Course: MET 330 Fluid Mechanics

Title: Exam #2

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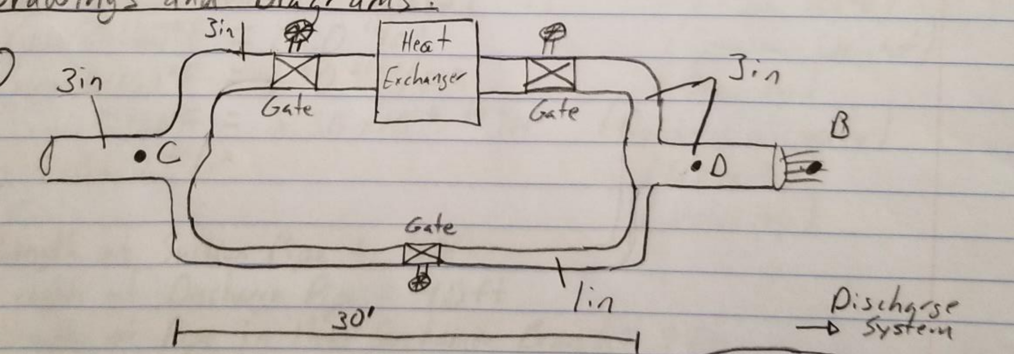
MET 330 Exam #2 Joey Cantrell

1. Purpose:

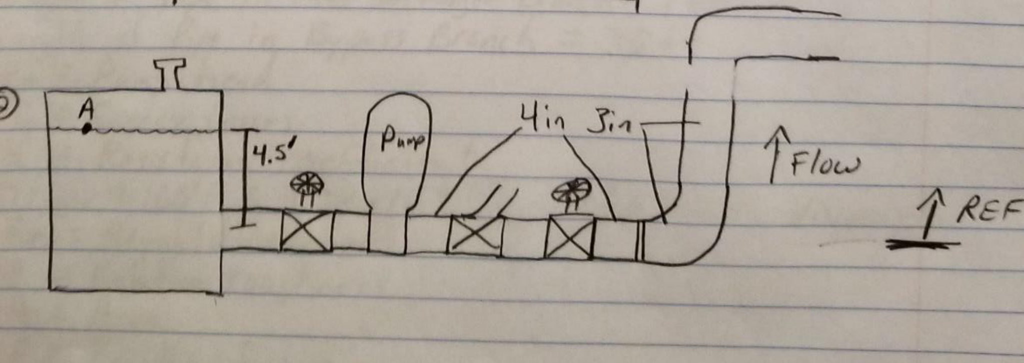
Determine the parameters associated with the system depicted in the figures in order to select the proper pump to use in the system.
① First, the flow rates in each branch of the discharge system using the Hardy-Cross method.
② Secondly, determine the pump head for the system.
③ Lastly, calculate the new total flow rate for the discharge system if it is operated with the gate valve of the bypass line closed to leave it only 1/4 open.

Drawings and Diagrams:

①



②



MET 330 Exam #2 — Joey Cantrell

Sources:

Mott, Robert L. and Untener, Joseph A. "Applied Fluid Mechanics", 7th edition Pearson Education, Inc. (2015)

Design Considerations:

- Incompressible fluid
- Steady state
- Isothermal process
- Schedule 40 steel pipe
- Water at 160°F
- Temperature remains constant throughout the system
- Angle of contraction of 30°

Data and Variables:

$Q_1 = 275\ \text{gal/min} = 0.6127\ \text{ft}^3/\text{s}$ (Flow Rate)

γ_{water} @ 160°F $= 61.0\ \text{lb/ft}^3$ (Specific Weight)

ρ_{water} @ 160°F $= 1.90\ \text{slugs/ft}^3$ (Density)

η_{water} @ 160°F $= 8.30 \times 10^{-6}\ \text{lb·s/ft}^2$ (Dynamic Viscosity)

$g = 32.2\ \text{ft/s}^2$ (Gravity)

V (Velocity)

