Name: ________________________________________.

MET 350 Thermal Applications
Dr. Orlando Ayala
Spring 2025
Test 1

Take home – Due Sunday February 23rd, 2025, before midnight.

# READ FIRST

1. RELAX!!!! DO NOT OVERTHINK THE PROBLEMS!!!! There is nothing hidden. The test was designed for you to pass and get the maximum number of points, while learning at the same time. HINT: THINK BEFORE TRYING TO USE/FIND EQUATIONS (OR EVEN FIND SIMILAR PROBLEMS)

2. The total points on this test are one hundred (100). Ten (10) points are from your HW assignments, and ten (10) other points are based on the basis of technical writing. The other eighty (80) points will come from the problem solutions. For the technical writing I will follow the attached rubric.

3. There are 2 problems. Each problem will be worth (80/2) points.

4. What you turn in should be only your own work. You cannot discuss the exam with anyone, except me. Call me, skype me, text me, email me, come to my office, if you have any question.

5. I do not read minds. You should be explicit and organized in your answers. Use drawings/figures. If you make a mistake, do not erase it. Rather use that opportunity to explain why you think it is a mistake and show the way to correct the problem.

6. You have to turn in your test ON TIME and ONLY through CANVAS. You must submit only one file and it has to be a pdf file. For the ePortfolio (which is optional) you are supposed to upload this artifact to your Google drive. I will provide more instructions later.

7. Do not start at the last minute so you can handle anything that could happen. Late tests will not be accepted. Test submitted through email will not be accepted either.

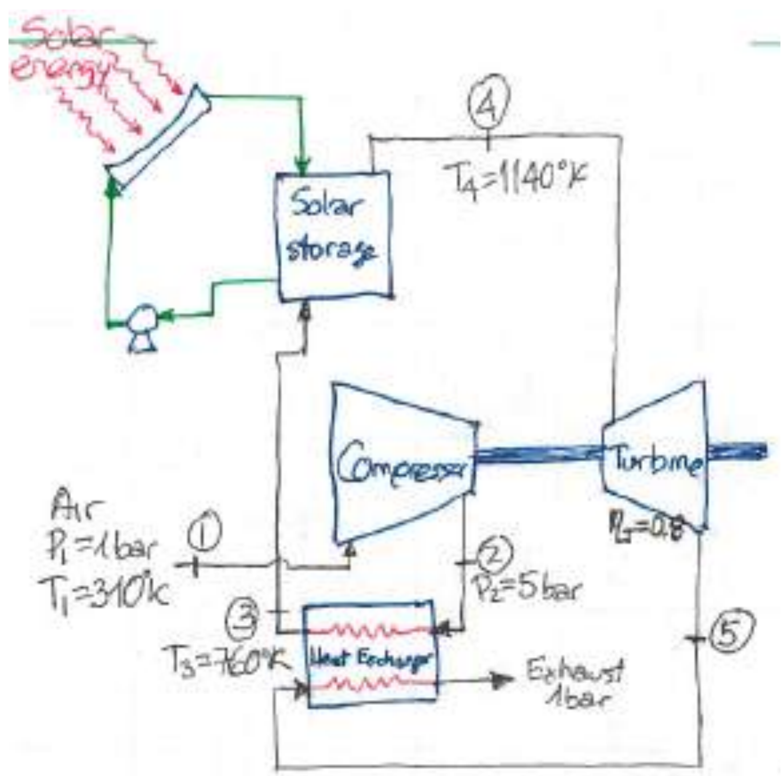
8. Cheating is completely wrong. The ODU Student Honor Pledge reads: "I pledge to support the honor system of Old Dominion University. I will refrain from any form of academic dishonesty or deception, such as cheating or plagiarism." By attending Old Dominion University you have accepted the responsibility to abide by this code. This is an institutional policy approved by the Board of Visitors. It is important to remind you the following part of the Honor Code:

    IX. PROHIBITED CONDUCT
    A. Academic Integrity violations, including:
    1. *Cheating*: Using unauthorized assistance, materials, study aids, or other information in any academic exercise (Examples of cheating include, but are not limited to, the following: using unapproved resources or assistance to complete an assignment, paper, project, quiz or exam; collaborating in violation of a faculty member's instructions; and submitting the same, or substantially the same, paper to more than one course for academic credit without first obtaining the approval of faculty).

