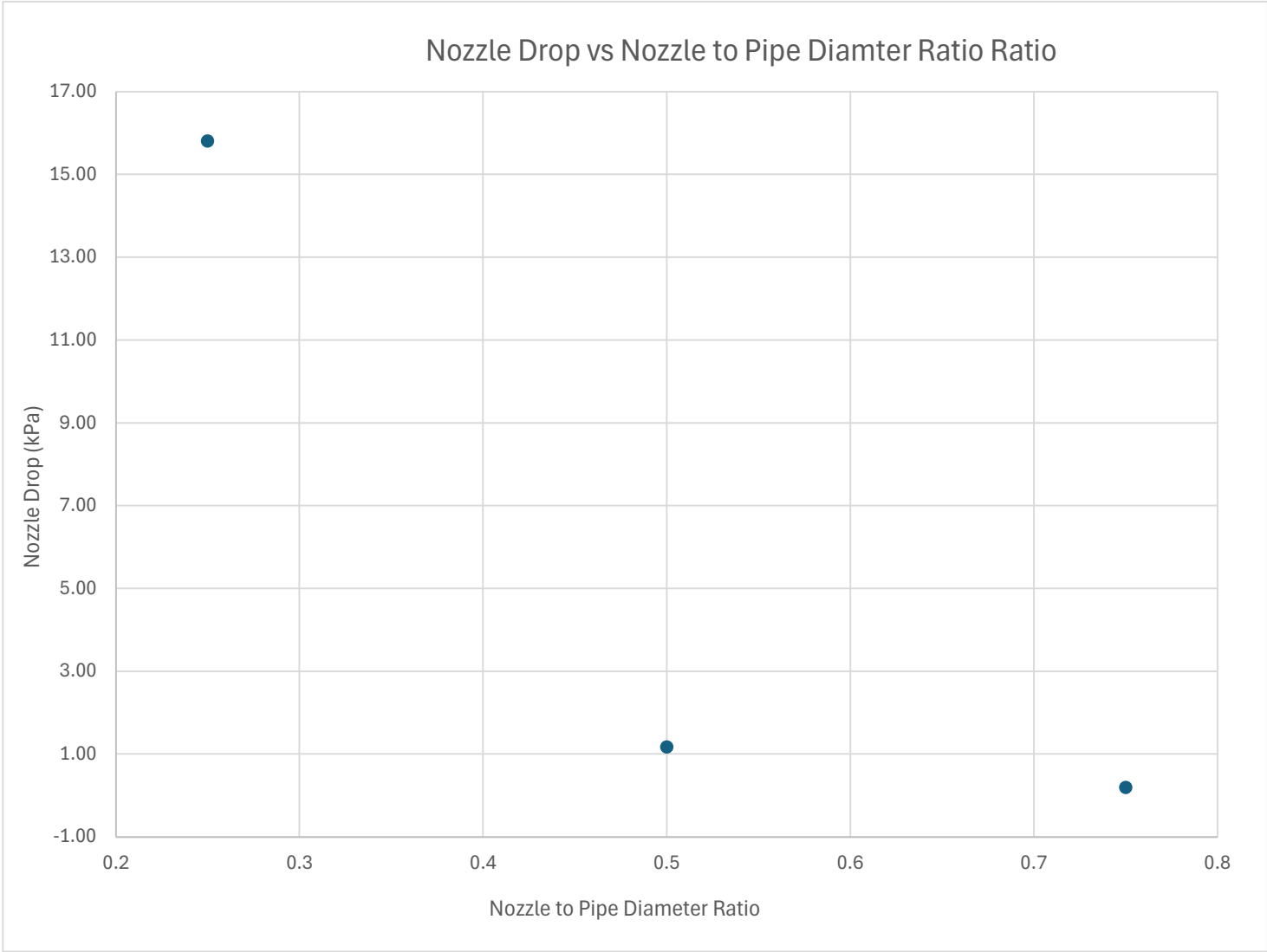
There is a nozzle drop situation.
Solving for ΔP_{loss} we used the Flow
nozzle equation $v = \frac{C \sqrt{2g(p_1 - p_2)/\gamma}}{\sqrt{(A_1/A_2)^2 - 1}}$.

By finding C and A_2 , the area of the inner
diameter of the nozzle, we could solve for
 $p_1 - p_2$. Comparing our figure C value of 0.9911
to $C \approx 1$ we could get a difference in
nozzle drop.