Length of Suction Pipe = 10 ft

Length of Discharge Pipe = 40 ft

Length of Pipe in Heat Exchanger Branch = 8 ft

Length of Pipe in Bypass Branch = 30 ft

h_A = Pump head

h_L = Energy Losses

K = Resistance coefficient

ν_{water} @ 160°F $= 4.38 \times 10^{-6}\ \text{ft}^2/\text{s}$ (Kinematic Viscosity)

Re = Reynold's number

ε = Relative roughness

A = Area

f = Friction factors

$\theta = 30°$ for contraction

D = Diameter

MET 330 Exam #2

Joey Cantrell

Procedure:

① We will use the Hardy-Cross method to determine the flow rates in each branch. We must first designate the branches and note the direction of flow for each pipe. Then we assume a flow rate for each pipe by making sure that the flow in matches the flow out. We then must calculate the energy losses in each pipe using Reynolds number and the relative roughness to determine appropriate friction factors. We will set up a spreadsheet to perform the iterative calculations of Reynolds number, friction factors, k values, and kQ^2. Lastly, we compute ΔQ which is used to determine the next iteration of Q (flow rate) to iterate to find the solution ($Q^{new} = Q^{old} - \Delta Q$). We calculate all the values again and continue the iteration process until the ratio $\frac{\Delta Q}{Q^{new}} < 1\%$ to be considered negligible.

② In order to calculate the pump head, we must use Bernoulli's equation and the flow rate established from the previous part to calculate h_A and h_L.

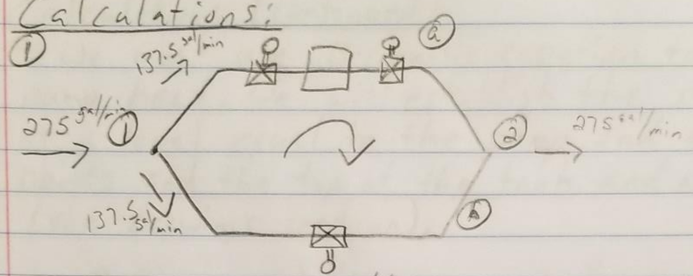
$$\frac{P_A}{\gamma} + z_A + \frac{V_A^2}{2g} + h_A - h_L = \frac{P_B}{\gamma} + z_B + \frac{V_B^2}{2g} \quad \text{(Bernoulli's equation)}$$

We will also use Reynold's number and relative roughness to determine the friction factors.

③ Using the value for pump head (h_A) obtained from the previous part, we must use the new value for the partially closed value to calculate a new flow rate for the Bypass branch. A gate valve that is 1/4 open changes the equivalent length from 8 to 900. The values for the flow rates in the branches will now be unknown, so we must use iterative techniques in Excel to calculate new values.

MET 330 Exam #2

Calculations:



Assume $Q_a = 137.5\ \text{gal/min} = 0.3064\ \text{ft}^3/\text{s}$

$Q_b = -137.5\ \text{gal/min} = -0.3064\ \text{ft}^3/\text{s}$

Now we plug values into Excel to perform iterations.

a) For the heat exchanger, since $K = 12$ and we know

$K = \frac{L_e}{D} f_T \rightarrow 12 = \frac{L_e}{0.25567\text{ft}}(0.017) \rightarrow L_e = 180.5\ \text{ft}$

For the elbows, $30 = \frac{L_e}{0.25567\text{ft}}(0.017) \rightarrow L_e = 451.2\text{ft}$

For the Tee, $60 = \frac{L_e}{0.25567\text{ft}}(0.017) \rightarrow L_e = 902.4\text{ft}$

For the gate valves, $8 = \frac{L_e}{0.25567\text{ft}}(0.017) \rightarrow L_e = 120.3\ \text{ft}$

b) For the gate valve, $8 = \frac{L_e}{0.08742\text{ft}}(0.022) \rightarrow L_e = 31.8\text{ft}$