**With that said, you are NOT authorized to use any online source of any type, unless is ODU related.**

1. Solar energy has been proposed as the source of heat addition in an ideal cold air-standard Brayton cycle, as shown in the figure. With the operating data provided in the figure, determine:

   (a) the thermal efficiency and the heat exchanger effectiveness.
   (b) the air mass flow rate, in kg/s, for a net power output of 500 kW.
   (c) what would be the thermal efficiency if the heat exchanger were operating at 100% effectiveness?




2. A turbojet aircraft flies with a velocity of 900 km/h at an altitude where the air temperature and pressure are -35 °C and 40 kPa. The combustion gases enter the turbine at 950 °C. The turbine produces 500 kJ/kg of work, all of which is used to drive the compressor. Assuming an isentropic efficiency of 80% for the compressor, an isentropic efficiency of 90% for the turbine, and using variable specific heats, determine:

   (a) the pressure of combustion gases at the turbine exit,
   (b) the velocity of the gases at the nozzle exit, and
   (c) the thrust for this engine if the diffuser inlet diameter is 1.6 m.

# Problem solution rubric

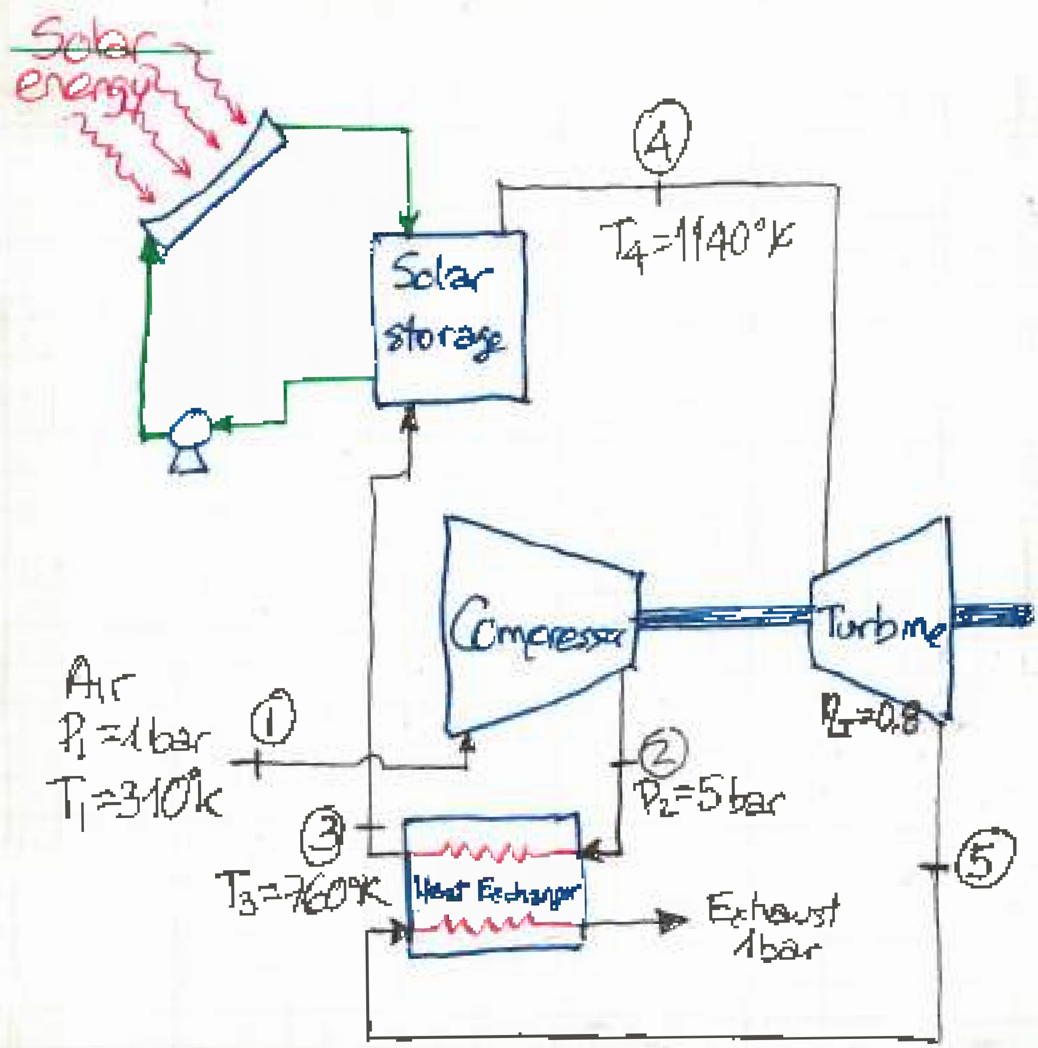
| | Exceeds Standard | Meets Standard | Approaches Standard | Needs Attention |
|---|---|---|---|---|
| | 4 | 3 | 2 | 1 |
| | **10 points** | **7 points** | **4 points** | **0 points** |
| **1. Purpose** 5% | The purpose of the section to be answered is clearly identified and stated. | The purpose of the section to be answered is identified, but is stated in a somewhat unclear manner. | The purpose of the section to be answered is partially identified, and is stated in a somewhat unclear manner. | The purpose of the section to be answered is erroneous or irrelevant. |
| **2. Drawings & Diagrams** 10% | Clear and accurate diagrams are included and make the section easier to understand. Diagrams are labeled neatly and accurately. | Diagrams are included and are labeled neatly and accurately. | Diagrams are included and are labeled. | Needed diagrams are missing OR are missing important labels. |
| **3. Sources** 5% | Several reputable background sources were used and cited correctly. | A few reputable background sources are used and cited correctly. | A few background sources are used and cited correctly, but some are not reputable sources. | Background sources are cited incorrectly. |
