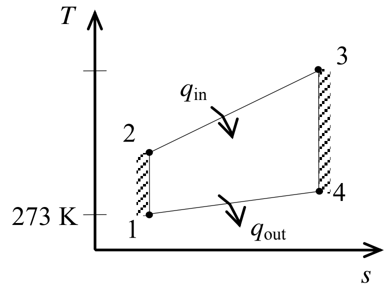
9-88 An aircraft engine operates as a simple ideal Brayton cycle with air as the working fluid. The pressure ratio and the rate of heat input are given. The net power and the thermal efficiency are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are c_p = 1.005 kJ/kg·K and k = 1.4 (Table A-2a).

Analysis For the isentropic compression process,

$$T_2 = T_1 r_p^{(k-1)/k} = (273\text{ K})(10)^{0.4/1.4} = 527.1\text{ K}$$



The heat addition is

$$q_{\text{in}} = \frac{\dot{Q}_{\text{in}}}{\dot{m}} = \frac{500\text{ kW}}{1\text{ kg/s}} = 500\text{ kJ/kg}$$

Applying the first law to the heat addition process,

$$q_{\text{in}} = c_p(T_3 - T_2)$$

$$T_3 = T_2 + \frac{q_{\text{in}}}{c_p} = 527.1\text{ K} + \frac{500\text{ kJ/kg}}{1.005\text{ kJ/kg}\cdot\text{K}} = 1025\text{ K}$$

The temperature at the exit of the turbine is

$$T_4 = T_3\left(\frac{1}{r_p}\right)^{(k-1)/k} = (1025\text{ K})\left(\frac{1}{10}\right)^{0.4/1.4} = 530.9\text{ K}$$

Applying the first law to the adiabatic turbine and the compressor produce

$$w_{\text{T}} = c_p(T_3 - T_4) = (1.005\text{ kJ/kg}\cdot\text{K})(1025 - 530.9)\text{K} = 496.6\text{ kJ/kg}$$

$$w_{\text{C}} = c_p(T_2 - T_1) = (1.005\text{ kJ/kg}\cdot\text{K})(527.1 - 273)\text{K} = 255.4\text{ kJ/kg}$$

The net power produced by the engine is then

$$\dot{W}_{\text{net}} = \dot{m}(w_{\text{T}} - w_{\text{C}}) = (1\text{ kg/s})(496.6 - 255.4)\text{kJ/kg} = \mathbf{241.2 kW}$$

Finally the thermal efficiency is

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{241.2\text{ kW}}{500\text{ kW}} = \mathbf{0.482}$$

9-99 A Brayton cycle with regeneration produces 115 kW power. The rates of heat addition and rejection are to be determined.

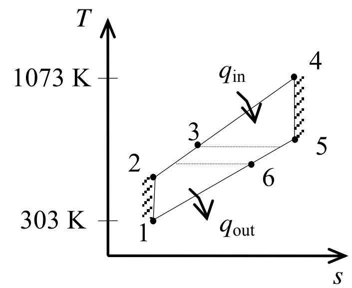
Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with constant specific heats at room temperature. **3** Kinetic and potential energy changes are negligible.

Properties The properties of air at room temperature are c_p = 1.005 kJ/kg.K and k = 1.4 (Table A-2a).

Analysis According to the isentropic process expressions for an ideal gas,

$$T_2 = T_1 r_p^{(k-1)/k} = (303\text{ K})(10)^{0.4/1.4} = 585.0\text{ K}$$

$$T_5 = T_4 \left(\frac{1}{r_p}\right)^{(k-1)/k} = (1073\text{ K})\left(\frac{1}{10}\right)^{0.4/1.4} = 555.8\text{ K}$$



When the first law is applied to the heat exchanger, the result is

$$T_3 - T_2 = T_5 - T_6$$

while the regenerator temperature specification gives

$$T_3 = T_5 - 10 = 555.8 - 10 = 545.8\text{ K}$$

The simultaneous solution of these two results gives

$$T_6 = T_5 - (T_3 - T_2) = 555.8 - (545.8 - 585.0) = 595.0\text{ K}$$

Application of the first law to the turbine and compressor gives

$$\begin{aligned} w_{\text{net}} &= c_p(T_4 - T_5) - c_p(T_2 - T_1) \\ &= (1.005\text{ kJ/kg}\cdot\text{K})(1073 - 555.8)\text{ K} - (1.005\text{ kJ/kg}\cdot\text{K})(585.0 - 303)\text{ K} \\ &= 236.4\text{ kJ/kg} \end{aligned}$$