For the elbows, $30 = \frac{L_e}{0.08742\text{ft}}(0.022) \rightarrow L_e = 119.2\text{ft}$

For the Tee, $60 = \frac{L_e}{0.08742\text{ft}}(0.022) \rightarrow L_e = 238.4\text{ft}$

For the contraction, $\frac{D_1}{D_2} = 3$ so from Figure 10.11, assuming $\theta = 30°$ we get $K = 0.05 \rightarrow 0.05 = \frac{L_e}{0.08742\text{ft}}(0.022)$

$\rightarrow L_e = 0.199\text{ft}$

So Branch a) $\Sigma L_e = 3128.3\ \text{ft}$ and Branch b) $\Sigma L_e = 747.2\text{ft}$

Then, using Excel with the Hardy-Cross method we get:

$$\boxed{\begin{aligned} Q_a &= 0.5468\ \text{ft}^3/\text{s} = 245.4\ \text{gal/min} \\ Q_b &= -0.0659\ \text{ft}^3/\text{s} = -29.6\ \text{gal/min} \end{aligned}}$$

Iteration	0															
Circuit	Pipe	Q (gal/mi	Q (ft3/s)	L (ft)	Le	D (ft)	e (ft)	Re	e/D	f	k	kQ	kQ^2			
I	a	137.5	0.306360	8	3128.3	0.25567	0.00015	348328.4	0.000587	0.018639	1347.028	412.6762	126.4276			-48.48%
	b	-137.5	-0.306360	30	747.2	0.08742	0.00015	1018727	0.001716	0.022775	87270.18	26736.13	-8190.89			48.48%
											SUM	54297.61	-8064.46	delta Q	-0.14852	
Iteration	1															
Circuit	Pipe	Q (gal/mi	Q (ft3/s)	L (ft)	Le	D (ft)	e (ft)	Re	e/D	f	k	kQ	kQ^2			
I	a		0.454884	8	3128.3	0.25567	0.00015	517197.8	0.000587	0.01827	1320.37	600.6149	273.21			-14.56%
	b		-0.157837	30	747.2	0.08742	0.00015	524848.7	0.001716	0.022997	88123.85	13909.21	-2195.39			41.97%
											SUM	29019.65	-1922.18	delta Q	-0.06624	
Iteration	2															
Circuit	Pipe	Q (gal/mi	Q (ft3/s)	L (ft)	Le	D (ft)	e (ft)	Re	e/D	f	k	kQ	kQ^2			
I	a		0.521121	8	3128.3	0.25567	0.00015	592508.7	0.000587	0.018167	1312.926	684.193	356.5473			-4.25%
	b		-0.091600	30	747.2	0.08742	0.00015	304593.2	0.001716	0.023303	89292.78	8179.21	-749.215			24.18%
											SUM	17726.81	-392.668	delta Q	-0.02215	
Iteration	3															
Circuit	Pipe	Q (gal/mi	Q (ft3/s)	L (ft)	Le	D (ft)	e (ft)	Re	e/D	f	k	kQ	kQ^2			
I	a		0.543272	8	3128.3	0.25567	0.00015	617694.2	0.000587	0.018138	1310.801	712.1214	386.8756			-0.63%
	b		-0.069449	30	747.2	0.08742	0.00015	230935.2	0.001716	0.02352	90125.45	6259.108	-434.688			4.94%
											SUM	13942.46	-47.8122	delta Q	-0.00343	
Iteration	4															
Circuit	Pipe	Q (gal/mi	Q (ft3/s)	L (ft)	Le	D (ft)	e (ft)	Re	e/D	f	k	kQ	kQ^2			
I	a		0.546701	8	3128.3	0.25567	0.00015	621593.2	0.000587	0.018133	1310.486	716.4442	391.6809			-0.03%
	b		-0.066020	30	747.2	0.08742	0.00015	219532.1	0.001716	0.023565	90299.29	5961.522	-393.577			0.22%
											SUM	13355.93	-1.89634	delta Q	-0.00014	
Iteration	5															
Circuit	Pipe	Q (gal/mi	Q (ft3/s)	L (ft)	Le	D (ft)	e (ft)	Re	e/D	f	k	kQ	kQ^2			
I	a		0.546843	8	3128.3	0.25567	0.00015	621754.7	0.000587	0.018133	1310.473	716.6232	391.8805			0.00%
	b		-0.065878	30	747.2	0.08742	0.00015	219060	0.001716	0.023567	90306.84	5949.199	-391.919			0.00%
											SUM	13331.64	-0.03844	delta Q	-2.9E-06	