| **4. Design considerations (assumptions, safety, cost, etc)** 10% | Design is carried out with applicable assumptions and full attention to safety and cost, etc. | Design is generally carried out with assumptions and attention to safety, cost, etc. | Design is carried out with some assumptions and some attention to safety, cost, etc. | Assumptions, safety and cost were ignored in the design. |
| **5. Data and variables** 5% | All data and variables are clearly described with all relevant details. | All data and variables are clearly described with most relevant details. | Most data and variables are clearly described with most relevant details. | Data and variables are not described OR the majority lack sufficient detail. |
| **6. Procedure** 25% | Procedure is described in clear steps. The step description is in a complete and easy to understand short paragraph. | Procedure is described in clear steps but the step description is not in a complete short paragraph. | Procedure is described in clear steps. The step description is in a complete short paragraph but it is difficult to understand. | Procedure is not described in clear steps at all. |
| **7. Calculations** 20% | All calculations are shown and the results are correct and labeled appropriately. The units of all values are shown. | Some calculations are shown and the results are correct and labeled appropriately. | Some calculations are shown and the results labeled appropriately. | No calculations are shown OR results are inaccurate or mislabeled. |
| **8. Summary** 5% | Summary describes the design, the relevant information and some future implications. | Summary describes the design and some relevant information. | Summary describes the design. | No summary is written. |
| **9. Materials** 5% | All materials used in the design are clearly and accurately described. | Almost all materials used in the design are clearly and accurately described. | Most of the materials used in the design are clearly and accurately described. | Many materials are described inaccurately OR are not described at all. |
| **10. Analysis** 10% | The design is discussed and analyzed. Argumentative predictions are made about what might happen in case of change in the operation and how the design could be change. | The design is discussed and analyzed. Argumentative predictions are made about what might happen in case of change in the operation. | The design is discussed and analyzed. No argumentative predictions are made about what might happen in case of change in the operation and how the design could be change. | The design is not discussed and analyzed. |