Then,

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{115\text{ kW}}{236.4\text{ kJ/kg}} = 0.4864\text{ kg/s}$$

Applying the first law to the combustion chamber produces

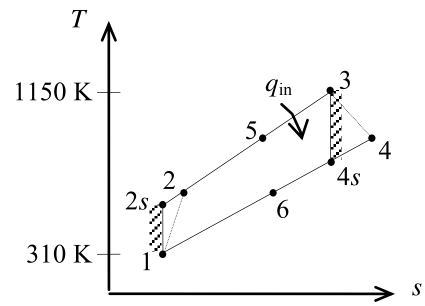
$$\dot{Q}_{\text{in}} = \dot{m}c_p(T_4 - T_3) = (0.4864\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(1073 - 545.8)\text{K} = \mathbf{258\ kW}$$

Similarly,

$$\dot{Q}_{\text{out}} = \dot{m}c_p(T_6 - T_1) = (0.4864\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(595.0 - 303)\text{K} = \mathbf{143\ kW}$$

9-107 A Brayton cycle with regeneration using air as the working fluid is considered. The air temperature at the turbine exit, the net work output, and the thermal efficiency are to be determined.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with variable specific heats. **3** Kinetic and potential energy changes are negligible.



Properties The properties of air are given in Table A-17.

Analysis (a) The properties of air at various states are

$$T_1 = 310 \text{ K} \longrightarrow \begin{array}{l} h_1 = 310.24 \text{ kJ/kg} \\ P_{r_1} = 1.5546 \end{array}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (7)(1.5546) = 10.88 \longrightarrow h_{2s} = 541.26 \text{ kJ/kg}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} \longrightarrow h_2 = h_1 + (h_{2s} - h_1)/\eta_C = 310.24 + (541.26 - 310.24)/(0.75) = 618.26 \text{ kJ/kg}$$

$$T_3 = 1150 \text{ K} \longrightarrow \begin{array}{l} h_3 = 1219.25 \text{ kJ/kg} \\ P_{r_3} = 200.15 \end{array}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{7}\right)(200.15) = 28.59 \longrightarrow h_{4s} = 711.80 \text{ kJ/kg}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow h_4 = h_3 - \eta_T (h_3 - h_{4s}) = 1219.25 - (0.82)(1219.25 - 711.80) = 803.14 \text{ kJ/kg}$$

Thus,

$$T_4 = \mathbf{782.8 \text{ K}}$$

(*b*)
$$\begin{aligned} w_{\text{net}} = w_{\text{T,out}} - w_{\text{C,in}} &= (h_3 - h_4) - (h_2 - h_1) \\ &= (1219.25 - 803.14) - (618.26 - 310.24) \\ &= \mathbf{108.09 \ kJ/kg} \end{aligned}$$

(*c*)
$$\varepsilon = \frac{h_5 - h_2}{h_4 - h_2} \longrightarrow \begin{aligned} h_5 &= h_2 + \varepsilon (h_4 - h_2) \\ &= 618.26 + (0.65)(803.14 - 618.26) \\ &= 738.43 \text{ kJ/kg} \end{aligned}$$

Then,

$$q_{\text{in}} = h_3 - h_5 = 1219.25 - 738.43 = 480.82 \text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{108.09 \text{ kJ/kg}}{480.82 \text{ kJ/kg}} = \mathbf{22.5\%}$$

9-119 A regenerative gas-turbine cycle with two stages of compression and two stages of expansion is considered. The minimum mass flow rate of air needed to develop a specified net power output is to be determined.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with variable specific heats. **3** Kinetic and potential energy changes are negligible.

Properties The properties of air are given in Table A-17.

Analysis The mass flow rate will be a minimum when the cycle is ideal. That is, the turbine and the compressors are isentropic, the regenerator has an effectiveness of 100%, and the compression ratios across each compression or expansion stage are identical. In our case it is $r_p = \sqrt{9} = 3$. Then the work inputs to each stage of compressor are identical, so are the work outputs of each stage of the turbine.

$$T_1 = 300 \text{ K} \longrightarrow h_1 = 300.19 \text{ kJ/kg}, \quad P_{r_1} = 1.386$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (3)(1.386) = 4.158 \longrightarrow h_2 = h_4 = 411.26 \text{ kJ/kg}$$

$$T_5 = 1200 \text{ K} \longrightarrow h_5 = h_7 = 1277.79 \text{ kJ/kg}, \quad P_{r_5} = 238$$

