Course: MET 330 Fluid Mechanics

Title: Exam #1

Date: 10/27/2019

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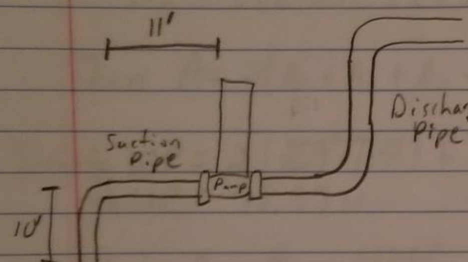
a. Purpose:

Find the proper horsepower of the pump needed to supply the required performance and determine the pipe diameter that will support the system.

Drawings and Diagrams:



Sources:

Mott, Robert L. and Untener, Joseph A. "Applied Fluid Mechanics", 7th edition Pearson Education, Inc. (2015)

Design Consideration:

1) Incompressible fluid
2) Isothermal process
3) Steady state
4) Steel pipe

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a. Data and Variables:

Density of water at 60°F - $\rho = 1.94 \frac{slugs}{ft^3}$

Suction pipe length = 11 ft

Discharge pipe length = 2500 ft

Velocity - $V \approx 3$ m/s

Head loss - h_L

Flow Rate - Q

Gravity - $g = 32.2$ ft/s²

Hydraulic Radius - R

Manning's n - $n = 0.050$ (Table 14.1)

Wetted Perimeter - WP

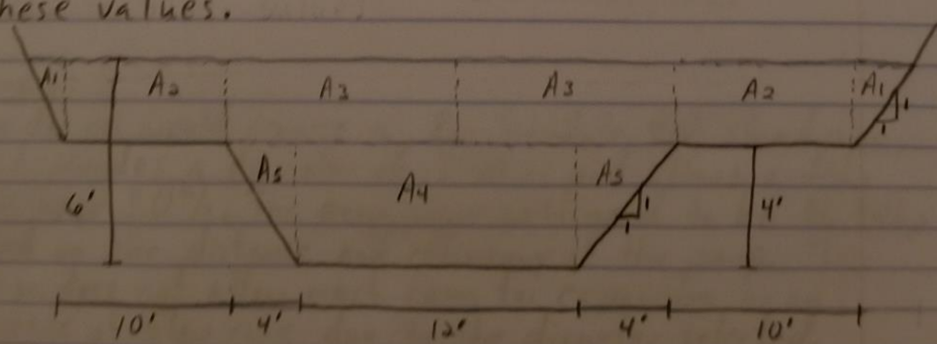
Slope - $S = 0.00015$

Procedure:

In order to calculate the power needed for the pump, we will use the equation $P = \rho g h_a Q$. This means we need to compute Reynold's number to obtain the head loss for the equation. First, we will have to calculate the flow rate in the lower channel to find the flow rate through the pipe. We will then select a pipe to use for the system that gets us near 3 m/s. Once we have the diameter, the remaining calculations will follow.

Calculations:

We know $Q = \frac{1.49}{n} A S^{1/2} R^{2/3}$ and $R = \frac{A}{WP}$. We will divide the channel into smaller pieces to calculate these values.



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a. Calculations (continued):

So $A_1 = \frac{1}{2}(b \times h) = \frac{1}{2}(2' \cdot 2') = 2\,ft^2$ (Since Δ is Δ)

$A_2 = L \times W = (10')(2') = 20\,ft^2$

$A_3 = L \times W = (10')(2') = 20\,ft^2$

$A_4 = L \times W = (12')(4') = 48\,ft^2$

$A_5 = \frac{1}{2}(b \times h) = \frac{1}{2}(4')(4') = 8\,ft^2$

Then $A = 2(A_1 + A_2 + A_3 + A_5) + A_4 = 148\,ft^2$

Similarly, $WP = 2\left(\sqrt{(2')^2+(2')^2}\right) + 2(10') + 2\left(\sqrt{(4')^2+(4')^2}\right) + 12'$