A

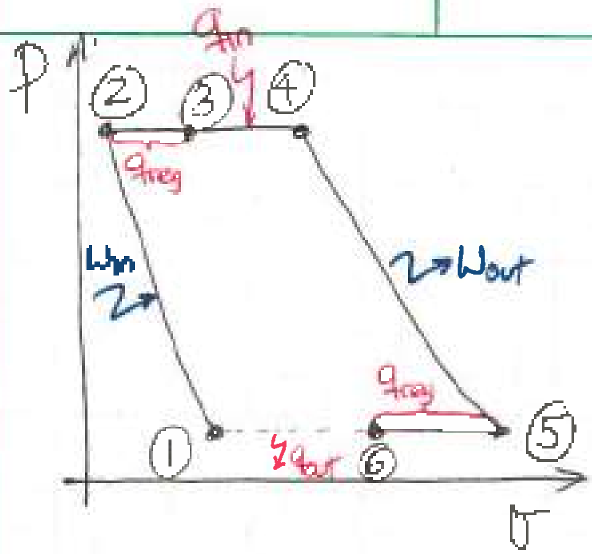
## 2 PURPOSE

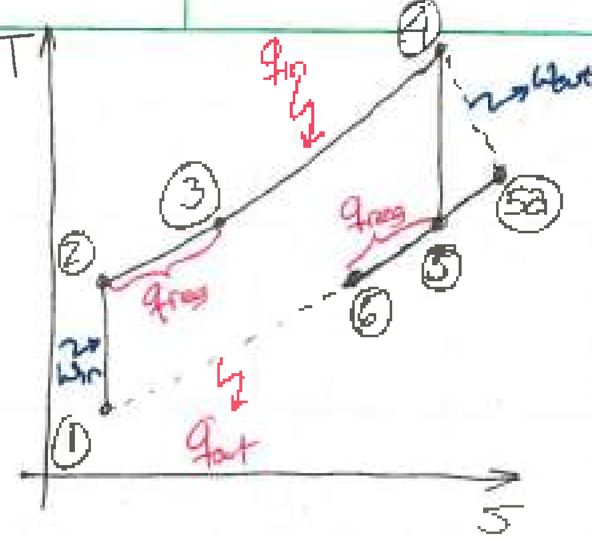
Find the thermal efficiency and the heat exchanger effectiveness for the cold-air standard Brayton cycle shown in the figure below. Also find the air mass flow rate to produce 500 kW.
Finally, find the thermal efficiency of that cycle assuming the heat exchanger was designed in such a way that its effectiveness is 100%.

## DRAWINGS & DIAGRAMS



B




## SOURCES

Cengel & Boles. Thermodynamics-An Engineering Approach, 8th edition. McGraw Hill. 2015.

## DESIGN CONSIDERATIONS

I assume the following:

1) Air behaves as ideal gas. $R=0.287 \frac{kJ}{kg \cdot K}$
2) $C_p$ & $C_v$ are constant & evaluated at 25°C
3) Isentropic efficiency of turbine is 0.8
4) Isentropic efficiency of compressor is 1.0
5) No heat losses in connecting pipes, neither fluid flow friction losses
6) $C_p=1.004926 \frac{kJ}{kg \cdot K}$, $C_v=0.717926 \frac{kJ}{kg \cdot K}$, $k=1.399763$ (TABLE A-2)

## DATA & VARIABLES

- $\eta_T=0.8$
- $\dot{W}_{net}=500$ kW
- State data given in the figure

MATERIALS: Air as ideal gas

C

# PROCEDURE & CALCULATIONS

I will start obtaining all the states in the cycle, then with all states, I will be able to compute the required work and heats to be able to get the thermal efficiency ($\eta_{th}$), HX effectiveness ($\varepsilon$), and mass flow rate ($\dot{m}$).