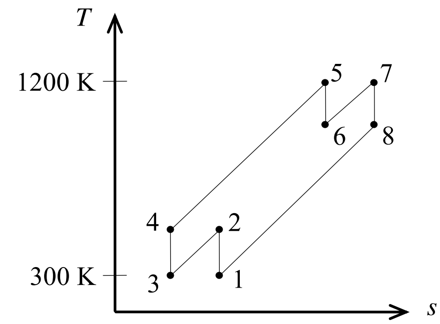
$$P_{r_6} = \frac{P_6}{P_5} P_{r_5} = \left(\frac{1}{3}\right)(238) = 79.33 \longrightarrow h_6 = h_8 = 946.36 \text{ kJ/kg}$$

$$w_{\text{C,in}} = 2(h_2 - h_1) = 2(411.26 - 300.19) = 222.14 \text{ kJ/kg}$$

$$w_{\text{T,out}} = 2(h_5 - h_6) = 2(1277.79 - 946.36) = 662.86 \text{ kJ/kg}$$

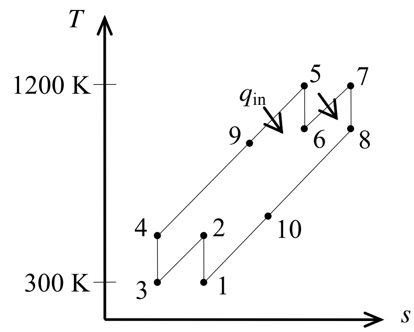
$$w_{\text{net}} = w_{\text{T,out}} - w_{\text{C,in}} = 662.86 - 222.14 = 440.72 \text{ kJ/kg}$$

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{110{,}000 \text{ kJ/s}}{440.72 \text{ kJ/kg}} = \mathbf{249.6\ kg/s}$$



9-121 An ideal gas-turbine cycle with two stages of compression and two stages of expansion is considered. The back work ratio and the thermal efficiency of the cycle are to be determined for the cases of with and without a regenerator.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with variable specific heats. **3** Kinetic and potential energy changes are negligible.



Properties The properties of air are given in Table A-17.

Analysis (*a*) The work inputs to each stage of compressor are identical, so are the work outputs of each stage of the turbine since this is an ideal cycle. Then,

$$T_1 = 300 \text{ K} \longrightarrow \begin{matrix} h_1 = 300.19 \text{ kJ/kg} \\ P_{r_1} = 1.386 \end{matrix}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (3)(1.386) = 4.158 \longrightarrow h_2 = h_4 = 411.26 \text{ kJ/kg}$$

$$T_5 = 1200 \text{ K} \longrightarrow \begin{matrix} h_5 = h_7 = 1277.79 \text{ kJ/kg} \\ P_{r_5} = 238 \end{matrix}$$

$$P_{r_6} = \frac{P_6}{P_5} P_{r_5} = \left(\frac{1}{3}\right)(238) = 79.33 \longrightarrow h_6 = h_8 = 946.36 \text{ kJ/kg}$$

$$w_{\text{C,in}} = 2(h_2 - h_1) = 2(411.26 - 300.19) = 222.14 \text{ kJ/kg}$$

$$w_{\text{T,out}} = 2(h_5 - h_6) = 2(1277.79 - 946.36) = 662.86 \text{ kJ/kg}$$

Thus,

$$r_{\text{bw}} = \frac{w_{\text{C,in}}}{w_{\text{T,out}}} = \frac{222.14 \text{ kJ/kg}}{662.86 \text{ kJ/kg}} = \mathbf{33.5\%}$$

$$q_{\text{in}} = (h_5 - h_4) + (h_7 - h_6) = (1277.79 - 411.26) + (1277.79 - 946.36) = 1197.96 \text{ kJ/kg}$$

$$w_{\text{net}} = w_{\text{T,out}} - w_{\text{C,in}} = 662.86 - 222.14 = 440.72 \text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{440.72 \text{ kJ/kg}}{1197.96 \text{ kJ/kg}} = \mathbf{36.8\%}$$

(*b*) When a regenerator is used, r_{bw} remains the same. The thermal efficiency in this case becomes

$$q_{\text{regen}} = \varepsilon(h_8 - h_4) = (0.75)(946.36 - 411.26) = 401.33 \text{ kJ/kg}$$

$$q_{\text{in}} = q_{\text{in,old}} - q_{\text{regen}} = 1197.96 - 401.33 = 796.63 \text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{440.72 \text{ kJ/kg}}{796.63 \text{ kJ/kg}} = \mathbf{55.3\%}$$