$\rightarrow WP = 48.97\,ft$

Then $R = \frac{A}{WP} = \frac{148\,ft^2}{48.97\,ft} = 3.022\,ft$

So $Q = \frac{1.49}{0.050}(148\,ft^2)(0.00015)^{1/2}(3.022\,ft)^{2/3}$

$\rightarrow Q = 112.91\,ft^3/s$

Then our flow rate needs to be $(0.03)(112.91\,ft^3/s) = 3.387\,ft^3/s$

In order to calculate the pipe diameter, we know

$Q = V \cdot A \rightarrow A = \frac{Q}{V} = \frac{3.387\,ft^3/s}{(3.0\,m/s)\left(\frac{3.28084\,ft}{1\,m}\right)}$ (Set $V = 3.0\,m/s$)

$\rightarrow A = 0.3441\,ft^2$

$\rightarrow \frac{\pi D^2}{4} = 0.3441\,ft^2$

$\rightarrow D^2 = 0.4382\,ft^2$

$\rightarrow D = 0.6619\,ft = 7.943\,in$

So we choose 8 in Schedule 40 pipe from Table F.1, which gives $V = \frac{Q}{A} = \frac{3.387\,ft^3/s}{(\pi(7.981\,in/(12\,in/ft))^2)/4} = 9.749\,ft/s = 2.972\,m/s$

8 in Schedule 40 steel pipe - $\boxed{D = 7.981\,in}$

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a. Calculations (continued):

To find h_L for the pump power, we need the Reynolds' number from $N_R = \frac{VD\rho}{\eta}$

So $N_R = \frac{(9.749\,ft/s)\left(\frac{7.981\,in}{12\,in}\cdot ft\right)(1.94\,slugs/ft^3)}{2.35\times10^{-5}\,lb\cdot s/ft^2} = 535266$

Then $\frac{D}{\varepsilon} = \frac{(7.981\,in/12\,in)(ft)}{1.5\times10^{-4}\,ft} = 4433.9$

So from Moody's Diagram (Figure 8.7) with $N_R = 535266$ and $\frac{D}{\varepsilon} = 4433.9$, we get a value for $f \doteq 0.016$

Then $h_L = f\cdot\frac{L}{D}\cdot\frac{V^2}{2g} = (0.016)\left(\frac{2500\,ft}{(7.981\,in/12\,in)(ft)}\right)\left(\frac{(9.749\,ft/s)^2}{2(32.2\,ft/s^2)}\right)$

$\rightarrow h_L = 88.76\,ft$

Using Bernoulli's to determine h_A.

$\cancel{\frac{p_1}{\gamma}} + z_1 + \cancel{\frac{V_1^2}{2g}} + h_A - h_L = \cancel{\frac{p_2}{\gamma}} + z_2 + \cancel{\frac{V_2^2}{2g}}$ (Since $P_1 = P_2$ + $V_1 = V_2$)

$\rightarrow h_A = h_L + z_2 - z_1 = 88.76\,ft + \left(10\,ft + 40\,ft - \frac{38\,in}{12\,in}(ft)\right) - 0$

$\rightarrow h_A = 135.59\,ft$

Finally, $P_A = (1.94\,\frac{slugs}{ft^3})(32.2\,ft/s^2)(135.59\,ft)(3.387\,ft^3/s)$

$\rightarrow P_A = 28688.7\,\frac{slugs\cdot ft^2}{s^3} = 28688.7\,\frac{lb\cdot ft}{s}\left(\frac{1\,hp}{550\,\frac{lb\cdot ft}{s}}\right) = 52.16\,hp$

Since the pump in 60% efficient we know

$e_M = \frac{P_A}{P_I} \rightarrow 0.60 = \frac{52.16\,hp}{P_I} \rightarrow \boxed{P_I = 86.94\,hp}$

Summary:

The design would require a 8in Schedule 40 steel pipe, which provides a diameter of 7.981 in and allows a flow rate near 3.0 m/s. The pump power will need to be 86.94hp based on the distance and efficiency of the pump. This design does not allow much room for expansion or an increase of flow rate due to the diameter selected.

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a. Materials:

- A pump that will allow 86.94 hp with 60% efficiency
- Approximately 2550 ft of 8in Schedule 40 steel pipe
- Other materials for remainder of design discussed in their respective sections.

Analysis:

The 8in Schedule 40 steel pipe will provide the required flow rate. However, there is little margin for increase in this system. The 86.94 hp pump should be replaced with a more efficient pump to reduce overall system cost. Also, relocating the upper channel closer to the lower channel will reduce the energy losses and increase the system performance overall.