GIVEN FROM PART A:		
Q =	3.3870	ft ³ /s
γ =	62.4100	lb/ft ³
n - 60% =	0.6000	
V =	9.7552	ft/s
Flow Area of 8" Pipe =	0.3472	ft ²
Estimated Area =	0.3472	ft ²
g =	32.2000	ft/s ²
Suction Pipe =	11.0000	ft
Discharge Pipe =	2500.0000	ft
Total Pipe Length	2511.0000	ft
ID 8" Pipe =	0.6651	ft
v =	0.0000	ft ² /s
Epsilon =	0.0002	ft
Re =	536212.65	

B1			B2		
Diff. P =	66.1847	kPa	Ploss	1.1812	kPa
Ptotal =	67.3658	kPa			
C =	0.9912				
A1 =	0.3472	ft ²			
A2 =	0.0868	ft ²			
d =	0.3326	ft			
B =	0.5000				

Nozzle Ratio	C	A1	A2	d	B	Ptotal	Diff. P	Ploss
0.25	0.9930	0.3472	0.0217	0.1663	0.2500	1140.3251	1124.5098	15.8153
0.5	0.9912	0.3472	0.0869	0.3326	0.5000	67.3226	66.1422	1.1804
0.75	0.9898	0.3472	0.1954	0.4988	0.7500	9.7194	9.5217	0.1977



Part C: Make sure there is enough water in the channel so it doesn't dry out.

C-1) Compute flow rate in the channel

$$Q = \left(\frac{1.49}{n} \right) A R^{2/3} S^{1/2}$$

Need to find n , flow area (A) and hydraulic radius (R)

$n = 0.050$ table 14.1 for natural channel U/light
brush

$$A = WD + 2(XD/2) + WD + 2(XD/2)$$

$$\rightarrow (12 \times 4 + 2(\frac{1}{2} 4 \times 4)) + (40 \times 2 + 2(\frac{1}{2} 2 \times 2))$$

$$A = 64 \text{ ft}^2 + 84 \text{ ft}^2$$

$$A = 148 \text{ ft}^2$$

$$\rightarrow 5.657$$

$$WP = 2(L_1) + 2(10) + 2(L_2) + 12 \quad L_1 = \sqrt{4^2 + 4^2} = 5.657$$

$$2(5.657) + 2(10) + 2(2.828) + 12 \quad L_2 = \sqrt{2^2 + 2^2} = 2.828$$

$$WP = 48.97 \text{ ft}$$

$$R = \frac{A}{WP} \rightarrow \frac{148 \text{ ft}^2}{48.97 \text{ ft}}$$

$$R = 3.022 \text{ ft}$$

$$Q = \left(\frac{1.49}{0.05} \right) \times 148 \times 3.022^{2/3} \times 0.00015^{1/2}$$

$$Q = 112.9067 \text{ ft}^3/\text{s}$$

(C-2) % of pumped water flow compared to lower channel flow.

Q_1 is from statement problem.

$$Q_1 = 3.387 \text{ ft}^3/\text{s}$$

Q_2 is flow from section C or lower open channel flow.

$$Q_2 = 112.9067 \text{ ft}^3/\text{s}$$

$$\frac{Q_1}{Q_2} \times 100 = Q\% \text{ pumped}$$

$$\frac{3.387 \text{ ft}^3/\text{s}}{112.9067 \text{ ft}^3/\text{s}} \times 100 = 2.99 \text{ of pumped H}_2\text{O flow}$$

Analysis:

We wanted to make sure our channel did not dry out from our design. To do so we needed to find the flow rate of the lower channel. Then compare the % of water pumped out. To do so the channel shape was broken into two trapezoids which we found the Area of then added together. Then we found the Wet Perimeter.

When we divide the Area by the Wet Perimeter we got the hydraulic radius. Using eq. $\left(\frac{1.49}{n}\right) AR^{2/3} S^{1/2}$

we found the flow to be $Q = 112.9067 \text{ ft}^3/\text{s}$. We only pump 3% of the channel

GIVEN FROM PARTS A & B & C		
Q =	3.3870	ft ³ /s
γ =	62.4100	lb/ft ³
n - 60% =	0.6000	
V =	9.7552	ft/s
Flow Area of 8" Pipe =	0.3472	ft ²
Estimated Area =	0.3472	ft ²
g =	32.2000	ft/s ²
Suction Pipe =	11.0000	ft
Discharge Pipe =	2500.0000	ft
Total Pipe Length	2511.0000	ft
ID 8" Pipe =	0.6651	ft
v =	0.0000121	ft ² /s
Epsilon =	0.0002	ft
Re =	536212.65	
W =	12.0000	ft
D =	4.0000	ft
L1 =	5.6568542	ft
L2 =	2.8284271	ft
S =	0.00015	ft
n =	0.05	

C1			C2		
Q =	112.9123	ft ³ /s	Q1 =	3.387	ft ³ /s
A =	148.0000	ft ²	Q2 =	112.9123	ft ³ /s
WP =	48.9706	ft	%Q Pumped =	0.029997	%
R =	3.0222	ft			

Pipe Depth (ft)	A	L1	L2	WP	R	Qx	Q1	Q%
4	64	5.6569	0.0000	23.3137	2.7452	45.7953	3.387	7.3960
5	105	5.6569	1.4142	46.1421	2.2756	66.2998	3.387	5.1086
6	148	5.6569	2.8284	48.9706	3.0222	112.9123	3.387	2.9997
7	193	5.6569	4.2426	51.7990	3.7259	169.2946	3.387	2.0007
8	240	5.6569	5.6569	54.6274	4.3934	234.9670	3.387	1.4415