## STATES

(1) —Isent→ (2) —P const, $q_{reg}$→ (3) —P const→ (4) —$\eta_T = 0.8$→ (5) (5a) —P const, $q_{reg}$→ (6)

**State 1**

$T_1 = 310K$

$P_1 = 101.3kPa$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$T_2 = T_1(5)^{\frac{k-1}{k}} \rightarrow T_2 = 490.89K$

**State 2**

$P_2 = 506.5kPa$

**State 3**

$T_3 = 760K$

$P_3 = P_2$

$P_3 = 506.5kPa$

**State 4**

$T_4 = 1140K$

$P_4 = P_3$

$P_4 = 506.5kPa$

**State 5**

$P_5 = P_6$

$P_5 = 101.3kPa$

$$\frac{T_5}{T_4} = \left(\frac{P_5}{P_4}\right)^{\frac{k-1}{k}}$$

$T_5 = 719.92K$

Actual condition:

$P_{5a} = 101.3kPa$

For $T_{5a}$:

$$\eta_T = \frac{T_4 - T_{5a}}{T_4 - T_5}$$

$T_{5a} = 803.94K$

**State 6**

$P_6 = 101.3kPa$

Applying 1st law to the HX:

$h_{5a} - h_6 = h_3 - h_2$

as cold-air standard assump

$T_{5a} - T_6 = T_3 - T_2$

Thus,

$T_6 = 550.44K$

D

(a) $\eta_{th} = \frac{W_{net}}{q_{in}}$

$$W_{net} = W_{out_{4-5a}} - W_{in_{1-2}}$$
$$= \eta_T W_{out_{4-5}} - W_{in_{1-2}}$$

$$W_{out_{4-5}} = h_4 - h_5 \text{ (1st law)}$$
$$= C_p(T_4 - T_5)$$

$$W_{out_{4-5}} = 442.15 \frac{kJ}{kg}$$

$$W_{in_{1-2}} = h_2 - h_1 \text{ (1st law)}$$
$$= C_p(T_2 - T_1)$$

$$W_{in_{1-2}} = 197.47 \frac{kJ}{kg}$$

$$W_{net} = 156.25 \frac{kJ}{kg}$$

$$q_{in} = h_4 - h_3 \text{ (1st law)}$$
$$= C_p(T_4 - T_3)$$

$$q_{in} = 381.87 \text{ kJ/kg}$$

~~$\eta_{th} = 0.409$~~

However, if we consider that the heat $q_{in}$ is provided by the SUN, this efficiency is higher than that (in terms of what we invest).

E

As far as the effectiveness of the heat exchanger,

$$\epsilon = \frac{h_3 - h_2}{h_{5a} - h_2} \quad \text{or} \quad \epsilon = \frac{T_3 - T_2}{T_{5a} - T_2}$$

due to Cp & Cv constants

$$\epsilon = \frac{760\,K - 490.89\,K}{803.94\,K - 490.89\,K}$$

$$\boxed{\epsilon = 0.86}$$

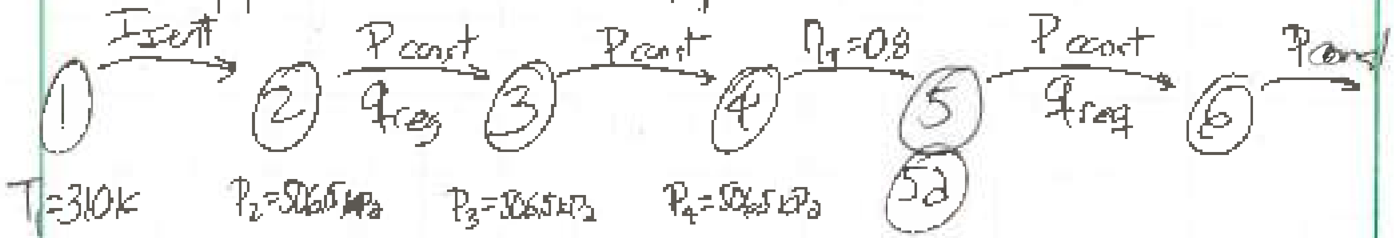
(b) $\dot{W}_{net} = \dot{m}\, w_{net}$

$$\dot{m} = \frac{\dot{W}_{net}}{w_{net}}$$

$$\boxed{\dot{m} = 3.2\ \text{kg/s}}$$

Now for part (c) of the problem, I will need to revise ALL states since the heat exchanger is operating at anothe effectiveness. States 2, 5, and 5a should still be the same but states 3 and 6 will be different.