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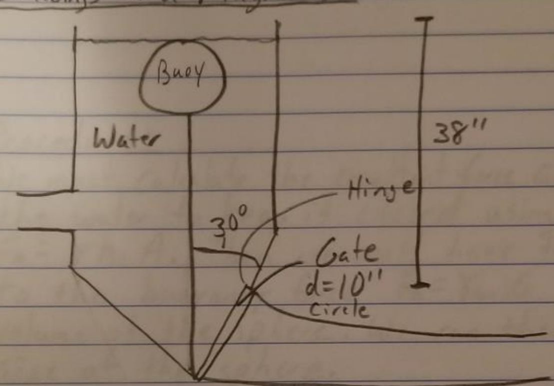
b. Purpose:

We need to determine the correct size of the buoy that will cause the gate to open at a certain water level. This will require buoyancy and stability calculations with the force needed to open the gate.

Drawings and Diagrams:



Sources:

· Mott, Robert L. and Untener, Joseph A. "Applied Fluid Mechanics", 7th edition Pearson Education, Inc. (2015)

Design Considerations:

1) Incompressible fluid
2) Isothermal process
3) Steady state
4) Neutral buoyancy
5) Neglect weight of gate and buoy

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b. Data and Variables:

Water Level - 38 in.

Diameter of gate - $D = 10\,in$

Angle is 30°

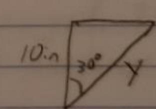
$\gamma_w = 62.4\ lb/ft^3$

Procedure:

We must calculate the resultant force acting the gate from the water to keep it closed using the equation $F_R = \gamma h_c A$. Then we will have to set that equal to the buoyancy force $F_b = \gamma_{H_2O} V_d$ to find the volume of the sphere. We can then determine the size of the sphere.

Calculations:

We must first find the distance to the centroid of the gate at the 30° angle.



So $y = \frac{D}{2} \cdot \cos(30°) = \frac{10in}{2} \cdot \cos(30°) = 4.33in$

Then $h_c = 38in + 4.33in = 42.33in$

So $L_c = \frac{h_c}{\cos(30°)} = \frac{42.33in}{\cos(30°)} = 48.88in$

We know $A = \frac{\pi D^2}{4} = \frac{\pi (10in)^2}{4} = 78.54in^2$

Then $F_R = \gamma h_c A = (62.4\ lb/ft^3)\left(\frac{42.33in}{12in} \cdot ft\right)\left(78.54in^2 \left(\frac{1ft^2}{12in^2}\right)\right)$

$\rightarrow F_R = 120.05\ lb$

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b. Calculations (continued):

From Appendix L, $I_c = \frac{\pi D^4}{64} = \frac{\pi (10in)^4}{64} = 490.87 in^4$

Then $L_p - L_c = \frac{I_c}{L_c A} = \frac{490.87 in^4}{(148.88 in)(78.54 in^2)} = 0.1279 in$

We know $\Sigma M = 0 \rightarrow F_R\left(\frac{D}{2} + (L_p - L_c)\right) - F_B\left(\frac{D}{2}\right) = 0$

$\rightarrow F_B(5in) = (120.05 lb)(5in + 0.1279 in)$

$\rightarrow F_B = 123.12 lb$

So we need to solve for V_d in buoyancy equation.

We have $123.12 lb = (62.4 \, lb/ft^3) V_d$

$\rightarrow V_d = 1.973 ft^3$

We know volume of a sphere is $V_d = \frac{4\pi r^3}{3}$

So $1.973 ft^3 = \frac{4\pi r^3}{3} \rightarrow r^3 = 0.4710 ft^3$

$\rightarrow r = 0.7781 ft = 9.337 in$

Then the buoy has a diameter of $\boxed{18.67 in}$

The sphere is stable due to the fact that it is spherical in shape and completely submerged. This causes the center of gravity and center of buoyancy coincident creating neutral stability.

Summary and Analysis:

The buoy needs to be a sphere with a diameter of 18.67in. to support the design. This design will hold assuming that the propeties of the water do not change dramatically.