MET 330 Exam #2 Joey Cantrell

Calculations (continued):

③ We must use Bernoulli's equation to obtain the pump head. We will establish the reference point at a level equal to the pump and assign our points and the top of the tank and at the discharge (see Drawings section).

So $\cancel{\frac{p_A}{\gamma}}^{0} + z_A + \cancel{\frac{V_A^2}{2g}}^{0} + h_A - h_L = \cancel{\frac{p_B}{\gamma}}^{0} + z_B + \frac{V_B^2}{2g}$

$\rightarrow h_A = (z_B - z_A) + h_L + \frac{V_B^2}{2g}$

We know that $Q_{Total} = Q_a + Q_b = 0.5468\ ft^3/s + (-0.0659\ ft^3/s)$
$= 0.481\ ft^3/s$

Then $V_B = \frac{Q_{Tot}}{A} = \frac{0.481\ ft^3/s}{(\pi(0.25567ft)^2/4)} = 9.367\ ft/s$

$\rightarrow \frac{V_B^2}{2g} = \frac{(9.367\ ft/s)^2}{2(32.2\ ft/s^2)} = 1.362ft$

So $h_A = (15ft - 4.5ft) + h_L + 1.362ft$

$h_A = h_L + 11.862ft$

So $h_L = f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} + K_{ent}\frac{V_1^2}{2g} + 2K_{Gate}\frac{V_1^2}{2g} + K_{Check}\frac{V_1^2}{2g}$
$+ K_{Red}\frac{V_2^2}{2g} + K_{elb}\frac{V_2^2}{2g} + \text{Elevation} + K_{elb}\frac{V_2^2}{2g} + f_2\frac{L_2}{D_2}\frac{V_2^2}{2g}$
$+ 2K_{Tee}\frac{V_2^2}{2g} + 2K_{Gate}\frac{V_a^2}{2g} + K_{HE}\frac{V_a^2}{2g} + 2K_{elb}\frac{V_a^2}{2g} + f_a\frac{L_a}{D_a}\frac{V_a^2}{2g}$

So $V_1 = \frac{Q_1}{A_1} = \frac{0.6127\ ft^3/s}{(\pi(0.3355ft)^2/4)} = 6.931\ ft/s \rightarrow \frac{V_1^2}{2g} = 0.7459ft$

$V_2 = \frac{Q_1}{A_2} = \frac{0.6127\ ft^3/s}{(\pi(0.25567ft)^2/4)} = 11.934\ ft/s \rightarrow \frac{V_2^2}{2g} = 2.2115ft$

$V_a = \frac{Q_a}{A_a} = \frac{0.5468\ ft^3/s}{(\pi(0.25567ft)^2/4)} = 10.651\ ft/s \rightarrow \frac{V_a^2}{2g} = 1.7615ft$

For the 4in pipe from the tank,

$Re_1 = \frac{V_1 D_1}{\nu} = \frac{(6.931\ ft/s)(0.3355ft)}{4.38\times10^{-6}\ ft^2/s} = 530902$ $\quad \frac{D_1}{\varepsilon_1} = \frac{0.3355ft}{1.5\times10^{-4}ft} = 2236.7$

$Re_2 = \frac{V_2 D_2}{\nu} = \frac{(11.934\ ft/s)(0.25567ft)}{4.38\times10^{-6}\ ft^2/s} = 696613$ $\quad \frac{D_2}{\varepsilon_2} = \frac{0.25567ft}{1.5\times10^{-4}ft} = 1704.5$

