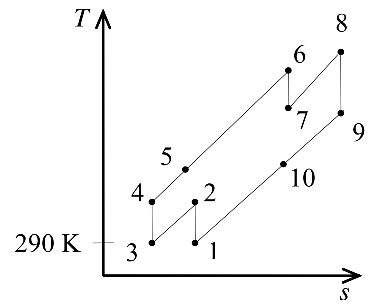
9-123 A regenerative gas-turbine cycle with two stages of compression and two stages of expansion is considered. The thermal efficiency of the cycle is to be determined.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with constant specific heats at room temperature. **3** Kinetic and potential energy changes are negligible.

Properties The properties of air at room temperature are c_p = 1.005 kJ/kg·K and k = 1.4 (Table A-2a).

Analysis The temperatures at various states are obtained as follows

$$T_2 = T_4 = T_1 r_p^{(k-1)/k} = (290\text{ K})(4)^{0.4/1.4} = 430.9\text{ K}$$



$$T_5 = T_4 + 20 = 430.9 + 20 = 450.9\text{ K}$$

$$q_{\text{in}} = c_p(T_6 - T_5)$$

$$T_6 = T_5 + \frac{q_{\text{in}}}{c_p} = 450.9\text{ K} + \frac{300\text{ kJ/kg}}{1.005\text{ kJ/kg}\cdot\text{K}} = 749.4\text{ K}$$

$$T_7 = T_6\left(\frac{1}{r_p}\right)^{(k-1)/k} = (749.4\text{ K})\left(\frac{1}{4}\right)^{0.4/1.4} = 504.3\text{ K}$$

$$T_8 = T_7 + \frac{q_{\text{in}}}{c_p} = 504.3\text{ K} + \frac{300\text{ kJ/kg}}{1.005\text{ kJ/kg}\cdot\text{K}} = 802.8\text{ K}$$

$$T_9 = T_8\left(\frac{1}{r_p}\right)^{(k-1)/k} = (802.8\text{ K})\left(\frac{1}{4}\right)^{0.4/1.4} = 540.2\text{ K}$$

$$T_{10} = T_9 - 20 = 540.2 - 20 = 520.2\text{ K}$$

The heat input is

$$q_{\text{in}} = 300 + 300 = 600\text{ kJ/kg}$$

The heat rejected is

$$\begin{aligned} q_{\text{out}} &= c_p(T_{10} - T_1) + c_p(T_2 - T_3) \\ &= (1.005\text{ kJ/kg}\cdot\text{K})(520.2 - 290 + 430.9 - 290)\text{ R} \\ &= 373.0\text{ kJ/kg} \end{aligned}$$

The thermal efficiency of the cycle is then

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{373.0}{600} = \mathbf{0.378}$$

9-129E A turbojet engine operating on an ideal cycle is flying at an altitude of 20,000 ft. The pressure at the turbine exit, the velocity of the exhaust gases, and the propulsive efficiency are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The air standard assumptions are applicable. **3** Air is an ideal gas with constant specific heats at room temperature. **4** Kinetic and potential energies are negligible, except at the diffuser inlet and the nozzle exit. **5** The turbine work output is equal to the compressor work input.

Properties The properties of air at room temperature are c_p = 0.24 Btu/lbm.R and k = 1.4 (Table A-2Ea).

Analysis (*a*) For convenience, we assume the aircraft is stationary and the air is moving towards the aircraft at a velocity of V_1 = 900 ft/s. Ideally, the air will leave the diffuser with a negligible velocity ($V_2 \cong 0$).

Diffuser:

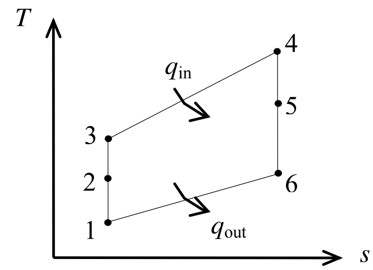
$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \Delta\dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$h_1 + V_1^2/2 = h_2 + V_2^2/2$$

$$0 = h_2 - h_1 + \frac{V_2^{2\,\nearrow 0} - V_1^2}{2}$$

$$0 = c_p(T_2 - T_1) - V_1^2/2$$



$$T_2 = T_1 + \frac{V_1^2}{2c_p} = 470 + \frac{(900\ \text{ft/s})^2}{(2)(0.24\ \text{Btu/lbm}\cdot\text{R})}\left(\frac{1\ \text{Btu/lbm}}{25{,}037\ \text{ft}^2/\text{s}^2}\right) = 537.4\ \text{R}$$

$$P_2 = P_1\left(\frac{T_2}{T_1}\right)^{k/(k-1)} = (7\ \text{psia})\left(\frac{537.3\ \text{R}}{470\ \text{R}}\right)^{1.4/0.4} = 11.19\ \text{psia}$$