Materials:

- 10in diameter gate
- 18.67in diameter spherical buoy

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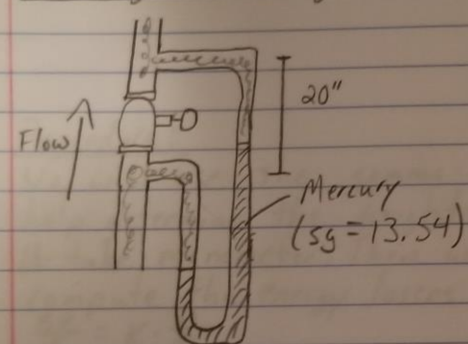
c. Purpose:

We must find the manometer reading that will tell the company whether the flow is maintaining a similar rate as the lower channel with no more than a 3% increase in flow through the valve.

Drawings and Diagrams:



Sources:

• Mott, Robert L. and Untener, Joseph A. "Applied Fluid Mechanics", 7th edition Pearson Education, Inc. (2015)

Design Considerations:

1) Incompressible fluid
2) Isothermal process
3) Steady state
4) Fluid in manometer is not moving

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C

Data and Variables:

Resistance coefficient - $K = 5.30$

Distance between U-tube manometer junctions - 20 in

Specific Gravity of Mercury = 13.54

$\gamma_{H_2O} = 62.4\ lb/ft^3$

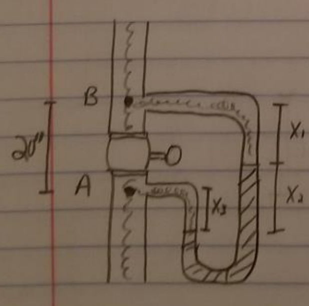
Gravity - $g = 32.2\ ft/s^2$

Velocity - $V = 9.749\ ft/s$

Procedure:

We will use the gamma-H equation $\Delta P = \gamma h$ to help determine the heights of the fluids in the U-tube manometer. Then we will be able to compute the energy losses per the equation given: $\frac{\Delta P}{\gamma} = K \cdot \frac{V^2}{2g}$.

Calculations:



We know $P_B + \gamma_w(x_1) + \gamma_{Hg}(x_2) - \gamma_w(x_3) = P_A$

$\rightarrow P_A - P_B = \gamma_w(x_1 - x_3) + \gamma_{Hg}(x_2)$

Using the energy loss equation:

$\frac{\Delta P}{\gamma_w} = K\frac{V^2}{2g} \rightarrow \Delta P = (62.4\ lb/ft^3)(5.30)\left(\frac{(9.749\ ft/s)^2}{2(32.2\ ft/s^2)}\right)$

$\rightarrow \Delta P = 488.08\ lb/ft^2$

$\rightarrow \Delta P = 3.3895$ psi

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C. Summary and Analysis:

The U-tube manometer allows us to calculate the difference in pressure from one side of the valve to the other. The value of $\Delta P = P_A - P_B$ is 3.3895 psi which tells us that the pressure before the valve is higher than the pressure after the valve. Thus, the valve does decrease the pressure due to the energy losses created by the valve.

Materials:

- A U-tube manometer to fit specifications
- A supply of Mercury to use inside the manometer

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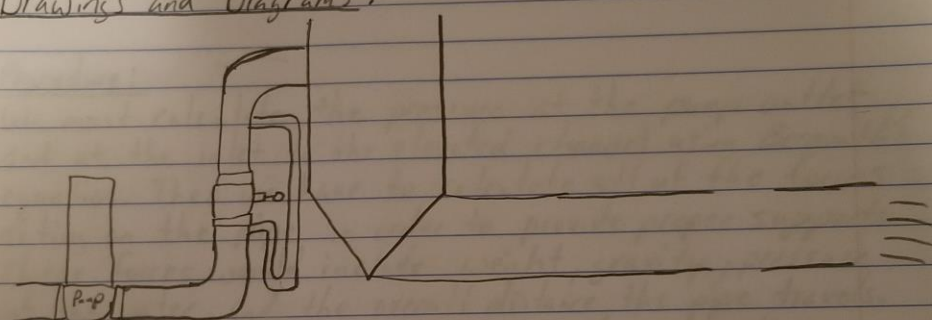
d Purpose:

We must calculate the forces acting on the pipe in order to determine the necessary supports needed to ensure that the pipe is off the ground and does not cause additional energy losses due to movement.