$Re_a = \frac{V_a D_a}{\nu} = \frac{(10.651\ ft/s)(0.25567ft)}{4.38\times10^{-6}\ ft^2/s} = 621722$ $\quad \frac{D_a}{\varepsilon_a} = 1704.5$

MET 330 Exam #2 Joey Cantrell

Calculations (continued):

2

Then $f_1 = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\varepsilon)} + \frac{5.74}{Re^{0.9}}\right)\right]^2} = 0.017$

Similarly $f_2 = 0.018$ and $f_a = 0.018$

So $h_L = \left[(0.017)\left(\frac{10ft}{0.3355ft}\right) + 0.5 + 2(8) + 100\right](0.7459ft)$

$+ \underbrace{\left[0.14\right.}_{\text{Assumed Sudden Contraction}} + 2(30) + (0.018)\left(\frac{40ft}{0.25567ft}\right) + \left.2(60)\right](2.2115ft)$

Figure 10.8

$+\left[2(30) + 2(8) + 12 + (0.018)\left(\frac{8ft}{0.25567ft}\right)\right](1.7615ft)$

$+ 15ft$

Then $h_L = 87.28ft + 404.61ft + 156.00\ ft + 15ft$

$h_L = 662.89$

Then $h_A = 662.89ft + 11.862ft = \boxed{674.75ft}$

MET 330 Exam # 2

Calculations (continued):

(3) We will have to use Bernoulli's equation twice (once for each brach) to set up equations to solve for flow rates in each. Then we will use iteration to obtain the flow rates. (Refer to figure (1) in Drawings section for labeled points C and D).

So $\frac{P_C}{\gamma} + \cancel{Z_C}^{0} + \cancel{\frac{V_C^2}{2g}} + h_A - h_L = \frac{P_D}{\gamma} + \cancel{Z_D}^{0} + \cancel{\frac{V_D^2}{2g}}$

$\rightarrow \frac{\Delta P_{CD}}{\gamma} = h_{La} - h_A$ (I)

Similarly, $\frac{\Delta P_{CD}}{\gamma} = h_{Lb} - h_A$ (II)

And $Q_C = Q_a + Q_b$ (III) (3 equations, 3 unknowns)

For (I), $\frac{\Delta P_{CD}}{\gamma} = 2f_2 \cdot K_{Tee} \cdot \frac{8}{g\pi^2 D_2^4} Q_1^2 + \left[2K_{elb} + 2K_{Gate} + K_{HE} + f_a \frac{L_a}{D_a}\right] \frac{8Q_a^2}{\pi^2 g D_a^4} - h_A$

Substituting known values $\rightarrow$ $\frac{\Delta P_{CD}}{\gamma} = 2(0.018)(60)\left(\frac{8(0.6127\,ft^3/s)^2}{(32.2\,ft/s^2)\pi^2(0.25567\,ft)^4}\right) + \left[2(30) + 2(8) + 12 + f_a\left(\frac{8\,ft}{0.25567\,ft}\right)\right] \frac{8Q_a^2}{\pi^2(32.2\,ft/s^2)(0.25567\,ft)^4} - 674.75\,ft$

$\rightarrow \frac{\Delta P_{CD}}{\gamma} = [518.44 + 184.34 f_a]\, Q_a^2 - 669.97\,ft$

For (II), $\frac{\Delta P_{CD}}{\gamma} = 2f_2 K_{Tee} \cdot \frac{8Q_1^2}{g\pi^2 D_2^4} + \left[2K_{elbow} + K_{Gate} + K_{cont} + f_b\left(\frac{L_b}{D_b}\right)\right] \frac{8Q_b^2}{\pi^2 g D_b^4} - h_A$