Compressor:

$$P_3 = P_4 = (r_p)(P_2) = (13)(11.19\ \text{psia}) = 145.5\ \text{psia}$$

$$T_3 = T_2\left(\frac{P_3}{P_2}\right)^{(k-1)/k} = (537.4\ \text{R})(13)^{0.4/1.4} = 1118.3\ \text{R}$$

Turbine:

$$w_{\text{comp,in}} = w_{\text{turb,out}} \longrightarrow h_3 - h_2 = h_4 - h_5 \longrightarrow c_p(T_3 - T_2) = c_p(T_4 - T_5)$$

or

$$T_5 = T_4 - T_3 + T_2 = 2400 - 1118.3 + 537.4 = 1819.1\ \text{R}$$

$$P_5 = P_4\left(\frac{T_5}{T_4}\right)^{k/(k-1)} = (145.5\ \text{psia})\left(\frac{1819.1\ \text{R}}{2400\ \text{R}}\right)^{1.4/0.4} = \mathbf{55.2\ psia}$$

(*b*) Nozzle:

$$T_6 = T_5\left(\frac{P_6}{P_5}\right)^{(k-1)/k} = (1819.1\ \text{R})\left(\frac{7\ \text{psia}}{55.2\ \text{psia}}\right)^{0.4/1.4} = 1008.6\ \text{R}$$

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \Delta\dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$h_5 + V_5^2/2 = h_6 + V_6^2/2$$

$$0 = h_6 - h_5 + \frac{V_6^2 - V_5^{2\nearrow 0}}{2}$$

$$0 = c_p(T_6 - T_5) + V_6^2/2$$

or

$$V_6 = \sqrt{(2)(0.240 \text{ Btu/lbm}\cdot\text{R})(1819.1 - 1008.6)\text{R}\left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/lbm}}\right)} = \mathbf{3121\ ft/s}$$

(*c*) The propulsive efficiency is the ratio of the propulsive work to the heat input,

$$w_p = (V_{\text{exit}} - V_{\text{inlet}})V_{\text{aircraft}}$$

$$= [(3121 - 900) \text{ ft/s}](900 \text{ ft/s})\left(\frac{1 \text{ Btu/lbm}}{25{,}037 \text{ ft}^2/\text{s}^2}\right) = 79.8 \text{ Btu/lbm}$$

$$q_{\text{in}} = h_4 - h_3 = c_p(T_4 - T_3) = (0.24 \text{ Btu/lbm}\cdot\text{R})(2400 - 1118.3)\text{R} = 307.6 \text{ Btu/lbm}$$

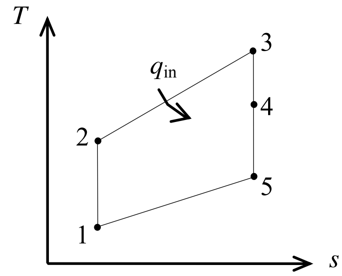
$$\eta_p = \frac{w_p}{q_{\text{in}}} = \frac{79.8 \text{ Btu/lbm}}{307.6 \text{ Btu/lbm}} = \mathbf{25.9\%}$$

9-135 A turbojet aircraft that has a pressure rate of 9 is stationary on the ground. The force that must be applied on the brakes to hold the plane stationary is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The air standard assumptions are applicable. **3** Air is an ideal gas with variable specific heats. **4** Kinetic and potential energies are negligible, except at the nozzle exit.

Properties The properties of air are given in Table A-17.