Drawings and Diagrams:



Sources:

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Design Considerations:

1) Incompressible fluid
2) Isothermal process
3) Steady state

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d. Data and Variables:

Length of discharge pipe - $L = 2500\ ft$

Gravity - $g = 32.2\ ft/s^2$

Pressure at upper channel - $P_2 = 0\ psi$

Velocity - $V = 9.749\ ft/s$

$h_A = 135.59 ft$

$h_L = 88.76 ft$

Procedure:

We must calculate the pressure at the pump outlet and at the inlet to the elevated channel using Bernoulli's equation. Then we have to calculate all of the forces acting on the pipe in order to provide proper supports. These forces would include weight, gravity, pressure of the water, and the overall distance the pipe travels. We use the sum of forces in x- and y-directions to obtain these forces.

Calculations:

We will use Bernoulli's equation to determine the pressures being applied to the elbows.

$$\frac{P_1}{\gamma} + \cancel{z_1}^{0} + \frac{V_1^2}{2g} + h_A - h_L = \cancel{\frac{P_2}{\gamma}}^{0} + z_2 + \frac{V_2^2}{2g}$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + h_A - h_L = z_2 + \frac{V_2^2}{2g}$$

$$\rightarrow P_1 = \gamma\left(z_2 + \frac{V_2^2 - V_1^2}{2g} - h_A + h_L\right)$$

$$\rightarrow P_1 = 62.4\ lb/ft^3\left(40ft + \frac{(9.749ft/s)^2}{2(32.2ft/s^2)} - 135.59ft + 88.76ft\right)$$

$$\rightarrow P_1 = -334.10\ lb/ft^2 = -2.32\ psi$$

Then $\Sigma F_x = 0 \rightarrow R_x - p_1 A_1 = 0$

$$\rightarrow R_x = p_1 A_1 = -2.32\ psi\left(\frac{\pi(7.891in)^2}{4}\right)$$

$$\rightarrow R_x = -113.47\ lb$$

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d Calculations (continued):

We know $\Sigma F_y = 0 \rightarrow R_y - W - p_2 A_2 = 0$
$\rightarrow R_y = W$

We know $W = 28.58\ lb/ft \cdot 2500ft = 71450\ lb$

Then $R_y = 71450\ lb$

Summary and Analysis:

The forces acting on the pipes in the horizontal and vertical directions are as follows: $R_x = -113.47\ lb$ and $R_y = 71450\ lb$. The force in the horizontal direction will not cause much problem since it is not terribly large. However, the civil engineers will have to plan for several sturdy supports to support the long discharge pipe over its entire length and at the elbows.

Materials:

No materials were used in this task. This task provides the civil engineers the needed forces to allow them to support the pipe system.

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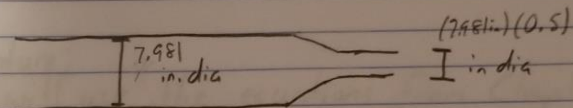
e Purpose:

We must determine how much difference in the pressure of the water before and after the nozzle to find out the drop in pressure.

Drawings and Diagrams:



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Design Considerations:

1) Incompressible fluid
2) Isothermal process
3) Steady state

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e. **Data and Variables:**

Nozzle to pipe diameter ratio - 0.5

Diameter of pipe - $D = 7.981 in$ (from a.)

$\gamma_{H_2O} = 62.4 \, lb/ft^3$

Gravity - $g = 32.2 \, ft/s^2$

Discharge Coefficient - C

Procedure:

We will use the equations from Chapter 15 on Flow Nozzles. The equation for V_1 can be rearranged to solve for the difference in pressure $(P_1 - P_2)$. We will also have to calculate the values for area of the pipe and nozzle, β, and Reynold's number.

Calculations:

We know the equation for Velocity $V_1 = C\sqrt{\frac{2g(P_1-P_2)/\gamma}{(A_1/A_2)^2-1}}$ can be rearranged to give:

$(P_1 - P_2) = \left(\frac{V_1}{C}\right)^2 \left(\frac{\gamma((A_1/A_2)^2-1)}{2g}\right)$ and $C = 0.9975 - 6.53\sqrt{\frac{\beta}{N_R}}$

So $\beta = \frac{d}{D} = 0.5$ from information given

Then $C = 0.9975 - 6.53\sqrt{\frac{0.5}{535266}}$ (N_R from part a.)