If the HX is 100% effective, $T_6$ will reach $T_2$ value and $T_3$ can be obtained from the effectiveness equation. So:




$T_1 = 310K$, $P_1 = 101.3 kPa$

$P_2 = 506.5 kPa$, $T_2 = 490.89K$

$P_3 = 506.5 kPa$

$P_4 = 506.5 kPa$, $T_4 = 1140K$

$P_5 = 101.3 kPa$, $T_5 = 719.92K$, $T_{5a} = 803.94K$

$P_6 = 101.3 kPa$

$T_6 = T_2$ as $\epsilon = 1.0$

$\boxed{T_6 = 490.89K}$

$$\epsilon = \frac{T_3 - T_2}{T_{5a} - T_2}$$

$$T_3 = (T_{5a} - T_2) + T_2$$

$$T_3 = T_{5a}$$

$$\boxed{T_3 = 803.94K}$$

Now, let us get $\eta_{th}$:

$$\eta_{th} = \frac{w_{net}}{q_{in}}$$

$$w_{net} = w_{out,\,4-5a} - w_{in,\,1-2}$$

$$= \underbrace{c_p(T_4 - T_{5a})}_{353.72 \frac{kJ}{kg}} - \underbrace{c_p(T_2 - T_1)}_{197.47 \frac{kJ}{kg}}$$

353.72 kJ/kg → from before; 197.47 kJ/kg → from before

$$q_{in} = c_p(T_4 - T_3) = 1.004926 \frac{kJ}{kg \cdot K}(1140K - 803.94K)$$
$$= 337.715 \frac{kJ}{kg}$$

G

So

$$\eta_{th} = \frac{445.15 \frac{kJ}{kg} - 197.47 \frac{kJ}{kg}}{337.715 \frac{kJ}{kg}}$$

$$\boxed{\eta_{th} = 0.463}$$

## SUMMARY

For the Brayton cycle shown in the figure, the thermal efficiency is 0.409 and the heat exchanger effectiveness is 0.86. By increasing the HX effectiveness to 100% the cycle thermal efficiency increasse to 0.463. this is a 13.2% of increase, which could represent a lot of savings in a long run.

## ANALYSIS

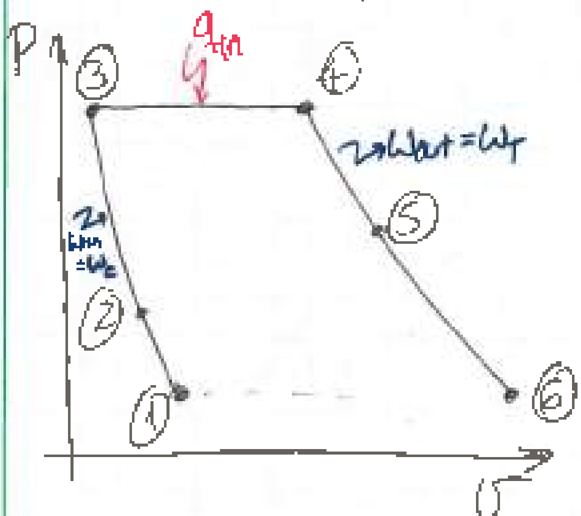
Improving the operation of any component in a cycle will increase the thermal efficiency of the cycle. If we had a 100% turbine efficiency, the thermal efficiency of the cycle would be 0.64 and with a HX effectiveness of 100%, the thermal efficiency would have been 0.73.