$\rightarrow \frac{\Delta P_{CD}}{\gamma} = 2(0.018)(60)\left(\frac{8(0.6127\,ft^3/s)^2}{(32.2\,ft/s^2)\pi^2(0.25567\,ft)^4}\right) + \left[2(30) + 900 + 0.5 + f_b\left(\frac{30\,ft}{0.08742\,ft}\right)\right] \frac{8Q_b^2}{\pi^2(32.2\,ft/s^2)(0.08742\,ft)^4} - 674.75\,ft$

$\rightarrow \frac{\Delta P_{CD}}{\gamma} = [413989 + 147911 f_b]\, Q_b^2 - 669.97\,ft$

Using equations (I), (II), and (III) ($Q_a = Q_1 - Q_b$) in Excel to perform iterations, the values obtained for $Q_a = 0.5918\,ft^3/s$ and $Q_b = 0.0209\,ft^3/s$

So $Q_{Tot} = Q_a + Q_b = 0.6127\,ft^3/s$

Iteration	0						
Guess Qa (ft3/s)	Va (ft/s)	Rea	D/e	fa	delta P / gamma (ft)		
0.5	9.739139	568494.5	1704.467	0.018197	-539.521375		
	fb	Qb (ft3/s)	Vb (ft/s)	Reb	D/e	fb	%Diff
	0.015	0.017704	2.949533	58869.44	582.8	0.025659	-71.0625
	0.025659	0.01767	2.943962	58758.26	582.8	0.025664	-0.01832
	0.025664	0.01767	2.94396	58758.21	582.8	0.025664	-8.1E-06
	0.025664	0.01767	2.94396	58758.21	582.8	0.025664	-3.6E-09
	0.025664	0.01767	2.94396	58758.21	582.8	0.025664	-1.6E-12
	0.025664	0.01767	2.94396	58758.21	582.8	0.025664	0
Iteration	1		%Diff Qa	-19.0059		Qa	0.59503
Guess Qa (ft3/s)	Va (ft/s)	Rea	D/e	fa	delta P / gamma (ft)		
0.55	10.71305	625343.9	1704.467	0.018129	-512.1309614		
	fb	Qb (ft3/s)	Vb (ft/s)	Reb	D/e	fb	%Diff
	0.025	0.019439	3.238697	64640.85	582.8	0.025434	-1.73762
	0.025434	0.019438	3.238448	64635.88	582.8	0.025435	-0.0007
	0.025435	0.019438	3.238448	64635.88	582.8	0.025435	-2.9E-07
	0.025435	0.019438	3.238448	64635.88	582.8	0.025435	-1.2E-10
	0.025435	0.019438	3.238448	64635.88	582.8	0.025435	-2.7E-14
	0.025435	0.019438	3.238448	64635.88	582.8	0.025435	0
Iteration	2		%Diff Qa	-7.86584		Qa	0.593262
Guess Qa (ft3/s)	Va (ft/s)	Rea	D/e	fa	delta P / gamma (ft)		
0.57	11.10262	648083.7	1704.467	0.018105	-500.4444967		
	fb	Qb (ft3/s)	Vb (ft/s)	Reb	D/e	fb	%Diff
	0.025	0.020146	3.356454	66991.14	582.8	0.025352	-1.40934
	0.025352	0.020145	3.356244	66986.96	582.8	0.025352	-0.00056
	0.025352	0.020145	3.356244	66986.96	582.8	0.025352	-2.2E-07
	0.025352	0.020145	3.356244	66986.96	582.8	0.025352	-9E-11
	0.025352	0.020145	3.356244	66986.96	582.8	0.025352	-1.4E-14
	0.025352	0.020145	3.356244	66986.96	582.8	0.025352	0
Iteration	3		%Diff Qa	-3.95704		Qa	0.592555
Guess Qa (ft3/s)	Va (ft/s)	Rea	D/e	fa	delta P / gamma (ft)		
0.59	11.49218	670823.5	1704.467	0.018082	-488.3407186		
	fb	Qb (ft3/s)	Vb (ft/s)	Reb	D/e	fb	%Diff
	0.025	0.020853	3.47421	69341.43	582.8	0.025275	-1.10019
	0.025275	0.020852	3.474041	69338.05	582.8	0.025275	-0.00043
	0.025275	0.020852	3.474041	69338.05	582.8	0.025275	-1.7E-07
	0.025275	0.020852	3.474041	69338.05	582.8	0.025275	-6.6E-11
	0.025275	0.020852	3.474041	69338.05	582.8	0.025275	0
	0.025275	0.020852	3.474041	69338.05	582.8	0.025275	0
Iteration	4		%Diff Qa	-0.31323		Qa	0.591848
Guess Qa (ft3/s)	Va (ft/s)	Rea	D/e	fa	delta P / gamma (ft)		
0.592	11.53114	673097.4	1704.467	0.01808	-487.1073885		
	fb	Qb (ft3/s)	Vb (ft/s)	Reb	D/e	fb	%Diff
	0.025	0.020924	3.485986	69576.46	582.8	0.025268	-1.07026
	0.025268	0.020923	3.485821	69573.16	582.8	0.025268	-0.00041
	0.025268	0.020923	3.485821	69573.16	582.8	0.025268	-1.6E-07
	0.025268	0.020923	3.485821	69573.16	582.8	0.025268	-6.3E-11
	0.025268	0.020923	3.485821	69573.16	582.8	0.025268	-2.7E-14
	0.025268	0.020923	3.485821	69573.16	582.8	0.025268	0
Iteration	5		%Diff Qa	0.037608		Qa	0.591777
Guess Qa (ft3/s)	Va (ft/s)	Rea	D/e	fa	delta P / gamma (ft)		
0.5918	11.52725	672870	1704.467	0.01808	-487.2309093		
	fb	Qb (ft3/s)	Vb (ft/s)	Reb	D/e	fb	%Diff
	0.025	0.020917	3.484808	69552.95	582.8	0.025268	-1.07325
	0.025268	0.020916	3.484643	69549.65	582.8	0.025268	-0.00042
	0.025268	0.020916	3.484643	69549.65	582.8	0.025268	-1.6E-07
	0.025268	0.020916	3.484643	69549.65	582.8	0.025268	-6.4E-11
	0.025268	0.020916	3.484643	69549.65	582.8	0.025268	-1.4E-14
	0.025268	0.020916	3.484643	69549.65	582.8	0.025268	0
Iteration	6		%Diff Qa	0.002631		Qa	0.591784