$\rightarrow C = 0.9911$

Then $A_1 = \frac{\pi(7.981in/12in(ft))^2}{4} = 0.3474 \, ft^2$ and $A_2 = \frac{\pi(3.9905in/12in(ft))^2}{4} = 0.0869 \, ft^2$

So $(P_1 - P_2) = \left(\frac{9.749 \, ft/s}{0.9911}\right)^2 \left(\frac{(62.4 \, lb/ft^3)[(0.3474ft^2/0.0869ft^2)^2-1]}{2(32.2 \, ft/s^2)}\right)$

$\rightarrow (P_1 - P_2) = 1404.57 \, lb/ft^2 \left(\frac{(ft)^2}{(12in)^2}\right) = \boxed{9.75 \, psi}$

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Summary:

The use of a flow nozzle that is one half the diameter of the pipe will cause a pressure drop across the nozzle of 9.75 psi. This is a fairly significant drop in pressure due to the restriction of the area by cutting the diameter in half.

Materials:

- A flow nozzle that is 3.9905 in in diameter.

Analysis:

The use of the flow nozzle will not drastically increase or impact the energy losses in the system. Their smooth design is purposeful in achieving this goal.

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f. Data and Variables:

Modulus elasticity of steel $= 200\ GPa$

Pipe diameter $- D = 7.981\ in$ (from a.)

Pipe Thickness $- \delta = 0.322\ in$

Correction factor $- Y = 0.40$ for steel

Bulk modulus of water $- E_o = 316000\ psi$

$\rho_w = 1.94\ slugs/ft^3$

Procedure:

We will calculate the change in pressure due to water hammer using the equation $\Delta P = \rho \cdot C \cdot V$. Then we will compare that to the maximum pressure the pipe we chose can withstand. In order to determine if cavitation exists, we must verify whether the operational pressure stays above the saturation pressure so that the liquid does not change phases.

Calculations:

We know that $C = \frac{\sqrt{E_o/\rho}}{\sqrt{1+E_o D/E\delta}}$ and $E_o = 316000\ psi$

So $\sqrt{E_o/\rho} = \sqrt{\frac{(316000\ lb/in^2)\left(\frac{12in}{1ft}\right)^2}{1.94\ slugs/ft^3}} = 4843.1$

And $\sqrt{1+\frac{E_o D}{E\delta}} = \sqrt{1+\frac{(316000\ lb/in^2)\left(\frac{12in}{1ft}\right)^2\left(\frac{7.981in}{12in}\right)(ft)}{(200\ GPa)\left(\frac{1000000000\ Pa}{1\ GPa}\right)\left(\frac{0.020885\ lb/ft^2}{1\ Pa}\right)\left(\frac{0.322in}{12in}\right)(ft)}} = 1.1256$

Then $C = \frac{4843.1}{1.1256} = 4302.7\ ft/s$

Then $\Delta P = (1.94\ slugs/ft^3)(4302.7\ ft)(9.749\ ft/s) = 81377.2\ lb/ft^2 = \boxed{565.1\ psi}$

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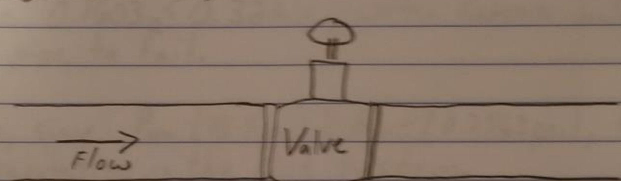
f. Purpose:
We must analyze the system to determine the pressure inside the pipe if the valve is suddenly closed. This will ensure that if water hammer or cavitation occurs then we would know if the pipes can withstand this pressure increase without failure.

Drawings and Diagrams:



Sources:

· Mott, Robert L. and Untener, Joseph A. "Applied Fluid Mechanics", 7th edition Pearson Education, Inc. (2015)

Design Considerations:
1) Incompressible fluid
2) Isothermal process
3) Steady state
4) Sudden valve closure

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f. Calculations (continued):

We need to know the pressure at the valve.