Analysis (*a*) Using variable specific heats for air,



Compressor:

$$T_1 = 290 \text{ K} \longrightarrow h_1 = 290.16 \text{ kJ/kg}$$
$$P_{r_1} = 1.2311$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (9)(1.2311) = 11.08 \longrightarrow h_2 = 544.07 \text{ kJ/kg}$$

$$\dot{Q}_{in} = \dot{m}_{fuel} \times \text{HV} = (0.5 \text{ kg/s})(42{,}700 \text{ kJ/kg}) = 21{,}350 \text{ kJ/s}$$

$$q_{in} = \frac{\dot{Q}_{in}}{\dot{m}} = \frac{21{,}350 \text{ kJ/s}}{20 \text{ kg/s}} = 1067.5 \text{ kJ/kg}$$

$$q_{in} = h_3 - h_2 \longrightarrow h_3 = h_2 + q_{in} = 544.07 + 1067.5 = 1611.6 \text{ kJ/kg} \longrightarrow P_{r_3} = 568.5$$

Turbine:

$$w_{comp,in} = w_{turb,out} \longrightarrow h_2 - h_1 = h_3 - h_4$$

or

$$h_4 = h_3 - h_2 + h_1 = 1611.6 - 544.07 + 290.16 = 1357.7 \text{ kJ/kg}$$

Nozzle:

$$P_{r_5} = P_{r_3}\left(\frac{P_5}{P_3}\right) = (568.5)\left(\frac{1}{9}\right) = 63.17 \longrightarrow h_5 = 888.56 \text{ kJ/kg}$$

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{system}^{\nearrow 0 \text{ (steady)}}$$
$$\dot{E}_{in} = \dot{E}_{out}$$
$$h_4 + V_4^2/2 = h_5 + V_5^2/2$$
$$0 = h_5 - h_4 + \frac{V_5^2 - V_4^{2\nearrow 0}}{2}$$

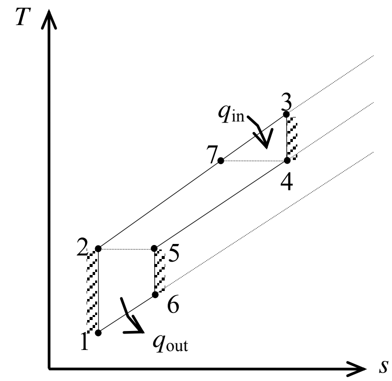
or

$$V_5 = \sqrt{2(h_4 - h_5)} = \sqrt{(2)(1357.7 - 888.56)\text{kJ/kg}\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 968.6 \text{ m/s}$$

$$\text{Brake force} = \text{Thrust} = \dot{m}(V_{exit} - V_{inlet}) = (20 \text{ kg/s})(968.6 - 0)\text{m/s}\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{19{,}370 N}$$

9-167 An ideal gas-turbine cycle with one stage of compression and two stages of expansion and regeneration is considered. The thermal efficiency of the cycle as a function of the compressor pressure ratio and the high-pressure turbine to compressor inlet temperature ratio is to be determined, and to be compared with the efficiency of the standard regenerative cycle.

Analysis The *T-s* diagram of the cycle is as shown in the figure. If the overall pressure ratio of the cycle is r_p, which is the pressure ratio across the compressor, then the pressure ratio across each turbine stage in the ideal case becomes $\sqrt{r_p}$. Using the isentropic relations, the temperatures at the compressor and turbine exit can be expressed as



$$T_5 = T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = T_1\left(r_p\right)^{(k-1)/k}$$

$$T_7 = T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = T_3\left(\frac{1}{\sqrt{r_p}}\right)^{(k-1)/k} = T_3 r_p^{(1-k)/2k}$$

$$T_6 = T_5\left(\frac{P_6}{P_5}\right)^{(k-1)/k} = T_5\left(\frac{1}{\sqrt{r_p}}\right)^{(k-1)/k} = T_2 r_p^{(1-k)/2k} = T_1 r_p^{(k-1)/k} r_p^{(1-k)/2k} = T_1 r_p^{(k-1)/2k}$$

Then,

$$q_{\text{in}} = h_3 - h_7 = c_p\left(T_3 - T_7\right) = c_p T_3\left(1 - r_p^{(1-k)/2k}\right)$$

$$q_{\text{out}} = h_6 - h_1 = c_p\left(T_6 - T_1\right) = c_p T_1\left(r_p^{(k-1)/2k} - 1\right)$$

and thus

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{c_p T_1\left(r_p^{(k-1)/2k} - 1\right)}{c_p T_3\left(1 - r_p^{(1-k)/2k}\right)}$$

which simplifies to

$$\eta_{\text{th}} = 1 - \frac{T_1}{T_3} r_p^{(k-1)/2k}$$

The thermal efficiency of the single stage ideal regenerative cycle is given as

$$\eta_{\text{th}} = 1 - \frac{T_1}{T_3} r_p^{(k-1)/k}$$

Therefore, the regenerative cycle with two stages of expansion has a higher thermal efficiency than the standard regenerative cycle with a single stage of expansion for any given value of the pressure ratio r_p.