Joey Cantrell

MET 330 Exam # 2

Summary and Analysis:

The system is very complex due to all of the features impacting the flow of the water throughout the entire system.

In order to determine the flow rates in each of the branches, it was assumed that the contraction was at a 30° angle in order to obtain a K value. We also implemented the Hardy-Cross method to determine the flow rates causing us to use equivalent length for the calculations of the flow rates. We determined the flow rates to be $Q_a = 0.5564\ ft^3/s$ (~250 gal/min) and $Q_b = -0.0564\ ft^3/s$ (~25 gal/min).

We were able to calculate the pump head by using all of the features that created energy losses after Bernoulli's equation was reduced to the remaining components. We had to break the system up into sections to separate the energy losses using the individual velocities from each section of the system.

To determine the new flow rates after the gate valve was partially closed in the bypass branch, we had to employ Bernoulli's equation. We were to reduce it down to pump head, energy losses, and the difference in the pressures. An iterative method was used in Excel to calculate the new flow rates.

The system will function as currently designed, however with all of the valves and the heat exchanger, there is concern about long term use of these components. These valves could create water hammer and potentially damage the pipes.

MET 330 Exam #2 Joey Cantrell

Materials:

- A tank open to the atmosphere to store the water
- 10 feet of Schedule 40 - 4in pipe
- 48 feet of Schedule 40 - 3in pipe
- 30 feet of Schedule 40 - 1in pipe
- 5 Gate valves
- 1 Check valve
- 1 Heat exchanger
- 2 - 1in elbows
- 4 - 3in elbows
- 2 T-junctions as specified
- 1 Pump capable of operating system