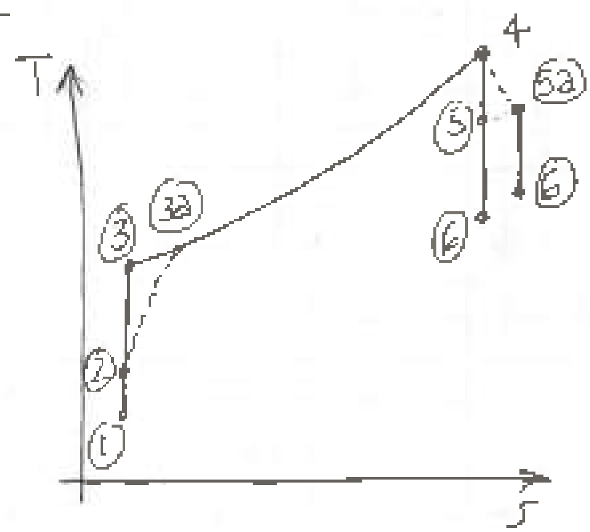
H

## 3) PURPOSE

Determine the pressure combustion gases at the exit of the turbine, the velocity of the gases at nozzle exit, and the thrust of a turbojet with inlet diameter of 1.6 m when flying at 900 Km/h with air conditions of −35°C and 40 kPa.

## DRAWINGS & DIAGRAMS




## SOURCES

Cengel & Boles. Thermodynamics – An Engineering Approach 8th edition McGraw Hill, 2015

## DESIGN CONSIDERATIONS

1) Air behaves as ideal gas ($R = 0.287 \frac{kJ}{kg \cdot K}$
2) Isentropic efficiencies of compressor & turbine are 0.8 & 0.9 respectively
3) Constant pressure combustion
4) Cp & Cv are variable
5) Work of turbine is all use to move compressor

## DATA & VARIABLES

* A turbojet with $V = 900\ km/h = 250\ m/s$
* $T_{amb} = -35°C = 238.15\ K$ ; $P_{amb} = 40\ kPa$
* $T_{turb\ inlet} = 950°C = 1223.15\ K$
* $\omega_T = 500\ \frac{kJ}{kg} = \omega_c$
* $\eta_c = 0.8$, $\eta_T = 0.9$
* $C_p$ & $C_v$ variable (Use table 17)

(a) $P_{out\ turb} = ?$

(b) $V_{nozzle\ exit} = ?$

(c) $F = ?$ $D_{diff\ inlet} = 1.6\ m$

## MATERIALS

Air as ideal gas

## PROCEDURE & CALCULATIONS

Let us start computing all the states in the turbojet

J

① —Isent, Diffuser→ ② —Isent, $\eta_c$→ ③ (3s) —P const→ ④ —Isent, $\eta_T$→ ⑤ (5s) —Isent, Nozzle→ ⑥

**State 1**

$P_1 = 40\,kPa$

$T_1 = 238.15\,K$

$V_1 = 250\,m/s$

Table 17

$h_1 = 238.17\,\frac{kJ}{kg}$

$P_{r_1} = 0.619257$

1st law

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$ ($V_2 \approx 0$)

$h_2 = 269.42\,\frac{kJ}{kg}$

Table 17 → $T_2 = 269.31\,K$

$P_{r_2} = 0.95084$

**State 2**

$V_2 \approx 0\,m/s$

$$\frac{P_2}{P_1} = \frac{P_{r_2}}{P_{r_1}}$$

$$P_2 = P_1 \frac{P_{r_2}}{P_{r_1}} \rightarrow P_2 = 61.42\,kPa$$

**State 3**

Now, $w_{c,real} = 500\,\frac{kJ}{kg}$

Using 1st law:

$$w_{c,real} = \frac{h_3 - h_2}{\eta_c}$$

$$h_3 = h_2 + w_{c,real}\,\eta_c$$

$h_3 = 669.42\,\frac{kJ}{kg}$

Table 17

$T_3 = 659.01\,K$

$P_{r_3} = 23.00$

$$\frac{P_3}{P_2} = \frac{P_{r_3}}{P_{r_2}}$$

$P_3 = 1485.697\,kPa$

**State 4**

$T_4 = 1223.15\,K$

Table 17

$h_4 = 1305.03\,\frac{kJ}{kg}$

$P_{r_4} = 257.47\,kPa$

$P_4 = P_3$

$P_4 = 1485.697\,kPa$

**State 5**

Now, $w_{T,real} = 500\,\frac{kJ}{kg}$

Using 1st law:

$$w_{T,real} = \eta_T (h_4 - h_5)$$

$$h_5 = h_4 - \frac{w_{T,real}}{\eta_T}$$

$h_5 = 749.47\,\frac{kJ}{kg}$

Table 17

$T_5 = 733.56\,K$

$P_{r_5} = 34.35$

$$\frac{P_5}{P_4} = \frac{P_{r_5}}{P_{r_4}}$$

$P_5 = 198.21\,kPa$

**State 6**

$P_6 = P_1$

$P_6 = 40\,kPa$

$$\frac{P_6}{P_5} = \frac{P_{r_6}}{P_{r_5}}$$

$P_{r_6} = 6.932$

Table 17

$T_6 = 473.61\,K$

$h_6 = 475.94\,\frac{kJ}{kg}$

The actual conditions at compressor and turbine exit are:

$$\eta_c = \frac{h_3 - h_2}{h_{3a} - h_2} \qquad \eta_T = \frac{h_4 - h_5}{h_4 - h_{5a}}$$

$h_{3a} = 769.42\,kJ/kg$ $\qquad$ $h_{5a} = 800.03\,kJ/kg$

Table 17: $T_{3a} = 751.96\,K$ $\qquad$ Table 17: $T_{5a} = 780.0\,K$

K

(a) $P_{exit\ turb} = P_5 = 198.12\,kPa$

(b) $V_{nozzle\ exit} = ?$ or $V_6 = ?$

1st law in the nozzle:

$$h_{5a} + \frac{V_{5a}^2}{2} = h_6 + \frac{V_6^2}{2}$$

$\approx 0$

$$V_6 = \sqrt{2(h_{5a} - h_6)}$$

$$V_6 = 805.10\,m/s$$

(c) $F = \dot{m}(V_{exit} - V_{inlet})$

$$\dot{m} = \rho_1 V_1 A_1 = \frac{V_1}{v_1}\frac{\pi}{4}D^2$$

$$v_1 = \frac{RT_1}{P_1}$$

$$F = \frac{V_1 P_1}{RT_1}\frac{\pi}{4}D^2(V_6 - V_1)$$

$$F = 163.293\,kN$$

L

## SUMMARY

The pressure at the turbine exit is 198.12 kPa. The velocity at the nozzle exit is 805.10 m/s, this represents an excess of 555.1 m/s (= 805.10 m/s − 250 m/s) that gives a kinetic energy lost to the atmosphere.
Finally, the thrust is 163.29 kN

## ANALYSIS

The propulsive power is

$$\dot{W}_p = F \times V_{aircraft} = 163.293\ kN \times 250\ m/s$$

$$\dot{W}_p = 40.823\ MW$$

and the heat provided is

$$\dot{Q}_{in} = \dot{m}(h_4 - h_{3a}) = \frac{V_1 P_1}{RT_1}\frac{\pi}{4}D^2(h_4 - h_{3a})$$

$$\dot{Q}_{in} = 157.559\ MW$$

Therefore the amount of energy from the combustion to propel the aircraft is ~25.9%. The rest goes to the atmosphere in the form of heat & excess kinetic energy

$$KE = \dot{m}\frac{V_{exc}^2}{2} = 294.169\ \frac{kg}{s}\frac{(555.1\ m/s)^2}{2} = 45.322\ MW \longrightarrow \sim 28.76\%$$

$$\dot{Q}_{out} = \dot{m}(h_6 - h_1) = 294.169\ \frac{kg}{s}\left(475.94\ \frac{kJ}{kg} - 238.17\ \frac{kJ}{kg}\right) = 69.944\ MW \longrightarrow \sim 44.39\%$$