$P_v = P_s + \Delta P$

$\rightarrow P_v = \rho \cdot g \cdot h_L + \Delta P = (1.94\ \text{slugs}/\text{ft}^3)(32.2\ \text{ft}/\text{s}^2)(88.76\ \text{ft}) + 565\ \text{psi}$

$\rightarrow P_v = 603.6\ \text{psi}$

Using equation 11-9, we have

$$t = \frac{PD}{2(S \cdot E + PY)} = \frac{(603.6\ \text{psi})(8.625\ \text{in})}{2((16000\ \text{psi})(1) + (603.6\ \text{psi})(0.40))} = 0.1603\ \text{in}$$

Since $0.1603\ \text{in} < 0.322\ \text{in}$, water hammer will not cause the pipe to fail.

Also, since $P_{oper}\ (38.5\ \text{psi}) > P_{sat}\ (0.2563\ \text{psi})$, there is no cavitation in the pipe system.

Summary and Analysis:

Based on the calculations above, it would take a large change to the system design to cause problems with water hammer or cavitation. The pipe selected seems very robust against these dangers in conjunction with this system.

Materials:

No additional materials were added in this task.

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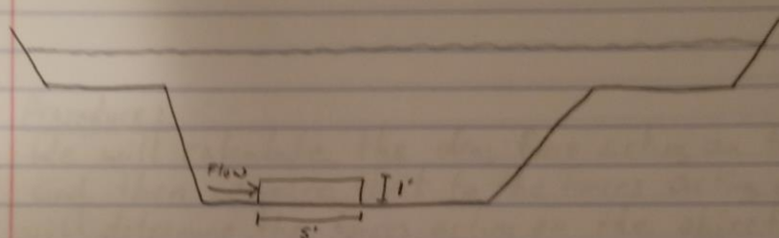
3 Purpose:

We must determine the maximum weight of an object on the bottom of the channel such that the water flow will be able to slide it along the bottom surface.

Drawings and Diagrams:



Sources:

- Mott, Robert L. and Untener, Joseph A. "Applied Fluid Mechanics", 7th edition Pearson Education, Inc. (2015)

Design Considerations:

1) Incompressible fluid
2) Isothermal process
3) Steady state
4) Object does not tumble

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g

Data and Variables:

Coefficient of Friction - $\mu = 0.60$

Object is square - 5' x 5' x 1'

Gravity - $g = 32.2\ ft/s^2$

$\gamma_{H_2O} = 62.4\ lb/ft^3$

$\rho_{H_2O} = 1.94\ slugs/ft^3$

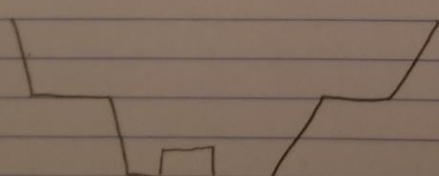
Water @ 60°F

Velocity - $V = 9.749\ ft/s$

Procedure:

We will calculate the drag force acting on the cylinder and then compare that to the forces acting on the cylinder. will determine the forces acting on the object such as weight, gravity, friction, and the normal force to see what is the weight that would be the maximum to still allow the water to slide it across the bottom.

Calculations:



We know $A = (5')(5') = 25ft^2$

We know $N_R = 535266$

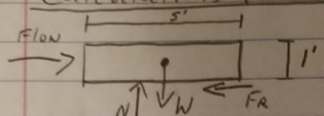
Then $C_D = 0.26$ (Figure 17.4)

Then $F_D = C_D\left(\frac{\rho V^2}{2}\right)A = (0.26)\left(\frac{(1.94\ slugs/ft^3)(9.749\ ft/s)^2}{2}\right)(25ft^2)$

$\rightarrow F_D = 599.2\ lb$

MET 330 Exam #1 Joey Cantrell

g Calculations (continued):



We know $\Sigma F_x = 0 \rightarrow F_D - F_R = 0 \rightarrow F_D = F_R = \mu N$

Then $599.2 lb = 0.60 (N)$

$\rightarrow \quad N = 998.7 lb$

Then $\Sigma F_y = 0 \rightarrow N - W = 0 \rightarrow N = W = \boxed{998.7 lb}$

Summary and Analysis:

The object can weigh as much as 998.7 lb before it becomes too heavy to move along the bottom based on the current flow rate. Obviously, if the flow rate changes in either direction, there is a direct impact to this calculation.

Materials:

- A square, cylindrical block that weighs no more than 998.7 lb.